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New Structures of Soft Permutation in Commutative Q-Algebras

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ABSTRACT: This paper contributes to the theory of soft sets. It investigates soft permutation commutative Q-algebras and their applications. A new method of combining permutation sets with soft sets is utilized to investigate new classes of algebra, including soft permutation commutative Q-algebra, soft permutation commutative G-part, and soft permutation commutative p-semisimple. This study elucidates an approach to find a relationship between the chemical structure of the atoms for some elements, like silver atom, sodium atom, and chlorine atom and some of our ideas presented here. Furthermore, we demonstrate that if (N, M) is a soft permutation commutative Q-algebra of $(X,\times,T)$ and the associative law is held for any $\lambda_i^\beta, \lambda_j^\beta, \lambda_k^\beta \in N(\lambda_m^\beta)$, then $(N(\lambda_m^\beta),\times)$ is a group, $\forall \lambda_m^\beta \in M$. Also, if (N, M) is a soft permutation commutative G-part of X. Then the left cancellation law is held for any equation $\lambda_i^\beta \times N(\lambda_j^\beta) = \lambda_i^\beta \times N(\lambda_k^\beta)$, where $\lambda_j^\beta, \lambda_k^\beta \in M$. Next, we prove that if $T \times N(\lambda_i^\beta) \in G(N, M)$. Then $\lambda_i^\beta \in G(N, M)$, Also, we show that $\lambda_j^\beta \in B(N, M)$ if and only if $\left(\lambda_i^\beta \times N(\lambda_j^\beta)\right) \times \lambda_i^\beta = T$, where $B(N, M) = \left\{\lambda_i^\beta \in M \mid T \times N(\lambda_i^\beta) = T\right\}$. Subsequently, the cases of orders of $(X,\times,T)$ are discussed. Moreover, precise outcomes relating to our new concepts as well.

Keywords: Symmetric groups, permutation sets, soft sets, Q-algebra, G-part, p-semisimple Q-algebra.

1. INTRODUCTION

Molodtsov's contribution [1] provides the essential components of soft sets theory, crucial for addressing challenges in computer science, economics, engineering, medicine, and other domains. The introduction of BCK-algebras [2] and BCI-algebras [3] offers further tools for practical problem-solving, with the well-known relationship that BCK-algebra is a proper subclass of BCI-algebra. Neggers and Kim [4] extend this framework by introducing d-algebras, a valuable generalization of BCK-algebras. Their subsequent study explores connections between d-algebras, BCK-algebras, and various interactions with oriented digraphs. In 2018, Khalil and Hassan [5] presented the concept of ρ-algebra, delving into intriguing properties of this algebraic structure. Explorations in diverse fields encompass symmetric and alternating groups, along with their permutations ([6]-[11]). Recent years have seen investigations into non-classical sets, including permutation sets ([12]), fuzzy sets ([13]-[16]), soft sets ([17],[18]), nano sets [19], and neutrosophic sets ([20]-[23]), addressing and examining various notions and features within the realm of mathematics.

Let β be a permutation in symmetric group S_n, then β can be written as product of disjoint cycles in an essentially unique way. As the form $\beta = (r_1^1, r_2^1, \ldots, r_{\alpha_1}^1)(r_1^2, r_2^2, \ldots, r_{\alpha_2}^2) \ldots \left(r_1^{c(\beta)}, r_2^{c(\beta)}, \ldots, r_{\alpha_{c(\beta)}}^{c(\beta)}\right)$ where for each $i \neq j$, thus $\{r_1^i, r_2^i, \ldots, r_{\alpha_i}^i\} \cap \{r_1^j, r_2^j, \ldots, r_{\alpha_j}^j\} = \emptyset$ [23]. Therefore $\beta = \lambda_1 \lambda_2 \ldots \lambda_{c(\beta)}$, where λ_I disjoint cycles of length $|\lambda_i| = \alpha_I$ and $c(\beta)$ is the number of disjoint cycle factors including the 1-cycle of β. Also, $\alpha = \alpha(\beta) = \left(\alpha_1(\beta), \alpha_2(\beta), \ldots, \alpha_{c(\beta)}(\beta)\right) = (\alpha_1, \alpha_2, \ldots, \alpha_{c(\beta)})$ is called the cycle type of β [24]. In 2022, some concepts on permutations are introduced and studied in some classes of algebras like BE-algebra [25], B-algebra [26], BH-algebra [27].

This work examines innovative algebraic topics, including the soft permutation commutative Q-algebra, soft permutation commutative G-part, and soft permutation commutative p-semisimple. The paper not only explores and discusses these concepts but also introduces a method for establishing a connection between the chemical structure of the silver atom and some of the algebraic principles expounded within. In addition, if (N,M) is a commutative soft permutation G-part of X. The left cancellation law is then applicable to any equation $\lambda_i^\beta \times \mathrm{N}(\lambda_j^\beta) = \lambda_i^\beta \times \mathrm{N}(\lambda_k^\beta)$, where $\lambda_j^\beta, \lambda_k^\beta \in \mathrm{M}$. Then we show that if $\mathrm{T} \times \mathrm{N}(\lambda_i^\beta) \in \mathrm{G}(\mathrm{N},\mathrm{M})$. Then $\lambda_i^\beta \in \mathrm{G}(\mathrm{N},\mathrm{M})$, Also, we prove that $\lambda_j^\beta \in \mathrm{B}(\mathrm{N},\mathrm{M})$ if and only if $\left(\lambda_i^\beta \times \mathrm{N}(\lambda_j^\beta)\right) \times \lambda_i^\beta = \mathrm{T}$, where $\mathrm{B}(\mathrm{N},\mathrm{M}) = \left\{\lambda_i^\beta \in \mathrm{M} \mid \mathrm{T} \times \mathrm{N}(\lambda_i^\beta) = \mathrm{T}\right\}$. The cases of orders of (X,,T) are then addressed. Furthermore, specific results relating to our new notions established and investigated.

2. PRELIMINARY

The definitions of Q-algebra and permutation sets are covered in this section.

Definition 1:[10]: For any permutation $\beta = \prod_{i=1}^{c(\beta)} \lambda_i$ in a symmetric group S_n, where $\{\lambda_i\}_{i=1}^{c(\beta)}$ is a composite of pairwise disjoint cycles $\{\lambda_i\}_{i=1}^{c(\beta)}$ where $\lambda_i = (t_1^i, t_2^i, \dots, t_{\alpha_i}^i), 1 \le i \le c(\beta)$, for some $1 \le \alpha_i,\ c(\beta) \le n$, where $c(\beta)$ is the number of disjoint cycle factors including the 1-cycle of β. If $\lambda = (t_1, t_2, \dots, t_k)$ is $k-$cycle in S_n, we define $\beta-$set as $\lambda^\beta = \{t_1, t_2, \dots, t_k\}$, It is called $\beta-$set of cycle λ. So the $\beta-$sets of $\{\lambda_i\}_{i=1}^{c(\beta)}$ are defined by $\{\lambda_i^\beta = \{t_1^i, t_2^i, \dots, t_{\alpha_i}^i\} | 1 \le i \le c(\beta)\}$.

Definition 2: [10]: Suppose that λ_i^β and λ_j^β are $\beta-$sets in X, where $|\lambda_i| = \sigma$ and $|\lambda_j| = \upsilon$, where $|\lambda_i|$ and $|\lambda_j|$ are the lengths of λ_i^β and λ_j^β, respectively. Then $\lambda_i^\beta = \lambda_j^\beta$, if $\sum_{k=1}^{\sigma} t_k^i = \sum_{k=1}^{\upsilon} t_k^j$ and there exists $1 \le d \le \sigma$, for each $1 \le r \le \upsilon$ such that $t_d^i = t_r^j$. Also, we call λ_i^β and λ_j^β are disjoint $\beta-$ sets in X, if and only if $\sum_{k=1}^{\sigma} t_k^i = \sum_{k=1}^{\upsilon} t_k^j$ and there exists $1 \le d \le \sigma$, for each $1 \le r \le \upsilon$ such that $t_d^i \ne t_r^j$. Moreover, $\lambda_i^\beta \ge \lambda_j^\beta$, if $\sum_{k=1}^{\sigma} t_k^i \ge \sum_{k=1}^{\upsilon} t_k^j$ and $|\lambda_i| \ge |\lambda_j|$.

Definition 3:[28] Let $X = \left\{\lambda_i^\beta\right\}_{i=1}^{c(\beta)} = \{t_1^i, t_2^i, \dots, t_{\alpha_i}^i\} | 1 \le i \le c(\beta)\}$ be a collection of β-sets, where β is a permutation in the symmetric group $G = S_n$. Let T= λ_k^β , for some $\lambda_k^\beta \in X$, where λ_k^β such that $\sum_{s=1}^{\alpha_k} t_s^k \le \sum_{s=1}^{\alpha_i} t_s^i$ & $|\lambda_k| \le |\lambda_i|$,$\forall(1 \le i \le c(\beta))$ .... $(*)$. Moreover, if there are at least two disjoint β-sets say λ_h^β , λ_g^β such that $(*)$. Then, let T= λ_h^β if there exists $t_r^h \in \lambda_h^\beta$ with $t_r^h \le t_s^g$, $\forall\ (1 \le s \le \alpha_g)$ or T= λ_g^β if there exists $t_r^g \in \lambda_g^\beta$ with $t_r^g \le t_s^h$, $\forall\ (1 \le s \le \alpha_h)$. Also, let $\times: X \times X \longrightarrow X$ be a binary operation. We can infer that $(X,\times,T)$ is a *permutation commutative Q-algebra* $(P-CQ-A)$, if $\times$ satisfies the conditions:

(1) $\lambda_i^\beta \times \lambda_i^\beta = T$,
(2) $\lambda_i^\beta \times T = \lambda_i^\beta$,
(3) $(\lambda_i^\beta \times \lambda_j^\beta) \times \lambda_k^\beta = (\lambda_i^\beta \times \lambda_k^\beta) \times \lambda_j^\beta$, $\forall \lambda_i^\beta, \lambda_j^\beta, \lambda_k^\beta \in X$.
(4) $\lambda_i^\beta \times \lambda_j^\beta = \lambda_j^\beta \times \lambda_i^\beta$, $\forall \lambda_i^\beta, \lambda_j^\beta \in X$.

Definition 4: [28] Let $(X,\times,T)$ be a $(P-CQ-A)$ and $I(\ne \emptyset) \subseteq X$. The set I is called a *permutation commutative Q-ideal* $(P-CQ-I)$ of X if for all $\lambda_i^\beta, \lambda_j^\beta, \lambda_k^\beta \in X$,

1) $T \in I$,
2) $\lambda_i^\beta \times \lambda_j^\beta \in I$ and $\lambda_j^\beta \in I \Longrightarrow \lambda_i^\beta \in I$.

Furthermore, X and $\{T\}$ are permutation commutative Q-ideals of X. Then X and $\{T\}$ are called *trivial and fixed* of X, respectively. A $(P-CQ-I)$ I is called *proper* if $I \ne X$. In addition, we say $I\ (\ne \emptyset) \subseteq X$ is a permutation commutative Q-subalgebra $(P-CQ-SA)$ of X.

Proposition 1: [28] If $(X,\times,T)$ is a $(P-CQ-A)$, then $\left(\lambda_i^\beta \times \left(\lambda_i^\beta \times \lambda_j^\beta\right)\right) \times \lambda_j^\beta = T, \forall \lambda_i^\beta, \lambda_j^\beta \in X$.

Definition 5: [1] Assume that $\nabla: U \to P(D)$ is a mapping of $U \subseteq D$ into $P(D)$, where D is a set of parameters and $P(D)$ is the power set of D. We say that (∇, U) is a soft set over D.

3. ON SOFT PERMUTATION COMMMUTATIVE Q-ALGEBRA

Definition 6: Assume that $(X,\times,T)$ is a $(P-CQ-A)$ and $\nabla: U \to P(X)$ is a multivalued function, where $U \subseteq X$ defined by $\nabla(u) = \{t \in P(X) | u \approx t\}, \forall u \in D$ where $\approx$ is any relation. We infer that (∇, U) is a soft permutation commutative Q-algebra $(SP-CQ-A)$ of X, if $(\nabla(u),\times,T)$ is a $(P-CQ-SA)$ of X, $\forall u \in D$.

Example 1:

Let (S_{12},o) be symmetric group and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & 4 & 1 & 2 & 5 & 7 & 6 & 10 & 9 & 8 & 11 & 12 \end{pmatrix}$ be a permutation in S_{12}. So, $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & 4 & 1 & 2 & 5 & 7 & 6 & 10 & 9 & 8 & 11 & 12 \end{pmatrix} = (1\ 3)(5)(2\ 4)(6\ 7)(8\ 10)(9)(11)(12)$. Therefore, we have $X = \left\{\lambda_i^\beta\right\}_{i=1}^4 = \{\{1,3\},\{5\},\{2,4\},\{6,7\},\{8,10\},\{9\},\{11\},\{12\}\}$ and T={1,3}. Let $K = \lambda_k^\beta$, $H = \lambda_h^\beta$, $G = \lambda_g^\beta$, $L = \lambda_l^\beta$, $J = \lambda_j^\beta$, $F = \lambda_f^\beta$, $D = \lambda_d^\beta$ for some $\lambda_k^\beta, \lambda_h^\beta, \lambda_g^\beta, \lambda_l^\beta, \lambda_j^\beta, \lambda_f^\beta, \lambda_d^\beta \in X$, where;

$\sum_{s=1}^{\alpha_k} t_s^k \geq \sum_{s=1}^{\alpha_i} t_s^i$ & $|\lambda_k| \geq |\lambda_i|, \forall (1 \leq i \leq c(\beta)) \ldots.(i)$,

$\sum_{s=1}^{\alpha_h} t_s^h \geq \sum_{s=1}^{\alpha_i} t_s^i$ & $|\lambda_h| \geq |\lambda_i|, \forall (i \in \{1,2,\ldots,c(\beta)\} - \{k\}) \ldots.(ii)$,

$\sum_{s=1}^{\alpha_g} t_s^g \geq \sum_{s=1}^{\alpha_i} t_s^i$ & $|\lambda_g| \geq |\lambda_i|, \forall (i \in \{1,2,\ldots,c(\beta)\} - \{k,h\}) \ldots.(iii)$,

$\sum_{s=1}^{\alpha_l} t_s^l \geq \sum_{s=1}^{\alpha_i} t_s^i$ & $|\lambda_l| \geq |\lambda_i|, \forall (i \in \{1,2,\ldots,c(\beta)\} - \{k,h,g\}) \ldots.(iv)$,

$\sum_{s=1}^{\alpha_j} t_s^j \geq \sum_{s=1}^{\alpha_i} t_s^i$ & $|\lambda_j| \geq |\lambda_i|, \forall (i \in \{1,2,\ldots,c(\beta)\} - \{k,h,g,l\}) \ldots.(v)$,

$\sum_{s=1}^{\alpha_f} t_s^f \geq \sum_{s=1}^{\alpha_i} t_s^i$ & $|\lambda_f| \geq |\lambda_i|, \forall (i \in \{1,2,\ldots,c(\beta)\} - \{k,h,g,l,j\}) \ldots.(vi)$,

$\sum_{s=1}^{\alpha_d} t_s^d \geq \sum_{s=1}^{\alpha_i} t_s^i$ & $|\lambda_d| \geq |\lambda_i|, \forall (i \in \{1,2,\ldots,c(\beta)\} - \{k,h,g,l,j,f\}) \ldots.(vii)$, where $\lambda_i, \lambda_k, \lambda_h, \lambda_g, \lambda_l, \lambda_j, \lambda_f$, and λ_d are cycles for $\lambda_i^\beta, \lambda_k^\beta, \lambda_h^\beta, \lambda_g^\beta, \lambda_l^\beta, \lambda_j^\beta, \lambda_f^\beta$, and λ_d^β, respectively.

Then, we consider that $K = \{8,10\}$ $H = \{6,7\}$, $G = \{12\}$, $L = \{11\}$, $J = \{9\}$, $F = \{2,4\}$, $D = \{5\}$.

Now, let $\lambda_i^\beta \in X$. Define $\times: X \times X \longrightarrow X$ by

$$T \times \lambda_i^\beta = \lambda_i^\beta,\ D \times \lambda_i^\beta = \begin{cases} L, if \lambda_i^\beta \in \{L,F\} \\ G, if \lambda_i^\beta \in \{G,J\} \\ K, if \lambda_i^\beta \in \{K,H\} \\ D, if \lambda_i^\beta = T \\ T, if\ \lambda_i^\beta = D \end{cases},\ F \times \lambda_i^\beta = \begin{cases} L, if \lambda_i^\beta \in \{L,D\} \\ J, if \lambda_i^\beta \in \{J,H\} \\ K, if \lambda_i^\beta \in \{K,G\} \\ F, if \lambda_i^\beta = T \\ T, if\ \lambda_i^\beta = F \end{cases},\ J \times \lambda_i^\beta = \begin{cases} G, if \lambda_i^\beta \in \{G,D\} \\ H, if \lambda_i^\beta \in \{H,F\} \\ K, if \lambda_i^\beta \in \{K,L\} \\ F, if \lambda_i^\beta = T \\ T, if\ \lambda_i^\beta = J \end{cases},$$

$L \times \lambda_i^\beta = \begin{cases} T, if \lambda_i^\beta = L \\ L, if\ \lambda_i^\beta \in \{T, D, F\} \\ K, if\ \lambda_i^\beta \in \{K, H, G, J\} \end{cases}$, $G \times \lambda_i^\beta = \begin{cases} T, if \lambda_i^\beta = G \\ G, if\ \lambda_i^\beta \in \{T, D, J\} \\ K, if\ \lambda_i^\beta \in \{K, H, L, F\} \end{cases}$, $H \times \lambda_i^\beta = \begin{cases} T, if \lambda_i^\beta = H \\ H, if\ \lambda_i^\beta \in \{T, F, J\} \\ K, if\ \lambda_i^\beta \in \{K, D, L, G\} \end{cases}$, and

$K \times \lambda_i^\beta = \begin{cases} T, if\ \lambda_i^\beta = K \\ K, if\ \lambda_i^\beta \neq K \end{cases}$.

Here we consider $(X,\times,T)$ is a $(P - CQ - A)$. See Table (1).

Table 1. $(X,\times,T)$ is a $(P - CQ - A)$.

×	{1,3}	{5}	{2,4}	{9}	{11}	{12}	{6,7}	{8,10}
{1,3}	{1,3}	{5}	{2,4}	{9}	{11}	{12}	{6,7}	{8,10}
{5}	{5}	{1,3}	{11}	{12}	{11}	{12}	{8,10}	{8,10}
{2,4}	{2,4}	{11}	{1,3}	{6,7}	{11}	{8,10}	{6,7}	{8,10}
{9}	{9}	{12}	{6,7}	{1,3}	{8,10}	{12}	{6,7}	{8,10}
{11}	{11}	{11}	{11}	{8,10}	{1,3}	{8,10}	{8,10}	{8,10}
{12}	{12}	{12}	{8,10}	{12}	{8,10}	{1,3}	{8,10}	{8,10}
{6,7}	{6,7}	{8,10}	{6,7}	{6,7}	{8,10}	{8,10}	{1,3}	{8,10}
{8,10}	{8,10}	{8,10}	{8,10}	{8,10}	{8,10}	{8,10}	{8,10}	{1,3}

and let $N_1: M_1 \to P(X)$, $N_2: M_2 \to P(X)$, where $M_1 = \{\{1,3\}, \{5\}, \{2,4\}, \{9\}\}, M_2 = X$ be defined by $N_1(h) = \{k \in P(X) | h \approx k \Leftrightarrow h = h \times k\}$, $\forall h \in M_1$, and $N_2(h) = \{k \in P(X) | k = h \times k\}, \forall h \in M_2$. Thus, $N_1(1,3\}) = N_1(\{5\}) = N_1(\{2,4\}) = N_1(\{9\}) = \{\{1,3\}\}$ which are $(P - CQ - SA)$ of X and hence (N_1, M_1) is a $(SP - CQ - A)$. But, (N_2, M_2) is not $(SP - CQ - A)$, since $N_2(\{5\}) = \{\{11\}, \{12\}, \{8,10\}\}$ is not $(P - CQ - SA)$.

Remark 1: Let (N, M) be a $(SP - CQ - A)$ and $\emptyset \neq U \subseteq M$, then $T \in N(\lambda_i^\beta), \forall \lambda_i^\beta \in M$ and (N, U) is a $(SP - CQ - A)$.

Proposition 2: Let (N, M) be a $(SP - CQ - A)$ of X, where $(M,\times,T)$ is $(P - CQ - A)$ of X. If $N: M \to P(X)$ satisfies $N(T) = T$ and $\lambda_i^\beta \times (\lambda_i^\beta \times \lambda_j^\beta) = \lambda_i^\beta \times \lambda_j^\beta$, $\forall \lambda_i^\beta, \lambda_j^\beta \in M$, then $N(\lambda_i^\beta) = T, \forall \lambda_i^\beta \in M$.

Proof: For any $\lambda_i^\beta, \lambda_j^\beta \in M$, we have $\lambda_i^\beta \times (\lambda_i^\beta \times \lambda_j^\beta) \in M$ and $\lambda_i^\beta \times \lambda_j^\beta \in M$ [since M is $(P - CQ - A)$ of X]. Substituting $\lambda_i^\beta = \lambda_j^\beta$ into the equation $\lambda_i^\beta \times (\lambda_i^\beta \times \lambda_j^\beta) = \lambda_i^\beta \times \lambda_j^\beta$, we have that $N(\lambda_i^\beta \times (\lambda_i^\beta \times \lambda_i^\beta)) = N(\lambda_i^\beta \times \lambda_i^\beta)$. So, $N(\lambda_i^\beta \times T) = N(T)$ (From (1) of Definition 3). Therefore $N(\lambda_i^\beta) = N(T)$ (From (2) of Definition 3), but $N(T) = T$. Hence $N(\lambda_i^\beta) = T, \forall \lambda_i^\beta \in M$.

Proposition 3: Let (N, M) be a $(SP - CQ - A)$ of $(X,\times,T)$. If the associative law holds for any $\lambda_i^\beta, \lambda_j^\beta, \lambda_k^\beta \in N(\lambda_m^\beta)$, then $(N(\lambda_m^\beta),\times)$ is a group for any $\lambda_i^\beta \in M$.

Proof: Since for any $\lambda_i^\beta, \lambda_j^\beta, \lambda_k^\beta \in N(\lambda_m^\beta)$, we have $(\lambda_i^\beta \times \lambda_j^\beta) \times \lambda_k^\beta = \lambda_i^\beta \times (\lambda_j^\beta \times \lambda_k^\beta)$, then we need only to show that $N(\lambda_m^\beta)$ has an identity which is T and for any $T \neq \lambda_i^\beta \in M$ has an inverse. By Substituting $\lambda_i^\beta = \lambda_j^\beta = \lambda_k^\beta$ into the associative law $(\lambda_i^\beta \times \lambda_j^\beta) \times \lambda_k^\beta = \lambda_i^\beta \times (\lambda_j^\beta \times \lambda_k^\beta)$. We have that $(\lambda_i^\beta \times \lambda_i^\beta) \times \lambda_i^\beta = \lambda_i^\beta \times (\lambda_i^\beta \times \lambda_i^\beta)$. Then $T \times \lambda_i^\beta = \lambda_i^\beta \times T = \lambda_i^\beta$, but $T \in N(\lambda_m^\beta), \forall \lambda_i^\beta \in M$. This means that T is the identity element of $N(\lambda_m^\beta)$. By (1) of Definition (3), every element $\lambda_i^\beta \in N(\lambda_m^\beta) \subseteq X$ has itself as its inverse. Therefore, $(N(\lambda_m^\beta),\times)$ is a group for any $\lambda_i^\beta \in M$.

Lemma 1: Let (N, M) be a $(SP - CQ - A)$ of $(X,\times,T)$. If $\lambda_i^\beta \times N(\lambda_j^\beta) = \lambda_i^\beta \times N(\lambda_k^\beta)$, $\forall \lambda_i^\beta \in X$ &$\forall \lambda_j^\beta, \lambda_k^\beta \in M$, then $T \times N(\lambda_j^\beta) = T \times N(\lambda_k^\beta)$.

Proof: We have

$$(\lambda_i^\beta \times N(\lambda_j^\beta)) \times \lambda_i^\beta = (\lambda_i^\beta \times \lambda_i^\beta) \times N(\lambda_j^\beta) \quad \text{(From (3) of Definition 3)}$$
$$= T \times N(\lambda_j^\beta) \quad \text{(From (2) of Definition 3)}$$

And

$$\left(\lambda_i^\beta \times N(\lambda_k^\beta)\right) \times \lambda_i^\beta = \left(\lambda_i^\beta \times \lambda_i^\beta\right) \times N(\lambda_k^\beta) \quad \text{(From (3) of Definition 3)}$$
$$= T \times N(\lambda_k^\beta)$$

Since $\lambda_i^\beta \times N(\lambda_j^\beta) = \lambda_i^\beta \times N(\lambda_k^\beta)$, then

$$T \times N(\lambda_j^\beta) = T \times N(\lambda_k^\beta).$$

Definition 7: Let (N, M) be a $(SP - CQ - A)$ of $(X,\times,T)$. We define

$$G(N, M) = \left\{\lambda_i^\beta \in M \mid T \times N(\lambda_i^\beta) = N(\lambda_i^\beta)\right\}.$$

Moreover, if $G(N, M) = X$, then (N, M) is called the soft *permutation commutative G-part* $(SP - CG - P)$ of X.

Example 2: Let $(X,\times,T)$ and (N_1, M_1) be $(P - CQ - A)$ and $(SP - CQ - A)$, respectively in Example (1), then $G(N_1, M_1) = \left\{\lambda_i^\beta \in M_1 \mid T \times N(\lambda_i^\beta) = N(\lambda_i^\beta)\right\} = \{\{5\}, \{7\}, \{1,4\}\} \neq X$, hence (N_1, M_1) is not $(SP - CG - P)$.

Example 3: Let (S_{11}, o) be symmetric group and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 3 & 10 & 7 & 2 & 6 & 5 & 1 & 9 & 11 & 4 & 8 \end{pmatrix}$ be a permutation in S_{11}. So, $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 3 & 10 & 7 & 2 & 3 & 7 & 1 & 9 & 11 & 4 & 8 \end{pmatrix} = (1\ 3\ 7)(2\ 10\ 4)(6\ 5)(8\ 9\ 11)$. Therefore, we have $X = \left\{\lambda_i^\beta\right\}_{i=1}^4 = \{\{1,3,7\}, \{2,4,10\}, \{5\ ,\ 6\}, \{8,9,11\})\}$ and $T = \{5,6\}$. Define $\times: X \times X \longrightarrow X$ by $\lambda_i^\beta \times \lambda_j^\beta =$

$$\begin{cases} T, if \left(\lambda_i^\beta = \lambda_j^\beta\right) \\ \lambda_i^\beta, if \left(\sum_{s=1}^{\alpha_i} t_s^i > \sum_{s=1}^{\alpha_j} t_s^j\right) \ or \ \left(\sum_{s=1}^{\alpha_i} t_s^i = \sum_{s=1}^{\alpha_j} t_s^j \ and \ |\lambda_i| > |\lambda_j|\right), \\ \lambda_j^\beta, if \left(\sum_{s=1}^{\alpha_i} t_s^i < \sum_{s=1}^{\alpha_j} t_s^j\right) or \ \left(\sum_{s=1}^{\alpha_i} t_s^i = \sum_{s=1}^{\alpha_j} t_s^j \ and \ \& |\lambda_i| < |\lambda_j|\right) \end{cases}$$

where λ_i and λ_j denote cycles for λ_i^β and λ_j^β, respectively. Here we consider $(X,\times,T)$ is a $(P - CQ - A)$. See table 2.

Table 2. $(X,\times,T)$ **is a** $(PC - Q - A)$

×	{5, 6}	{1, 3, 7}	{2, 4, 10}	{8, 9, 11}
{5, 6}	{5,6}	{1,3,7}	{2,4,10}	{8,9,11}
{1, 3, 7}	{1,3,7}	{5,6}	{2,4,10}	{8,9, 11}
{2, 4, 10}	{2,4,10}	{2,4,10}	{5,6}	{8,9, 11}
{8, 9, 11}	{8,9, 11}	{8,9, 11}	{8,9, 11}	{5,6}

If we define $N: M \rightarrow P(X)$ by $N(h) = \{k \in P(X) | h \approx k \Leftrightarrow h = k \underset{\times}{} h\}$, $\forall h \in M$, where $M = X$. Then (N, M) is $(SPC - G - A)$ of X , since each one of $N(\{5,6\}) = N(\{1,3,7\}\}) = \{5,6\}$, $N(\{2,4,10\}) = \{\{5,6\}, \{1,3,7\}\}$, $N(\{8,9, 11\}) = \{\{5,6\}, \{1,3,7\}, \{2,4,10\}\}$ is $(P - CQ - SA)$ of X, and $G(N, M) = \left\{\lambda_i^\beta \in M \mid T \times N(\lambda_i^\beta) = N(\lambda_i^\beta)\right\} = X$. Then (N, M) is $(SP - CG - P)$ of X.

Example 4:

The chemical element silver, denoted as (Ag), possesses 47 protons in its nucleus, and its electron count equals the proton count, resulting in 47 electrons revolving around it. Additionally, there are five atomic shells labelled as $\{K, L, M, N, O\}$ surrounding the nucleus, each containing a specific number of electrons, as illustrated in fig 1.

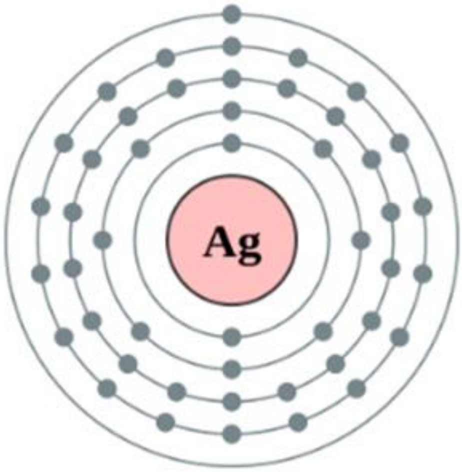

Fig (1): Atomic shells for Silver

Hence $K = \{e_1, e_2\}$, $L = \{e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$,

$M = \{e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23}, e_{24}, e_{25}, e_{26}, e_{27}, e_{28}\}$,

$N = \{e_{29}, e_{30}, e_{31}, e_{32}, e_{33}, e_{34}, e_{35}, e_{36}, e_{37}, e_{38}, e_{39}, e_{40}, e_{41}, e_{42}, e_{43}, e_{44}, e_{45}, e_{46}\}$,

$O = \{e_{47}\}$.

Let $\beta = (1\ 2)(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10)(11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 24\ 25\ 26\ 27\ 28)$

$(29\ 30\ 31\ 32\ 33\ 34\ 35\ 36\ 37\ 38\ 39\ 40\ 41\ 42\ 43\ 44\ 45\ 46)(47)$ be a permutation in S_{47} .Therefore, we have $X = \left\{\lambda_i^\beta\right\}_{i=1}^{5} = \{\{1,2\}, \{3, 4, 5, 6, 7, 8, 9, 10\}, \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21,$

$22, 23, 24, 25, 26, 27, 28\}, \{29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46\}\{47\}\}$. Define a map $f: Ag \to X$ by $f(A) = \{i, j, \dots, r\}$, $\forall A = \{e_i, e_j, \dots, e_r\} \in Ag$. Then $f(K) = \lambda_1^\beta$, $f(L) = \lambda_2^\beta$, $f(M) = \lambda_3^\beta$, $f(N) = \lambda_4^\beta$, $f(O) = \lambda_5^\beta$. For any $1 \le i \le 5$ and $A \in Ag$, let $\overline{f(A)} = \overline{\lambda_i^\beta} = \sum_{k=1}^{\sigma} t_k^i$, where $|\lambda_i| = \sigma$. For any $f(A), f(B) \in f(Ag) = \{f(K), f(L), f(M),$

$f(N), f(O)\}$ define $\# : f(Ag) \times f(Ag) \to f(Ag)$ by $f(A)\#f(B) = \begin{cases} f(A), & if\ \overline{f(A)} > \overline{f(B)}, \\ f(B), & if\ \overline{f(A)} < \overline{f(B)}, \\ f(K), & if\ \overline{f(A)} = \overline{f(B)}, \end{cases}$ Therefore we can consider table (2).

Table (3): $(f(Ag), \#, f(K))$ is a $(PSC-Q-A)$.

$\#$	$f(K)$	$f(L)$	$f(M)$	$f(N)$	$f(O)$
$f(K)$	$f(K)$	$f(L)$	$f(M)$	$f(N)$	$f(O)$
$f(L)$	$f(L)$	$f(K)$	$f(M)$	$f(N)$	$f(O)$
$f(M)$	$f(M)$	$f(M)$	$f(K)$	$f(N)$	$f(M)$
$f(N)$	$f(N)$	$f(N)$	$f(N)$	$f(K)$	$f(N)$
$f(O)$	$f(O)$	$f(O)$	$f(M)$	$f(N)$	$f(K)$

Defined $W: D \to P(f(Ag))$ by $W(d) = \{h \in P(f(Ag)) | d \approx h \Leftrightarrow d = d\ \#\ h\}, \forall d \in D$, where $D = f(Ag)$. Then (W, D) is $(SPSC-G-A)$ of $f(Ag)$, since each one of $W(f(K)) = W(f(L)) = f(K) = W(f(M)) = \{f(K), f(L), f(O)\}, W(f(N)) = \{f(K), f(L), f(M), f(O)\},$
$W(f(O)) = \{f(K), f(L)\}$ is $(P-CQ-SA)$ of $f(Ag)$, and $G(W, D) = \{d \in D \mid f(K)\ \#\ W(d) = W(d)\} = f(Ag)$. Therefore (W, D) is $(SP-CG-P)$ of $f(Ag)$.

Corollary 1: Let (N, M) be a $(SP - CG - P)$ of X. If $\lambda_i^\beta \times N(\lambda_j^\beta) = \lambda_i^\beta \times N(\lambda_k^\beta)$, then the left cancellation law is held.
Proof: Let $\lambda_i^\beta \times N(\lambda_j^\beta) = \lambda_i^\beta \times N(\lambda_k^\beta)$. From Lemma 3.6, we have that $T \times N(\lambda_j^\beta) = T \times N(\lambda_k^\beta)$. Also, $\lambda_j^\beta, \lambda_j^\beta \in X = G(N, M)$ [Since (N, M) is a $(SP - CG - P)$]. Thus $T \times N(\lambda_j^\beta) = N(\lambda_j^\beta)$ and $T \times N(\lambda_k^\beta) = N(\lambda_k^\beta)$ and hence $N(\lambda_j^\beta) = N(\lambda_k^\beta)$.
Proposition 4: Let (N, M) be a $(SP - CG - P)$ of X. Then $\lambda_i^\beta \in G(N, M)$ if $T \times N(\lambda_i^\beta) \in G(N, M)$.
Proof: If $\lambda_i^\beta \in G(N, M)$, then $T \times N(\lambda_i^\beta) = N(\lambda_i^\beta)$ and $T \times \left(T \times N(\lambda_i^\beta)\right) = T \times N(\lambda_i^\beta)$. Hence $T \times N(\lambda_i^\beta) \in G(N, M)$.
Remark 2: If (N, M) is a $(SP - CG - P)$ of X. Then the converse of Proposition (4) is held.
Definition 8: Let (N, M) be a $(SP - CG - A)$ of X. We define

$$B(N, M) = \left\{\lambda_i^\beta \in M \mid T \times N(\lambda_i^\beta) = T\right\}.$$

Moreover, if $B(N, M) = \{T\}$, then (N, M) is referred to as soft *permutation commutative p-semisimple* Q-algebra $(SP - CPSQ - A)$.

Remark 3: $G(N, M) \cap B(N, M) = \{T\}$.
Proposition 5: Let (N, M) be a $(SP - CG - P)$ of X and $\lambda_i^\beta, \lambda_j^\beta \in X$, then $\lambda_j^\beta \in B(N, M) \Leftrightarrow \left(\lambda_i^\beta \times N(\lambda_j^\beta)\right) \times \lambda_i^\beta = T$.
Proof: Let $\lambda_j^\beta \in B(N, M)$. Then $T \times N(\lambda_j^\beta) = T$, but $\lambda_i^\beta \times \lambda_i^\beta = T$ (From (1) of Definition 3) and hence $T = T \times N(\lambda_j^\beta) = \left(\lambda_i^\beta \times \lambda_i^\beta\right) \times N(\lambda_j^\beta) = \left(\lambda_i^\beta \times N(\lambda_j^\beta)\right) \times \lambda_i^\beta$ (From (3) of Definition 3). Then $\left(\lambda_i^\beta \times N(\lambda_j^\beta)\right) \times \lambda_i^\beta = T$.

Conversely, assume that $\left(\lambda_i^\beta \times N(\lambda_j^\beta)\right) \times \lambda_i^\beta = T$. From [Definition 3-(3)], we have $\left(\lambda_i^\beta \times \lambda_i^\beta\right) \times N(\lambda_j^\beta) = \left(\lambda_i^\beta \times N(\lambda_j^\beta)\right) \times \lambda_i^\beta$. Thus $\left(\lambda_i^\beta \times \lambda_i^\beta\right) \times N(\lambda_j^\beta) = T$ and hence $T \times N(\lambda_j^\beta) = T$. Then $\lambda_j^\beta \in B(N, M)$.

Proposition 6: Let (N, M) be a $(SP - CQ - A)$ of $(X, \times, T)$. Then $B(N, M)$ is a $(P - CQ - I)$ of X if $N(T) = T$.
Proof: Since $(T \times N(T)) \times T = (T \times T) \times T = T \times T = T$, from Proposition 5, $T \in B(N, M)$. Let $\lambda_i^\beta \times \lambda_j^\beta \in B(N, M)$ and $\lambda_j^\beta \in B(N, M)$. Then by Proposition 5, $\left(\left(\lambda_i^\beta \times \lambda_j^\beta\right) \times N(\lambda_i^\beta)\right) \times \left(\lambda_i^\beta \times \lambda_j^\beta\right) = T$ ...(a). Also, By (3) of Definition (3), we have that $\left(\left(\lambda_i^\beta \times \lambda_j^\beta\right) \times N(\lambda_i^\beta)\right) \times \left(\lambda_i^\beta \times \lambda_j^\beta\right) = \left(\left(\lambda_i^\beta \times \lambda_j^\beta\right) \times \left(\lambda_i^\beta \times \lambda_j^\beta\right)\right) \times N(\lambda_i^\beta) = T \times N(\lambda_i^\beta)$ ……..(b) (From (1) of Definition 3). Thus from (a) and (b), we have $T \times N(\lambda_i^\beta) = T$. Hence $\lambda_i^\beta \in B(N, M)$. Therefore $B(N, M)$ is a $(P - CQ - I)$ of X.
Proposition 7: Let (N, X) be a $(SP - CQ - A)$ of $(X, \times, T)$ and M be a $(P - CQ - SA)$ of X, then $G(N, X) \cap M = G(N, M)$.
Proof: If $\lambda_i^\beta \in G(N, M)$, then $T \times N(\lambda_i^\beta) = N(\lambda_i^\beta)$ and $\lambda_i^\beta \in M \subseteq X$. Then $\lambda_i^\beta \in G(N, X)$ and so $\lambda_i^\beta \in G(N, X) \cap M$, thus $G(N, M) \subseteq G(N, X) \cap M$. Also, If $\lambda_i^\beta \in G(N, X) \cap M$, then $\lambda_i^\beta \in G(N, X)$ and $\lambda_i^\beta \in M$, so $T \times N(\lambda_i^\beta) = N(\lambda_i^\beta)$ and $\lambda_i^\beta \in M$. Hence $\lambda_i^\beta \in G(N, M)$ which proves the Proposition.

Proposition 8: If (N, M) is a $(SP - CG - P)$ of X, then (N, M) is a $(SP - CPSQ - A)$.
Proof: Assume that (N, M) is a $(SP - CG - P)$of X, then $G(N, M) = X$. By Remark (2) $T = G(N, M) \cap B(N, M) = X \cap B(N, M) = B(N, M)$. Hence (N, M) is a $(SP - CPSQ - A)$.

Proposition 9: Let (N, X) be a $(SP - CQ - A)$ of $(X, \times, T)$ with $N(h) = h, \forall h \in X$. If $(X, \times, T)$ of order 3, then $|G(N, X)| \neq 3$, that is, $G(N, X) \neq X$.
Proof: Let's assume that $X = \left\{T, \lambda_i^\beta, \lambda_j^\beta\right\}$ is a $(P - CQ - A)$. Assume that $|G(N, M)| = 3$, that is, $G(N, M) = X$. Then $T \times N(T) = N(T)$, hence $T \times T = T$, $T \times N(\lambda_i^\beta) = N(\lambda_i^\beta)$, then $T \times \lambda_i^\beta = \lambda_i^\beta$ and $T \times N(\lambda_j^\beta) = N(\lambda_j^\beta)$, thus $T \times \lambda_j^\beta = \lambda_j^\beta$. From (1) and (2) of Definition (2.3), we have that $\lambda_i^\beta \times \lambda_i^\beta = T, \lambda_j^\beta \times \lambda_j^\beta = T, \lambda_i^\beta \times T = \lambda_i^\beta$, and $\lambda_j^\beta \times T = \lambda_j^\beta$. Now let $\lambda_i^\beta \times \lambda_j^\beta = T$. Then T, λ_i^β and λ_j^β are elements of the computation.

If $\lambda_j^\beta \times \lambda_i^\beta = T$, then $\lambda_i^\beta \times \lambda_j^\beta = T = \lambda_j^\beta \times \lambda_i^\beta$ and so $\left(\lambda_i^\beta \times \lambda_j^\beta\right) \times \lambda_i^\beta = \left(\lambda_j^\beta \times \lambda_i^\beta\right) \times \lambda_i^\beta$. By (3) of Definition (3), $\left(\lambda_i^\beta \times \lambda_i^\beta\right) \times \lambda_j^\beta = \left(\lambda_j^\beta \times \lambda_i^\beta\right) \times \lambda_i^\beta$. Hence $T \times \lambda_j^\beta = T \times \lambda_i^\beta$ and hence $\lambda_i^\beta = \lambda_j^\beta$[Since the cancellation law is held in $G(X)$], but it is a contradiction. Also, if $\lambda_j^\beta \times \lambda_i^\beta = \lambda_i^\beta$, then $\lambda_i^\beta = \lambda_j^\beta \times \lambda_i^\beta = \left(T \times \lambda_j^\beta\right) \times \lambda_i^\beta = \left(T \times \lambda_i^\beta\right) \times \lambda_j^\beta = \lambda_i^\beta \times \lambda_j^\beta = T$, a contradiction. For the case $\lambda_j^\beta \times \lambda_i^\beta = \lambda_j^\beta$, we have that $\lambda_j^\beta = \lambda_j^\beta \times \lambda_i^\beta = \left(T \times \lambda_j^\beta\right) \times \lambda_i^\beta = \left(T \times \lambda_i^\beta\right) \times$

$\lambda_j^\beta = \lambda_i^\beta \times \lambda_j^\beta = T$. Which is also a contradiction. Next, if $\lambda_i^\beta \times \lambda_j^\beta = \lambda_i^\beta$, then $\left(\lambda_i^\beta \times \left(\lambda_i^\beta \times \lambda_j^\beta\right)\right) \times \lambda_j^\beta = \left(\lambda_i^\beta \times \lambda_i^\beta\right) \times \lambda_j^\beta = T \times \lambda_j^\beta = \lambda_j^\beta \neq T$. This gives the conclusion that Proposition (1) is not satisfy, and this a contradiction. Now, suppose that $\lambda_i^\beta \times \lambda_j^\beta = \lambda_j^\beta$. If $\lambda_j^\beta \times \lambda_i^\beta = T$, then $\lambda_j^\beta = \lambda_i^\beta \times \lambda_j^\beta = \left(T \times \lambda_i^\beta\right) \times \lambda_j^\beta = \left(T \times \lambda_j^\beta\right) \times \lambda_i^\beta = \lambda_j^\beta \times \lambda_i^\beta = T$, a contradiction. If $\lambda_j^\beta \times \lambda_i^\beta = T$, then $\lambda_j^\beta = \lambda_i^\beta \times \lambda_j^\beta = \left(T \times \lambda_i^\beta\right) \times \lambda_j^\beta = \left(T \times \lambda_j^\beta\right) \times \lambda_i^\beta = \lambda_j^\beta \times \lambda_i^\beta = T$, a contradiction. For the case $\lambda_j^\beta \times \lambda_i^\beta = \lambda_j^\beta$, we have $\lambda_i^\beta = T \times \lambda_i^\beta = \left(\lambda_j^\beta \times \lambda_i^\beta\right) \times \lambda_i^\beta = \left(\lambda_j^\beta \times \lambda_i^\beta\right) \times \lambda_j^\beta = \lambda_j^\beta \times \lambda_j^\beta = T$. This is yet another contradiction. This brings the proof to a close.

Proposition 10: Let (N, X) be a $(SP - CQ - A)$ of $(X,\times,T)$. If $(X,\times,T)$ of order 2, then in every case, the set $G(N, M)$ is a $(P - CQ - I)$ of X.

Proof: Let $|X| = 2$. Then either $G(N, M) = \{T\}$ or $G(N, M) = X$. In any case, $G(N, M)$ is a $(P - CQ - I)$ of X.

Proposition 11: Let (N, X) be a $(SP - CQ - A)$ of $(X,\times,T)$ and $(X,\times,T)$ of order 3. Then $G(N, M)$ is a $(P - CQ - I)$ of X if and only if $|G(N, M)| = 1$.

Proof: Let $X = \left\{T, \lambda_i^\beta, \lambda_j^\beta\right\}$ be a $(P - CQ - A)$. If $|G(N, M)| = 1$, then $G(N, M) = \{T\}$ is the trivial $(P - CQ - I)$ of X. Now, let $G(N, M)$ be a $(P - CQ - I)$ of X. By Proposition (9), we know that either $|G(N, M)| = 1$ or $|G(N, M)| = 2$. Suppose that $|G(N, M)| = 2$. Then either $G(N, M) = \left\{T, \lambda_i^\beta\right\}$ or $G(N, M) = \left\{T, \lambda_j^\beta\right\}$. If $G(N, M) = \left\{T, \lambda_i^\beta\right\}$, then $\lambda_j^\beta \times \lambda_i^\beta \notin G(N, M)$ because $G(N, M)$ is a $(P - CQ - I)$ of X. Hence $\lambda_j^\beta \times \lambda_i^\beta = \lambda_j^\beta$. Then $\lambda_i^\beta = T \times \lambda_i^\beta = \left(\lambda_j^\beta \times \lambda_j^\beta\right) \times \lambda_i^\beta = \left(\lambda_j^\beta \times \lambda_i^\beta\right) \times \lambda_j^\beta = \lambda_j^\beta \times \lambda_j^\beta = T$. This is a contradiction. Similarly, $G(N, M) = \left\{T, \lambda_j^\beta\right\}$ gives a contradiction. Hence $|G(N, M)| \neq 2$ and so $|G(N, M)| = 1$.

Definition 9.
Let $\beta = \begin{pmatrix} 1 & 2 & 3 & . & . & . & m \\ \beta(1) & \beta(2) & \beta(3) & . & . & . & \beta(m) \end{pmatrix} = \lambda_1 \lambda_2 \dots \lambda_{c(\beta)}$ and $\gamma = \begin{pmatrix} 1 & 2 & 3 & . & . & . & n \\ \gamma(1) & \gamma(2) & \gamma(3) & . & . & . & \gamma(n) \end{pmatrix} = f_1 f_2 \dots f_{c(\gamma)}$ be two permutations in S_m and S_n, respectively, where $n \le m$. Assume that $\{\lambda_i^\beta = \{t_1^i, t_2^i, \dots, t_{\alpha_i}^i\} | 1 \le i \le c(\beta)\}$ and $\{f_i^\gamma = \{d_1^i, d_2^i, \dots, d_{\alpha_i}^i\} | 1 \le i \le c(\gamma)\}$ are two families of permutation sets for β and γ, respectively. Define $\omega_{\beta,\gamma} =$

$$\begin{pmatrix} 1 & 2 & 3 & . & . & . & m & m+1 & m+2 & m+3 & . & . & . & m+n \\ \omega_{\beta,\gamma}(1) & \omega_{\beta,\gamma}(2) & \omega_{\beta,\gamma}(3) & . & . & . & \omega_{\beta,\gamma}(m) & \omega_{\beta,\gamma}(m+1) & \omega_{\beta,\gamma}(m+2) & \omega_{\beta,\gamma}(m+3) & . & . & . & \omega_{\beta,\gamma}(m+n) \end{pmatrix}$$

in S_{n+m} by the following:

(1) If $n \le m$, then $\omega_{\beta,\gamma}(j) = \begin{cases} \beta(j), & if\ 1 \le j \le m \\ \gamma(m+n+1-j) + m, & if\ m+1 \le j \le m+n \end{cases}$.

(2) If $n > m$, then $\omega_{\beta,\gamma}(j) = \begin{cases} \gamma(j), & if\ 1 \le j \le n \\ \beta(m+n+1-j) + n, & if\ n+1 \le j \le m+n \end{cases}$.

So, $\omega_{\beta,\gamma} = \prod_{i=1}^{c(\omega_{\beta,\gamma})} \delta_i$ where $\prod_{i=1}^{c(\omega_{\beta,\gamma})} \delta_i$ is a composite of pairwise disjoint cycles $\{\delta_i\}_{i=1}^{c(\omega_{\beta,\gamma})}$. We say $\omega_{\beta,\gamma}$ is an extension permutation of β and γ.

Example 5.
Let $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 9 & 1 & 2 & 8 & 3 & 6 & 7 \end{pmatrix}$ & $\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 5 & 2 & 3 \end{pmatrix}$ be two permutations in $S_m = S_9$ and $S_n = S_5$, respectively. Since $\beta = (1\ 4)(2\ 5)(3\ 9\ 7)(6\ 8)$ and $\gamma = (1)(2\ 4)(3\ 5)$. Hence, we get $X = \left\{\lambda_i^\beta\right\}_{i=1}^4 = \{\{1,4\}, \{2,5\}, \{3,7,9\}, \{6,8\}\}$ and $Y = \left\{\lambda_i^\beta\right\}_{i=1}^3 = \{\{1\}, \{2,4\}, \{3,5\}\}$. Since $n < m$, we consider that $\omega_{\beta,\gamma}(j) = \begin{cases} \beta(j), & if\ 1 \le j \le m \\ \gamma(m+n+1-j) + m, & if\ m+1 \le j \le m+n \end{cases}$.

Therefore, $\omega_{\beta,\gamma} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 4 & 5 & 9 & 1 & 2 & 8 & 3 & 6 & 7 & 12 & 11 & 14 & 13 & 10 \end{pmatrix}$ is an extension permutation in S_{14} of β and γ.

Example 6.
The Sodium atom (Na) and Chlorine atom (Cl), have 11 and 17 electrons, respectively. Furthermore, three atomic shells $Na = \{K_{Na}, M_{Na}, N_{Na}\}$ and $Cl = \{K_{Cl}, M_{Cl}, N_{Cl}\}$ exist around nucleus (Na) and (Cl), respectively. See fig (2).

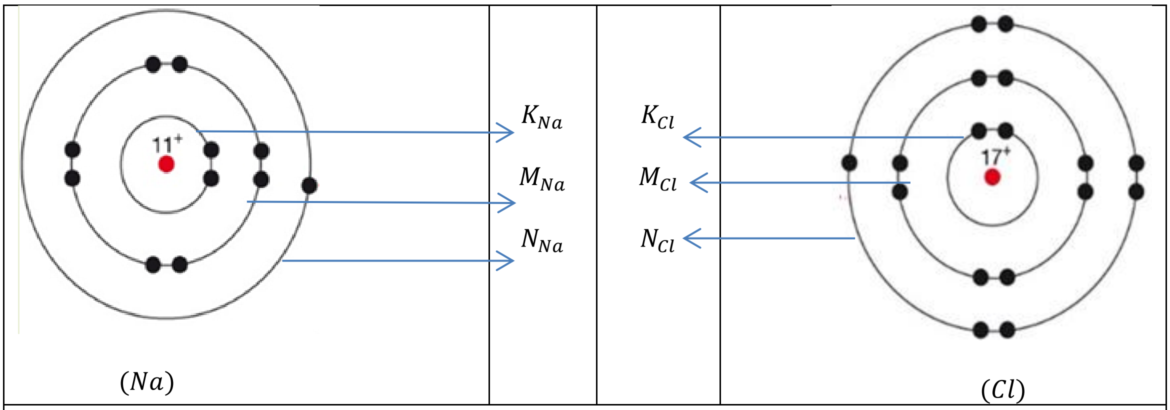

Fig (2): Atomic shells for Sodium and Chlorine

Hence $K_{Na} = \{e_1, e_2\}$, $L_{Na} = \{e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$, $M_{Na} = \{e_{11}\}$, $K_{Cl} = \{é_1, é_2\}$, $L_{Cl} = \{é_3, é_4, é_5, é_6, é_7, é_8, é_9, é_{10}\}$, and $M_{Cl} = \{é_{11}, é_{12}, é_{13}, é_{14}, é_{15}, é_{16}, é_{17}\}$.
Let $\beta = (1\ 2)(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10)(11)$ be a permutation in $S_m = S_{11}$ and $\gamma = (1\ 2)(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10)(11\ 12\ 13\ 14\ 15\ 16\ 17)$ be a permutation in $S_n = S_{17}$. Therefore, we have $X = \left\{\lambda_i^{\beta}\right\}_{i=1}^{3} = \{\lambda_1^{\beta} = \{1,2\}, \lambda_2^{\beta} = \{3,4,5,6,7,8,9,10\}, \lambda_3^{\beta} = \{11\}\}$ and $Y = \{t_i^{\gamma}\}_{i=1}^{3} = \{t_1^{\gamma} = \{1,2\}, t_2^{\gamma} = \{3,4,5,6,7,8,9,10\}, t_3^{\gamma} = \{11,12,13,14,15,16,17\}$. Define a map $f_1: Na \to X$ by $f_1(A) = \{i, j, \dots, r\}$, $\forall A = \{e_i, e_j, \dots, e_r\} \in Na$ and $f_2: Cl \to Y$ by $f_2(A) = \{i, j, \dots, r\}$, $\forall A = \{é_i, é_j, \dots, é_r\} \in Cl$. Then $f_1(K_{Na}) = f_2(K_{Cl}) = \lambda_1^{\beta} = t_1^{\gamma}$, $f_1(L_{Na}) = f_2(L_{Cl}) = \lambda_2^{\beta} = t_2^{\gamma}$, $f_1(M_{Na}) = \lambda_3^{\beta}$, $f_2(M_{Cl}) = t_3^{\gamma}$. Define $f: \{Na, Cl\} \to \{X, Y\}$ by $f(A) = \begin{cases} f_1(A), & if\ A \in Na - Cl \\ f_2(A), & if\ A \in Cl - Na \\ f_1(A) = f_2(A), & if\ A \in Na \cap Cl \end{cases}$. Since $17 = n > m = 11$, then we consider that $\omega_{\beta,\gamma}(j) = \begin{cases} \gamma(j), & if\ 1 \le j \le n \\ \beta(m + n + 1 - j), & if\ n + 1 \le j \le m + n \end{cases}$. As a consequence, we obtain the permutation $\omega_{\beta,\gamma} = \begin{pmatrix} 1 & 2 & 3 & . & . & . & 17 & 18 & 19 & 20 & . & . & . & 28 \\ 1 & 2 & 3 & . & . & . & 17 & 28 & 27 & 26 & . & . & . & 18 \end{pmatrix}$ in S_{28} is an extension permutation of β and γ.

Definition 10. Let $\omega_{\beta,\gamma}$ be an extension permutation of β and γ. For some $T = \delta_k^{\omega_{\beta,\gamma}} \in X = \{\delta_i^{\omega_{\beta,\gamma}} = \{\delta_1^i, \delta_2^i, \dots, \delta_{\alpha_i}^i\} | 1 \le i \le c(\omega_{\beta,\gamma})\}$ and the binary operation $\times$ on X, we say $(X, \times, \delta_k^{\omega_{\beta,\gamma}})$ is an extension permutation Q-algebra $(EP - Q - A)$ of β and γ if .

(5) $\delta_i^{\omega_{\beta,\gamma}} \times \delta_i^{\omega_{\beta,\gamma}} = \delta_k^{\omega_{\beta,\gamma}}$,

(6) $\delta_i^{\omega_{\beta,\gamma}} \times \delta_k^{\omega_{\beta,\gamma}} = \delta_i^{\omega_{\beta,\gamma}}$,

(7) $\delta_i^{\omega_{\beta,\gamma}} \times \delta_j^{\omega_{\beta,\gamma}}) \times \delta_h^{\omega_{\beta,\gamma}} = (\delta_i^{\omega_{\beta,\gamma}} \times \delta_h^{\omega_{\beta,\gamma}}) \times \delta_j^{\omega_{\beta,\gamma}}$, $\forall \delta_i^{\omega_{\beta,\gamma}}, \delta_j^{\omega_{\beta,\gamma}}, \delta_h^{\omega_{\beta,\gamma}} \in X$.

Example 7.
Suppose that β and γ are two permutations in Example 5. Hence $\omega_{\beta,\gamma} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 4 & 5 & 9 & 1 & 2 & 8 & 3 & 6 & 7 & 12 & 11 & 14 & 13 & 10 \end{pmatrix}$ is an extension permutation of β and γ. Also, $\omega_{\beta,\gamma} = (1\ 4)(2\ 5)(3\ 9\ 7)(6\ 8)(10\ 12\ 14)(13)$. Therefore, we have $X = \left\{\delta_i^{\omega_{\beta,\gamma}}\right\}_{i=1}^{6} = \{\{1,4\}, \{2,5\}, \{3,7,9\}, \{6,8\}, \{10,12,14\}, \{13\}\}$ and $T = \{1,4\}$. Define $\times: X \times X \longrightarrow X$ by table (4).

Table (4): $(X, \times, T)$ is a $(EP - Q - A)$.

$\times$	{1,4}	{2, 5}	{3,7,9}	{6,8}	{10, 12, 14}	{13}
{1,4}	{1,4}	{1,4}	{1,4}	{1,4}	{1,4}	{1,4}
{2, 5}	{2, 5}	{1,4}	{2, 5}	{1,4}	{1,4}	{1,4}
{3,7,9}	{3,7,9}	{3,7,9}	{1,4}	{1,4}	{1,4}	{1,4}
{6,8}	{6,8}	{1,4}	{6,8}	{1,4}	{1,4}	{6,8}
{10, 12, 14}	{10, 12, 14}	{10, 12, 14}	{10, 12, 14}	{10, 12, 14}	{1,4}	{10, 12, 14}
{13}	{13}	{3,7,9}	{2, 5}	{1,4}	{1,4}	{1,4}

Hence, $(X,\times,T)$ is a $(EP-Q-A)$.

Definition 11. Let $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ be $(EP-Q-A)$. We say that X is bounded with unit $\delta_h{}^{\omega\beta,\gamma}$, if $\delta_h{}^{\omega\beta,\gamma}\in X$ such that $\delta_i{}^{\omega\beta,\gamma}\times\delta_h{}^{\omega\beta,\gamma}=\delta_k{}^{\omega\beta,\gamma}$, $\forall\delta_i{}^{\omega\beta,\gamma}\in X$.

Example 8. See Example (7), we consider that $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ is $(EP-Q-A)$ of β and γ and bounded with unit {10, 12, 14}.

Definition 12. Let $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ be $(EP-Q-A)$. We say X is an extension permutation commutative Q-algebra $(EP-CQ-A)$ of β and γ, if $\delta_i{}^{\omega\beta,\gamma}\times\delta_j{}^{\omega\beta,\gamma}=\delta_j{}^{\omega\beta,\gamma}\times\delta_i{}^{\omega\beta,\gamma}$, $\forall\delta_j{}^{\omega\beta,\gamma},\delta_i{}^{\omega\beta,\gamma}\in X$.

Example 9.

Let $\beta=\begin{pmatrix}1&2&3&4&5\\5&4&2&3&1\end{pmatrix}$ & $\gamma=\begin{pmatrix}1&2&3&4&5&6&7&8\\8&3&5&6&2&4&7&1\end{pmatrix}$ be two permutations in $S_m=S_5$ and $S_n=S_8$, respectively. Since $\beta=(1\ 5)(2\ 4\ 5)$ and $\gamma=(1\ 8)(6\ 4)(2\ 3\ 5)(7)$. Since $n>m$, then we consider that $\omega_{\beta,\gamma}(j)=\begin{cases}\gamma(j), & if\ 1\le j\le n\\ \beta(m+n+1-j)+n, & if\ n+1\le j\le m+n\end{cases}$. Therefore, $\omega_{\beta,\gamma}=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13\\8&3&5&6&2&4&7&1&9&11&10&12&13\end{pmatrix}$ is an extension permutation in S_{13} of β and γ. Also, $\omega_{\beta,\gamma}=(1\ 8)(6\ 4)(2\ 3\ 5)(7)(9)(10\ 11)(12)(13)$. Therefore, we have $X=\left\{\delta_i^{\omega_{\beta,\gamma}}\right\}_{i=1}^{6}=$ {{7},{9}, {12}, {13},{10,11}, {1,8}, {4,6}, { 2,3,5}} and $T=\{7\}$. Define $\times: X\times X\longrightarrow X$ by table (5).

Table (5): $(X,\times,T)$ is a $(EP-CQ-A)$.

×	{7}	{9}	{12}	{13}	{10,11}	{1,8}	{4, 6}	{2,3,5}
{7}	{7}	{9}	{12}	{13}	{10,11}	{1,8}	{4, 6}	{2,3,5}
{9}	{9}	{7}	{10,11}	{1,8}	{10,11}	{1,8}	{2,3,5}	{2,3,5}
{12}	{12}	{10,11}	{7}	{4, 6}	{10,11}	{2,3,5}	{4, 6}	{2,3,5}
{13}	{13}	{1,8}	{4, 6}	{7}	{2,3,5}	{1,8}	{4, 6}	{2,3,5}
{10,11}	{10,11}	{10,11}	{10,11}	{2,3,5}	{7}	{2,3,5}	{2,3,5}	{2,3,5}
{1,8}	{1,8}	{1,8}	{2,3,5}	{1,8}	{2,3,5}	{7}	{2,3,5}	{2,3,5}
{4, 6}	{4, 6}	{2,3,5}	{4, 6}	{4, 6}	{2,3,5}	{2,3,5}	{7}	{2,3,5}
{2,3,5}	{2,3,5}	{2,3,5}	{2,3,5}	{2,3,5}	{2,3,5}	{2,3,5}	{2,3,5}	{7}

Hence, $(X,\times,T)$ is a $(EP-CQ-A)$.

Proposition 12. Let $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ be $(EP-Q-A)$. If X is bounded with unit $\delta_h{}^{\omega\beta,\gamma}$, then $\delta_h{}^{\omega\beta,\gamma}\neq\delta_k{}^{\omega\beta,\gamma}$.

Proof. Suppose that $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ is $(EP-Q-A)$ and X is bounded with unit $\delta_h{}^{\omega\beta,\gamma}$. Then $\delta_i{}^{\omega\beta,\gamma}\times\delta_h{}^{\omega\beta,\gamma}=\delta_k{}^{\omega\beta,\gamma}$, $\forall\delta_i{}^{\omega\beta,\gamma}\in X$. If $\delta_h{}^{\omega\beta,\gamma}=\delta_k{}^{\omega\beta,\gamma}$, then for any $\delta_i^{\omega\beta,\gamma}\in X-\delta_k{}^{\omega\beta,\gamma}$, we get $\delta_i^{\omega\beta,\gamma}\times\delta_h{}^{\omega\beta,\gamma}=\delta_i^{\omega\beta,\gamma}\times\delta_k{}^{\omega\beta,\gamma}=\delta_i^{\omega\beta,\gamma}$ [from (2) in Definition (10)]. However, $\delta_i{}^{\omega\beta,\gamma}\times\delta_h{}^{\omega\beta,\gamma}=\delta_k{}^{\omega\beta,\gamma}$[Since $\delta_h{}^{\omega\beta,\gamma}$ is unit] and this implies that $\delta_i^{\omega\beta,\gamma}=\delta_k{}^{\omega\beta,\gamma}$, but this contradiction with ($\delta_i^{\omega\beta,\gamma}\in X-\delta_k{}^{\omega\beta,\gamma}$). Hence $\delta_h{}^{\omega\beta,\gamma}\neq\delta_k{}^{\omega\beta,\gamma}$.

Proposition 13. Let $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ be $(EP-Q-A)$. If X is bounded with unit $\delta_h{}^{\omega\beta,\gamma}$, then $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ is not $(EP-CQ-A)$.

Proof. Assume that $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ is $(EP-Q-A)$ and X is bounded with unit $\delta_h{}^{\omega\beta,\gamma}$. Then $\delta_h{}^{\omega\beta,\gamma}\neq\delta_k{}^{\omega\beta,\gamma}$ [from Proposition 12]. Also, $\delta_k{}^{\omega\beta,\gamma}\times\delta_h{}^{\omega\beta,\gamma}=\delta_k{}^{\omega\beta,\gamma}$ [Since $\delta_h{}^{\omega\beta,\gamma}$ is unit] and $\delta_h{}^{\omega\beta,\gamma}\times\delta_k{}^{\omega\beta,\gamma}=\delta_h{}^{\omega\beta,\gamma}$ [from (2) in Definition (10)]. Now, assume that $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ is $(EP-CQ-A)$. Then $\delta_k{}^{\omega\beta,\gamma}\times\delta_h{}^{\omega\beta,\gamma}=\delta_h{}^{\omega\beta,\gamma}\times\delta_k{}^{\omega\beta,\gamma}$ [from Definition 12]. Then $\delta_k{}^{\omega\beta,\gamma}=\delta_h{}^{\omega\beta,\gamma}$, but this contradiction with Proposition 12. Hence $(X,\times,\delta_k{}^{\omega\beta,\gamma})$ is not $(EP-CQ-A)$.

4. CONCLUSION

This paper presents numerous novel extensions to Q-algebras, investigating their properties through the lens of non-classical sets, particularly soft sets. A method for establishing a correlation between the chemical structure of the silver atom and the concepts introduced here is detailed. Instead of relying on permutation sets in future research, the paper advocates the utilisation of non-classical sets like fuzzy sets and neutrosophic sets to enhance the precision of thoughts and outcomes. Furthermore, the research aims to introduce a fresh approach for exploring the relationships within the chemical structure of other elements, accompanied by the exploration of new mathematical concepts. Specifically, the paper endeavours to employ this knowledge to calculate the number of electrons in each electron shell, a crucial aspect for comprehending and discussing the properties of these elements.

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CONFLICTS OF INTEREST

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