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Fuzzy Decision by Opinion Score Method (FDOSM): A Systematic Review

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ABSTRACT: Multiple criterion decision-making (MCDM) has been widely utilized in everyday life in various ways, with countless success stories aiding in the analysis of complicated problems and the provision of an accurate decision process. To date, MCDM remains the best strategy to provide the finest solutions to solve complex problems in the field of specialized systems. Even so, several challenges faced MCDM approaches, as mentioned in academic literature. The most significant challenges are uncertainty and ambiguity. The fuzzy decision by opinion score method (FDOSM) is one of the most recent MCDM methods. Consequently, the purpose of this study was to examine and assess publications regarding the various types of developments of FDOSM in recent years and to collect the essential literature findings. Fundamentally, based on systematic review protocol, the searching process was conducted across four major databases, including IEEE Xplore (IEEE), ScienceDirect (SD), Scopus, and PubMed from the date of publishing of FDOSM article, based on particular Query "(MCDM OR 'Multi-criteria decision-making') AND (FDOSM OR 'Fuzzy decision by opinion score method') AND (Fuzzy) AND ('development method' OR 'developing'). The data collection process started on 23 August 2023 and ended on 30 October 2023, which included several scientific studies related to development FDOSM across several fuzzy types. These indexes are considered extensive and dependable enough to have the scope of our literature review. A final set of articles, n=22, was selected depended on predefined inclusion and exclusion criteria for this study. A coherent classification for current studies has been formed according to the development of FDOSM based on a new extension of a fuzzy set. Other aspects, such as the research method followed, the protocol adopted by the systematic review, and demographic statistics of the literature distribution, were included. Exciting patterns were observed, which were compiled and analyzed in tabular format according to their importance. The results of this literature systematic review provide a precise summary of each recent development concerning the FDOSM and its use, including: 1- Extracting the development types employed in the FDOSM method, 2- Extracting aggregation operator types, 3- Integration Method with FDOSM (hybrid with other methods), and 4- Case study types showing how FDOSM approaches may help decision-makers in numerous decisions. Finally, this study highlights research opportunities and encourages current and future efforts to better comprehend this field.

1. INTRODUCTION

Overall, machine learning (ML) and Decision-making (DM) play essential part in human activities because they have strengths in data analysis, prediction, optimization, and decision support, they are required in all aspects of life because of today's complicated fast world [1] [2] [3]. The decision-makers (DMs) face difficult choices with wide-ranging repercussions that involve significant risks and many factors [4]. A decision can be defined as a choice based on the facts at hand or a technique used to address a specific decision problem, taking into account the preferences, knowledge, and relevant data of the DMs [4, 5]. Multi-Criteria Decision Making (MCDM) can be mentioned that it is a method used to solve decision-making problems by utilizing numerous criteria [6] [7]. Also, understanding the varying development goals and purposes behind MCDM methods is crucial due to their variety [8]. Different of its techniques serve other functions [9] [10]. MCDM is a subspecialty in operations research [11], that aims to provide mathematical and computational tools to support decision-making processes to assist decision-makers in choosing the best alternative based on a set of preferences and priorities [12, 13].

The usage of MCDM is rapidly increasing due to its ability to improve decisions quality by making process decisions more effective, reasonable, and explicit when compared to traditional approaches [14]. It is possible to evaluate a limited number of decision alternatives based on a limited set of performance criteria due to the advanced

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methods of MCDM, also it faces various challenges [15] [16]. Fuzzy sets are preferable to precise values for calculating criteria to overcome these challenges, as proposed by Zadeh et al. [17].

MCDM methods can be categorized into two primary approaches [18], each of which gives criterion weighting called Fuzzy-Weighted Zero-Inconsistency (FWZIC), and/or alternative ranking called Fuzzy Decision by Opinion Score Method (FDOSM) [19]. The fuzzy decision by opinion score method FDOSM has been proposed to address these challenges [15, 20]. For MCDM in a fuzzy environment for ranking the alternatives, it's a new [21, 22], effective, and powerful method that was published in 2020 [23, 24]. FDOSM has been used in a wide range of studies to solve many multi-criteria decision-making problems [25, 26]. FDOSM addresses the challenges faced by both human and mathematical MCDM approaches [27, 28] (e.g., technique for order of preference by similarity to ideal solution [TOPSIS], VlseKriterijumska Optimizacija I Kompromisno Resenje [VIKOR], which included: (1) Inconsistency, which is the primary issue with the human approach [29], (2) reducing time consuming when performing comparisons [30], (3) avoiding unnatural comparisons among criteria [31], (4) resolving vagueness in the data by using fuzzy numbers [19], (5) resolving the distance measurement problem using positive and negative ideal solutions [32], and (6) in mathematical approaches, the issue of criterion weighting was eliminated [33]. FDOSM provides logical decisions based on expert opinions [34].

The objective of this study is to provide a comprehensive review one of the latest methods in multi-criteria decision- making namely Fuzzy decision by opinion score method (FDOSM) to find new directions for researchers. Furthermore, it is crucial to scan and gather existing information despite the relatively small number of studies in this field, in order to explore various techniques for developing MCDM methods using FDOSM alongside other approaches.

1.1 FDOSM Method Phases

FDOSM method depends on three phases: *Data Input Unit, Data Transformation Unit, and Data Processing Unit* [30] [35]. These phases will be described with the steps and mathematical equations. Figure 1 shows the flowchart of FDOSM phases, which can be presented below:

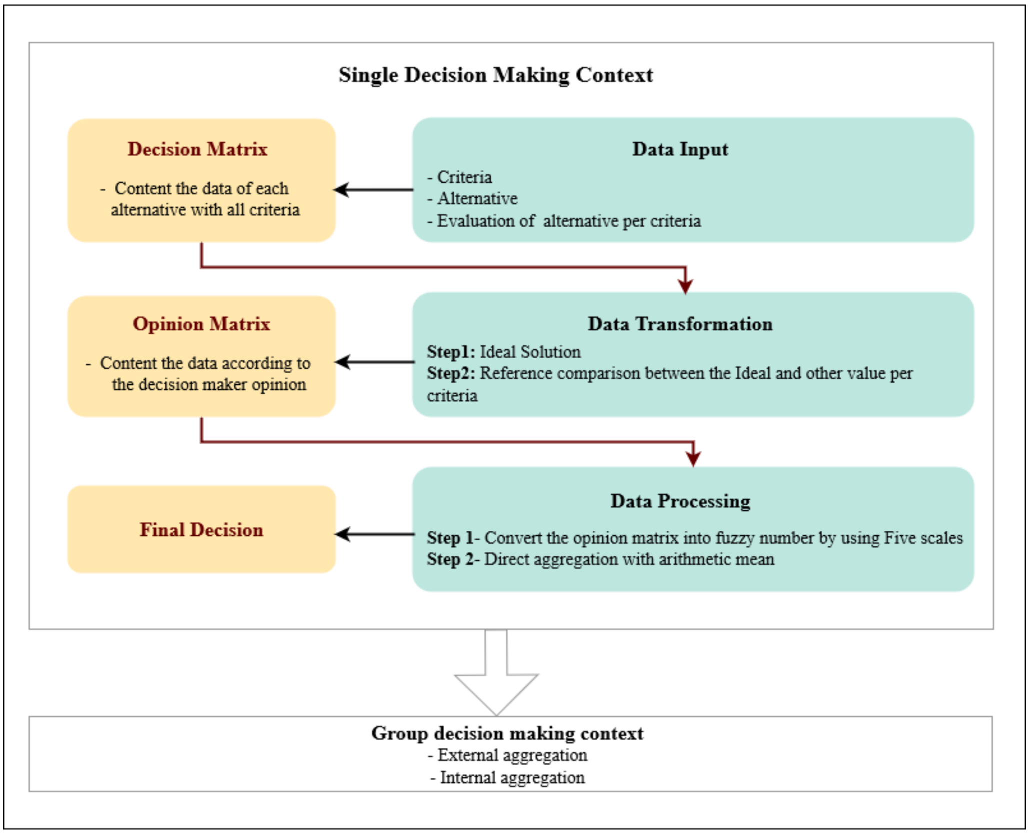

FIGURE 1. -FDOSM Phases

1.1.1 Phase 1: Data Input Unit

Like previous MCDM approaches, this approach also addresses MCDM issues involving (m) alternatives (A1, ..., Am) and (n) decision criteria set (C1, ..., Cn). Both of these components comprise the decision matrix.

$$D = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{array}{c} A_1 \\ \vdots \\ A_m \end{array} & \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \end{array} \quad (1)$$

Where x is any element in the decision matrix.
A decision matrix results from the first step, which is transformed into an opinion matrix in the subsequent stage.

1.1.2 Phase 2: Data Transformation Unit

Following the decision matrix created in the first block, FDOSM in this unit selects a three-parameter ideal solution (minimum, maximum, and critical values). The minimum value is used when dealing with cost criteria because a lower value indicates a better solution. In contrast, the maximum value is used for benefit criteria (higher values mean better solutions). However, in many real-life problems the ideal solution is not always benefit or cost criteria. Therefore, the critical value technique is used when the ideal solution does not the minimum or maximum values, like blood pressure. The following steps will be presented and explained at this stage:

Step 1: Select the ideal solution: As a result, the ideal solution is defined as follows:

$$A^* = \left\{\left[\left(\max_i v_{ij} \mid j \in J\right), \left(\min_i v_{ij} \mid j \in J\right), \left(Op_{ij} \in IJ\right) \mid i = 1,2,3, \dots . m\right]\right\} \quad (2)$$

Max denotes the ideal value for the benefit criterion, min for the costing criterion, and op_{ij} is the critical value when the ideal value resides between max and min, and (i) refers to the alternatives, and (j) for the criteria.
Step 2: Reference comparison: After the selection of the optimum solution in the previous step, the expert compares it to the remainder of the alternatives for each criterion, and the difference is characterized using one of the following linguistic terms: (No difference, slight difference, difference, big difference, and huge difference). The alternatives are compared with the ideal solution after the ideal solution chosen process.

$$Op_{\text{Lang}} = \left\{\left(\left(\tilde{v}_{ij} \otimes v_{ij} \mid j \in J\right) \cdot \mid i = 1.2.3 \dots . . m\right)\right\} \quad (3)$$

Where $\otimes$ refers to a reference comparison between both the ideal solution and the alternatives.

The result of this step is the linguistic term opinion matrix, which is currently ready to be converted into fuzzy numbers by fuzzy membership.

$$Op\_\text{Lang} = \begin{array}{c} A_1 \\ \vdots \\ A_m \end{array} \begin{bmatrix} op_{11} & \cdots & op_{1n} \\ \vdots & \ddots & \vdots \\ op_{m1} & \cdots & op_{mn} \end{bmatrix} \quad (4)$$

1.1.3 Phase 3: Data-Processing Unit

The following steps will describe this unit:

Step 1: The opinion terms in the opinion matrix are replaced with triangular fuzzy numbers (TFNs) to create a fuzzy decision matrix. The resulting matrix is the fuzzy opinion decision matrix (FDij).

Step 2: Aggregate the results of the preceding step for every alternative using an aggregation operator such as the arithmetic mean, after completing the fuzzy decision matrix, the aggregation process is used to identify the best alternative using one of these aggregating operators:

$$\text{Arithmetic mean } A_{m(x)} = \frac{\sum_{i=1}^{n} x_i}{n} \quad (5)$$

$$A_{m(x)} = \frac{\sum(a_f + a_m + a_l)(b_f + b_m + b_l)(c_f + c_m + c_l)}{n} \quad (6)$$

Step 3: Defuzzification of the aggregation results, using the centroid method, can be calculated as follows:

$$\frac{(a+b+c)}{3} \tag{7}$$

1.1.4 Group Decision Making

Group MCDM (G-MCDM) includes situation in which more than one decision maker is required to determine the best alternative. G-MCDM techniques methodically collect and combine the expertise and judgments of specialists from various disciplines [30]. External and internal aggregations are the two most common patterns proposed in the scholarly literature on group decision making. This study used both G-MCDM strategies to aggregate the implicit weights received from each decision maker and offer a final ranking of alternatives in order to provide a full view and thorough analysis of the FDOSM method [36].

FDOSM uses individual and group decision-making platforms. The FDOSM method is based on the ideal solution principle [37]. It allows experts (decision makers) to determine and choose the best value and compare it to others depending on the same criterion [22]. As a consequence, a different mathematical operation must be carried out to obtain the final rank and choose the best alternative from a group of accessible alternatives [38]. FDOSM attempts to handle ambiguous and fuzzy information utilizing triangular fuzzy numbers (TFNs) [39]. TFN, on the other hand, cannot address some real-world issues. Experts' opinions can be subjective and fuzzy, leading to ambiguous and uncertain information [40]. Various methods have been developed to address the complexities and conflicts inherent in real-world tasks, including fuzzy set (FS) methods, which can successfully manage such decision-making issues [41]. Therefore, FDOSM should be improved and extended to other fuzzy environments such as (Pythagorean fuzzy set (PFS) [42], T-spherical fuzzy sets (T-SFSs) [43], Q-rung orthopair fuzzy sets (q-ROFS) [20], Cubic Pythagorean fuzzy sets (CPFS) [25], neutrosophic fuzzy sets (NFSs) [40], interval type-2 trapezoidal (IT2T) [44], Fermatean fuzzy set (FFS) [27], q-rung orthopair hesitant FS (q-ROHFS) [41], 2-tuple fuzzy set [45], intuitionistic fuzzy sets (IFSs) [24], Fermatean probabilistic hesitant-fuzzy sets (FPHFSs) [46], spherical fuzzy rough set (SFRSs) [47], q-ROF rough sets (q-ROFRSs) [48], dual hesitant fuzzy sets (DHFS) [15], Rough Fermatean fuzzy sets [49],Hexagonal-Fuzzy Number [HFN] [21], z-cloud rough numbers [37], to effectively address ambiguity issues and obtain extra useful information [40].

Following up on research findings and methodologies developed and integrated with other instruments has become a time-consuming activity that takes much of the study effort. Also, it is considered a big challenge for a researcher to track thousands of academic publications. This study aims to provide comprehensive and up-to-date articles on FDOSM under a fuzzy environment. Identify and classify existing methods, discuss their benefits, challenges, and limitations, and highlight the literature recommendations. This systematized review could present a starting point for many researchers based on the study's findings, which are intended to assist scholars in understanding and advancing the MCDM field and provide decision-makers with a toolkit for addressing complex decision problems in a fuzzy environment.

1.2 Paper Organization

The arrangement of this paper is organized as follows: Section 2 describes the technique used for this systematic review protocol, focusing on Information Source, Search Strategy, Study Selection, and Inclusion and Exclusion Criteria. Discussion in section 3 presents the acquired results and their corresponding consequences in subsections: Fuzzy Set Number, Aggregation Operators, The Integration Method, and Case Studies. Then, Section 4 presents the recommendations of the literature that extracting ideas for the current issues. Section 5 presents the limitations of this study. Finally, Conclusion summarizes the Leverage existing vital information to explore various approaches for enhancing MCDM methods by integrating FDOSM with other techniques.

2. SYSTEMATIC REVIEW PROTOCOL

In performing an extensive significant search and comprehensive coverage of articles during a systematic literature review study, reliance on searching in a single literature database should be avoided, as it is unlikely that all relevant references will be in a single database [50]. The systematic literature review is a well-organized protocol compared to traditional review methods and improves research synthesis by identifying relevant studies based on predetermined criteria. It's method process consists of several major steps: establishing the scope, research mechanism development, choosing and extracting the study, and merging the information. As a result, it is an advanced strategy that can be used in a wide range of research fields and scientific disciplines [51]. It is adopted in this review to study FDOSM extension in many fields and highlighted the most effective methods that integrated with FDOSM to reduce the most MCDM challenges.

2.1 Information Source

The four reliable databases IEEE Xplore (IEEE), ScienceDirect (SD), Scopus, and PubMed have been used in this study to search for relevant articles. This review provides evident and precise information for additional research and analysis required by researchers in their field of expertise and the extent of its development and interaction with other disciplines. Only studies that used FDOSM method were included in this review. These databases comprehensively cover scientific and technological research conducted in this field. The four databases selected for the study were highly relevant, academically resilient, scientifically sound, considered adequate, and most appropriate for the current review.

2.2 Search Strategy

The search started on 23 August 2023 and concluded on 30 October 2023 using the suitable Query: "(MCDM OR 'Multi-criteria decision-making') AND (FDOSM OR 'Fuzzy decision by opinion score method') AND (Fuzzy) AND ('development method' OR 'developing'). Specific pre-defined keywords were chosen based on the most papers that utilized FDOSM development terms. The search was conducted via four scientifically reliable databases: IEEE Xplore (IEEE), ScienceDirect (SD), Scopus, and PubMed to apply the study that involved: searching, downloading, filtering, extracting, and drafting. To gain the most recent studies from 2020 to 2023 in the specific study topic by exact keywords, which has been adopted by performing it many times before building the final review query. The limitations of this study, only a few articles that introduced MCDM development methods to apply the systematic review scanning, which is not enough to provide a clear ideal view for solving MCDM problems involving developed approaches with FDOSM in a fuzzy environment. Also, three related topic papers couldn't be downloaded.

2.3 Study Selection

Three steps have been achieved in the study selection. The first step was to remove duplicates that were (5) articles from all the studies initially collected (74) for further investigation. To do so, Endnote reference management software was used to automatically identify and exclude articles that appear in more detail. Second step, the extracted articles' titles and abstracts are scanned n=8, using the study inclusion and exclusion criteria to select related studies in the final round. Third step, for each paper that fulfills our inclusion criteria, full-text reading was performed to obtain useful information and examine it for this review by creating a valuable table that contains the topic's info. Accordingly, the final set of relevant articles for this review consists of (22), where (39) articles do not match our inclusion criteria and are excluded, as shown in Figure 2.

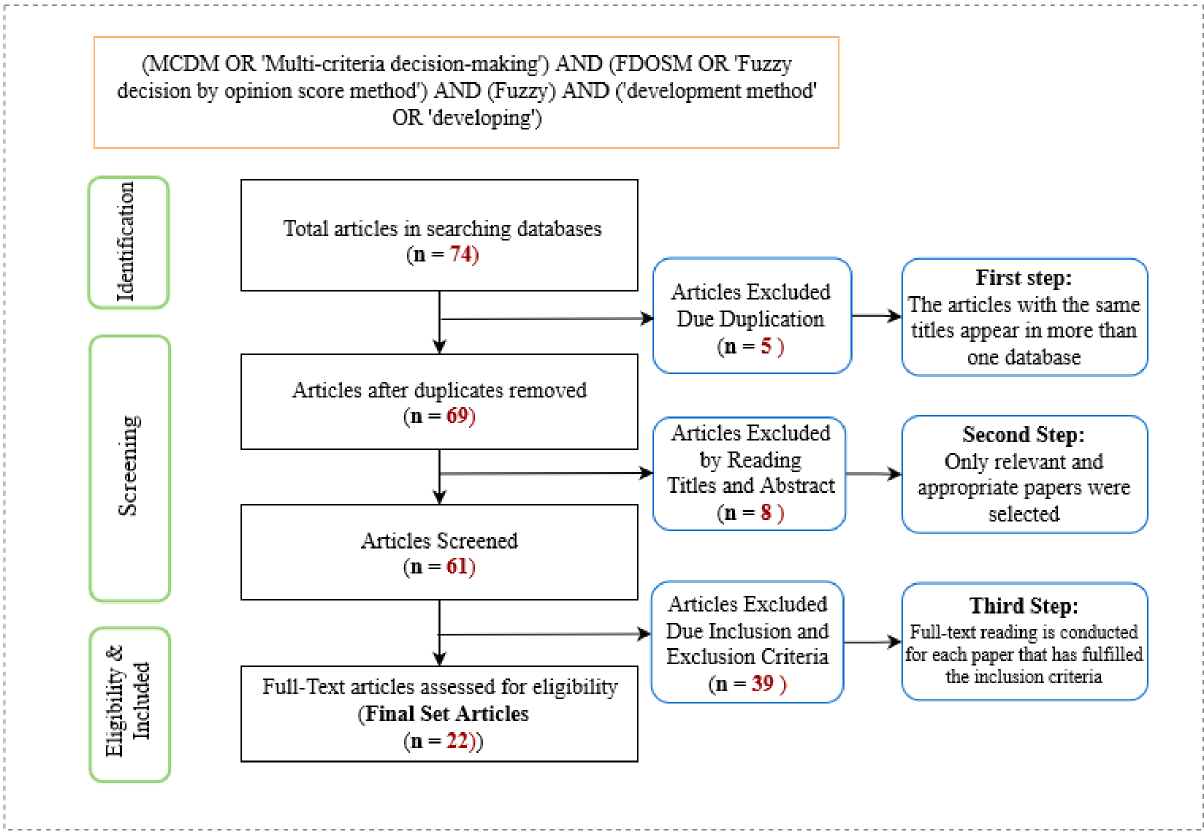

Figure 2. - Flowchart of the Search Query and Inclusion Criteria with Filtering Process

Table 1 presents a comprehensive summary of the selected articles within the scope of our study.

Table 1. - The Related Studies

	Research Ref	Year	The integrated method
1	[20]	2022	FDOSM+FWZIC
2	[56]	2022	FDOSM+FWZIC
3	[19]	2022	FDOSM+FWZIC
4	[43]	2021	FDOSM+FWZIC
5	[25]	2021	FDOSM+FWZIC
6	[40]	2022	FDOSM+FWZIC
7	[44]	2021	N/A
8	[27]	2022	N/A
9	[41]	2023	FDOSM+FWZIC + MULTIMOORA
10	[45]	2022	N/A
11	[24]	2021	N/A
12	[46]	2023	FDOSM+FWZIC + multi attributive ideal-real comparative analysis (MAIRCA)
13	[47]	2023	FDOSM+FWZIC
14	[48]	2023	FDOSM+FWZIC
15	[15]	2023	FDOSM+FWZIC
16	[49]	2023	FDOSM+FWZIC
17	[21]	2023	N/A
18	[37]	2023	FDOSM+FWZIC
19	[22]	2022	FDOSM+FWZIC
20	[38]	2022	FDOSM+ CRITIC
21	[32]	2023	FDOSM+ CRITIC
22	[42]	2023	N/A

2.4 Inclusion and Exclusion Criteria

The selection criteria utilized in this systematic review were critical to ensuring that the review focused on high-quality related studies. Table 2 outlines the criteria used for determining the inclusion or exclusion related to articles concentrated on MCDM methods and the development of FDOSM methods. The table is constructed based on three columns: Main Criteria, Inclusion Criteria, and Exclusion Criteria. The main column contains the core classifiers related to the topic used to choose or exclude articles such as Selected Databases, Language, Topic, Direct Application, Paper Types, Subject Areas, Access Type, and Published Years. The Inclusion and Exclusion Criteria columns refer to specific information based on the core classifiers that must be taken in the study or not, as shown below:

Table 2. - Inclusion and Exclusion Criteria

Main Criteria	Inclusion Criteria	Exclusion Criteria
Selected Databases	IEEE Xplore (IEEE), ScienceDirect (SD), Scopus, and PubMed only	Other
Language	English language Articles only	Non- English language Articles
Topic	Articles that focused on a specific part of MCDM	Other
Direct Application	Articles that used development of MCDM methods with FDOSM	Does not include FDOSM method
Paper Types	Review Articles, Research Articles, and Conferance Papers	Other
Subject Areas	Computer Science	Other
Access Type	Open access	Non-Open access
Published Years	2020-2023	Older than 2020

The table above defined the particular criteria used to narrow down the selection of articles for the study, ensuring that only relevant and appropriate papers were selected for analysis. This facilitated the reviewers' discovery and selection of several studies aligned with the primary goals and objectives of the systematic review. The use of inclusion criteria contributed to ensuring a thorough and severe assessment, which enhanced the value of the results in the MCDM field.

3. DISCUSSION

The discussions for the conducted state-of-the-art studies of FDOSM developed with MCDM methods in a fuzzy environment have been presented in this section. Practically, the initial result of the search query showed (74) relevant articles, but the final set became (22) after applying two filtering processes. The collected final set of articles was studied comprehensively to cope with all the technical and scientific techniques of the current study topic. Concerns, such as Fuzzy Set Number, Aggregation Operators, and The Integration Method, were identified and categorized in the literature of this study, as presented below:

3.1 Fuzzy Set Number

Zadeh was the first to propose fuzzy set theory (1965) [52] [53], To deal with the ambiguity and imprecision that are inherent in human judgment. the fuzzy set was formed also with the goal of improving information accuracy and decision-making reliability [25].

3.1.1 Fuzzy Sets

To comprehend the principles of fuzzy sets, need to first understand the fundamental concept in classical set theory. A classical set is a relatively simple concept in mathematics. A set is a collection of well-defined items. These objects, which can comprise a variety of items, can be included or excluded from the set [54] .The function μA(x) determines if a member x belongs to the classical set A or not [21].

$$\mu_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$

Hence $\mu_A(x) \in [0, 1]$. The function $\mu_A(x)$ takes only the values 1 or 0

while the concept of a fuzzy set can be defined more precisely as one of the different forms of logic used to express a specific thing that cannot be compensated for with an exact value , by applying a function known as a membership function, fuzzy logic sets a numerical value between 0 and 1 to reflect the degree of membership of items in order to infer uncertain items more efficiently and accurately [55].

A fuzzy set R is described:

$$R = \{(x, \mu R(x))/x \in A, \mu R(x) \in [0,1]\} \quad (8)$$

Where μR(x) is a membership function, μR(x) determines the grade to which every element of A belongs to the fuzzy set R [54].

The development types that are used in the FDOSM method based on the Fuzzy Set that is investigated in this study are as follows:

1- q-rung orthopair fuzzy set (q-ROFS)

The q-rung orthopair fuzzy set (q-ROFS) is utilized to address the drawbacks of information expression in conventional fuzzy sets (i.e., intuitionistic fuzzy set (IFS) and Pythagorean fuzzy set (PFS)) [56]. In q-ROFSs, the limitation of other fuzzy sets is eliminated, and the sum of the q powers of membership and non-membership grades are real numbers between the [0, 1]. As a result, the DMs are free to choose any grade for μ and υ ($\mu \in [0,1]$" and " $v \in [0,1]$) and any place [57]. Utilizing the benefits of q-ROFS in handling with unsure conditions by offering more scope in data representation and structuring efficiently [20]. The value of each linguistic term with q-ROFS is shown in Table 3.

Table 3. - q-ROF Opinion Matrix [20]

Linguistic scale	q-ROFS
No Difference	(0.90, 0.20)
Slight Difference	(0.80, 0.45)
Difference	(0.65, 0.50
Big Difference	(0.40, 0.60)
Huge Difference	(0.20, 0.90)

The following points describe the fundamental arithmetic operations with q-ROFS.

(1) The fuzzy opinion matrices resulting are aggregated using the equation of the q-rung orthopair fuzzy weighted arithmetic mean (q-ROFWA) aggregation operation (9).

$$q-\text{ROFWA}\,(\tilde{a}_1,\tilde{a}_2,\dots,\tilde{a}_n)=\left\langle\left(1-\prod_{k=1}^{n}\left(1-\mu_k^q\right)^{wk}\right)^{1/q},\prod_{k=1}^{n}v_k^{w_k}\right\rangle \tag{9}$$

(2) The q-ROFS division operation is shown in (10) as follows:

$$p_1/p_2=\left(\frac{\mu_1}{\mu_2},\sqrt{\frac{v_1^q-v_2^q}{1-v_2^q}}\right),\text{ if } \mu_1\le min\left\{\mu_2,\frac{\mu_2\pi_1}{\pi_2}\right\},v_1\ge v_2. \tag{10}$$

(3) Shows the equation of q-ROFS division on a crisp value.

$$p/\lambda=\left(\sqrt[4]{1-\left(1-(\mu_p)^q\right)^{\frac{1}{t}}},(v_p)^{\frac{1}{\lambda}}\right),\lambda > 0. \tag{11}$$

2- Pythagorean fuzzy number (PFN)

Pythagorean fuzzy number (PFN) is a novel evaluation format determined by membership and non-membership status, the total of which is less than or equal to 1 [58]. The PFN was recently developed as a modern version of an intuitionistic fuzzy set to address the troublesome uncertainty in group decision problems [59]. PFN appeared as an effective approach to represent the fuzziness and uncertainty of MCDM issues, with the Pythagorean fuzzy set (PFS) being more general than the intuitionistic fuzzy set (IFS) [60].

About the distinctness, the PFS must satisfy the condition that the square sum of the membership degree and the non-membership degree is equal to or less than one. At the same time, the IFS needs to achieve the condition that the sum of both degrees is equal to or less than 1[19]. The value of every linguistic term with PFS is shown in Table 4.

Table 4. - Pythagorean Fuzzy Opinion Matrix [19]

Linguistic scale	PFNs
No Difference	(0.90, 0.20)
Slight Difference	(0.80, 0.45)
Difference	(0.65, 0.50)
Big Difference	(0.40, 0.60)
Huge Difference	(0.20, 0.90)

The following are the definitions of PFNs and their operational laws:

Definition 1 [42]. Let Z be a universal set, and a PFS P is defined as:

$$P=\{\langle z,(\mu_P(z),v_P(z))\rangle \mid z\in Z\}, \tag{12}$$

Where $\mu_P(z)$ and $v_P(z)$ express the membership and non-membership grades of the element z ∈ Z in the set P. The functions $\mu_P(z)$, $v_P(z)$:

$Z \rightarrow [0,1]$ satisfy the condition $0 \leq (\mu_P(z))^2 + (v_P(z))^2 \leq 1$. The indeterminacy grade is πP(z) = $\sqrt{1-(\mu_P(z))^2-(v_P(z))^2}$. PFN is represented by α = (μα, vα) a PFN, where μα ∈ [0, 1], vα ∈ [0, 1] and 0 ≤ $\mu_\alpha^2 + v_\alpha^2 \leq 1$.

Definition 2 [61]. Let α = (μα, vα), α1 = $(\mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$ be three PFNs, and their operational laws are appear as following:

i) αc = (vα, μα),

ii) $\alpha_1 \oplus \alpha_2 = \left(\sqrt{(\mu_{\alpha_1})^2 + (\mu_{\alpha_2})^2 - (\mu_{\alpha_1})^2(\mu_{\alpha_2})^2}, v_{\alpha_1}v_{\alpha_2}\right)$,

iii) $\alpha_1 \otimes \alpha_2 = \left(\mu_{\alpha_1}\mu_{\alpha_2}, \sqrt{(v_{\alpha_1})^2 + (v_{\alpha_2})^2 - (v_{\alpha_1})^2(v_{\alpha_2})^2}\right)$,

iv) $\lambda\alpha = \left(\sqrt{1-(1-(\mu_\alpha)^2)^\lambda}, (v_\alpha)^\lambda\right), \lambda > 0$,

v) $\alpha^\lambda = \left((\mu_\alpha)^\lambda, \sqrt{1-(1-(v_\alpha)^2)^\lambda}\right), \lambda >$ 0.

Let αi= (μαi, vα1) (i =1, 2, . . . , n) be the set of PFNs and p, q ≥ 0 where

$$\varphi_i = \frac{(1+T(\alpha_i))}{\sum_{i=1}^{n}(1+T(\alpha_i))} \,. \tag{13}$$

$$T(\alpha_i) = \sum_{j=1, j\neq i}^{n} \mathrm{Sup}(\alpha_i, \alpha_j), (i = 1,2, \ldots, n). \tag{14}$$

$$\mathrm{Sup}(\alpha_i, \alpha_j) = \left(1 - d(\alpha_i, \alpha_j) + \left|v_{\alpha_i}{}^2 - v_{\alpha_j}{}^2\right| + \left|\pi_{\alpha_i}{}^2 - \pi_{\alpha_j}{}^2\right|\right). \tag{15}$$

$$d(\alpha_i, \alpha_j) = \frac{1}{2}\left|\mu_{\alpha_i}{}^2 - \mu_{\alpha_j}{}^2\right|. \tag{16}$$

$$\pi_{\alpha_i} = \sqrt{1 - \mu_{\alpha_i}{}^2 - v_{\alpha_i}{}^2} \tag{17}$$

The fuzzy opinion matrices resulting aggregated using the formulation of the AM operator in (18).

$$\mathrm{PFWA}(p(m_1), p(m_2), \cdots, p(m_n)) = \left(\sum_{i=1}^{n} w_i \mu_p(m_i), \sum_{i=1}^{n} w_i v_p(m_i)\right) \tag{18}$$

3- T-spherical fuzzy sets (T-SFSs)

Due to the constraints faced by the Pythagorean fuzzy number and other kinds of fuzzy sets, a novel concept of T-spherical fuzzy sets (T-SFSs) has been developed. The T-SFS structure is broader and more general, with no constraints on their constants, and it can handle uncertainty in data to capture information with a greater degree of freedom [62]. T-SFS is widely used in various fields to solve many MCDM problems, and this method is better capable of addressing and representing unknown information in unknown environments. As a result, to stay up with the current status of solving uncertainty and vagueness issues, FDOSM methods must be extended into the T-SFSs environment (referred to as T-spherical FDOSM [T-SFDOSM]) [43]. Table 5 shows the value of every linguistic term with T-SFSs.

Table 5. - T-SF Opinion Matrix [43]

Linguistic scale	T-SFS
No difference	(0.85, 0.15, 0.1)
Slight difference	(0.75, 0.25, 0.2)
Difference	(0.55, 0.5, 0.25)
Big difference	(0.25, 0.75, 0.2)
Huge difference	(0.15, 0.85, 0.1)

The T-SFS is defined in (19) and (20).

$$P = \{\langle m, (\mu_d(m), v_d(m), s_d(m))\rangle \mid m \in M\}, \tag{19}$$

Where $u_d, M \to [0,1]$ is the membership function, and $v_d: M \to [0,1]$ is a non-membership function of element $m \in M$, and $s_d: M \to [0,1]$.

$$0 < (\mu_d(m))^T + (v_d(m))^T + (s_d(m))^T \leq 1, \tag{20}$$

where T ≥ 1

The applied arithmetic operation using T-SFS use the following equations. T-SFS summation and aggregation operations can be seen in (21).

$$\text{T- SWAM}(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_n) = \left\{\left[1 - \prod_{i=1}^{n} \left(1 - \mu_{\tilde{p}_i}^2\right)^{w_i}\right]^{1/T} \prod_{i=1}^{n} v_{\tilde{p}_i}^{w_i}, \left[\prod_{i=1}^{n} \left(1 - \mu_{\tilde{p}_i}^2\right)^{w_i} - \prod_{i=1}^{n} \left(1 - \mu_{\tilde{p}_i}^2 - s_{\tilde{p}_i}^2\right)^{w_i}\right]^{1/T}\right\} \tag{21}$$

The division operation was performed using (22) [63].

$$p_1 \oslash p_2 = \left(\left(\frac{(\mu_{p1}^{T}(2-\mu_{p2}^{T})}{1-(1-\mu_{p1}^{T})\cdot(1-\mu_{p2}^{T})}\right)^{\frac{1}{T}} \frac{(v_{p1}^{T}-v_{p2}^{T})^{\frac{1}{T}}}{(1-v_{p1}^{T}\cdot v_{p2}^{T})^{\frac{1}{T}}}, \frac{(s_{p1}^{T}-s_{p2}^{T})^{\frac{1}{T}}}{(1-s_{p1}^{T}\cdot s_{p2}^{T})^{\frac{1}{T}}}\right) \quad \text{if } \frac{\mu_{p2}^{T}}{\mu_{p1}^{T}} \geq \frac{1-s_{p2}^{T}}{1-s_{p1}^{T}} \frac{1+s_{p1}^{T}}{1+s_{p2}^{T}} \geq 1 \tag{22}$$

(23) shows the equation of T-SFS division on crisp value.

$$\tilde{P} \oslash \lambda = \left\{\left(1 - \left(1 - \mu_{\tilde{P}}^{T}\right)^{1/\lambda}\right)^{1/T}, v_{\tilde{P}}^{1/\lambda}, \quad s_{\tilde{P}}^{1/\lambda}\right\} \tag{23}$$

where $\lambda > 0$.

4- Cubic Pythagorean Fuzzy Sets (CPFS)

CPFS is one of the most powerful methods for dealing with uncertainty issues, particularly in complicated cases. It was developed by utilizing interval-valued Pythagorean fuzzy sets (IVPFSs) and PFSs to express vagueness or ill-defined information [64]. The preceding benefits make CPFS a strong tool, and CPFS offers more sophisticated mathematical expressions that use both PFS and IVPFS all at once [25]. Table 6 presents the value of each linguistic term using *CPFS*.

Table 6. - Linguistic Terms with Equivalent CPFNs [25]

Linguistic scale	CPFNs
Extremely no difference	(0.9, 1, 0, 0.1, 1, 0)
Huge no difference	0.8, 0.9, 0.2, 0.25, 0.9, 0.2)
Very no difference	(0.7, 0.8, 0.25, 0.35, 0.8, 0.25)
Medium no difference	(0.6, 0.7, 0.35, 0.5, 0.7, 0.35)
No difference	(0.5, 0.6, 0.5, 0.6, 0.6, 0.5)
Difference	(0.4, 0.5, 0.6, 0.7, 0.5, 0.6)
Medium difference	(0.35, 0.45, 0.7, 0.75, 0.45, 0.7)
Very difference	(0.3, 0.4, 0.75, 0.8, 0.4, 0.75)
Huge difference	(0.1, 0.2, 0.8, 0.9, 0.2, 0.8)
Extremely difference	(0, 0.1, 0.9, 1, 0.1, 0.9)

(24) and (25) are the equations that are used to define CPFS.

$$= (\tilde{p}_{\tilde{c}}, p_{\tilde{c}}) = \left(\langle [\mu^L_{\tilde{c}}, v^L_{c}], [\mu^U_{\tilde{c}}, v^U_{\tilde{c}}] \rangle ; (\mu_{\tilde{c}}, v_{\tilde{c}})\right), \tag{24}$$

Where $\mu^L_{\tilde{c}}, \mu^U_{\tilde{c}} : M \to [0,1]$is the lower and upper bound of the membership function, whereas $v^L_{\tilde{c}}, v^U_{\tilde{c}} : M \to [0,1]$ is the lower and upper bound of the non-membership function. of element $m \in M$to p, and it must satisfy the constraint shown in (25).

$$0 < \mu^L_{\tilde{c}}(m)^2 + v^L_{c}(m)^2 \leq 1, 0 < \mu^U_{\tilde{c}}(m)^2 + v^U_{c}(m)^2 \leq 1$$
$$\text{and } 0 < \mu_{\tilde{c}}(m)^2 + v_{c}(m)^2 \leq 1. \tag{25}$$

Cubic Pythagorean fuzzy average mean (CPFA) Aggregation Operation can be seen in (26) as following:

$$\mathrm{CPFWA}(\mathrm{e}_1 e_2 \ldots, \mathrm{e}_\mathrm{h}) = \begin{pmatrix} \left\langle \left[\sqrt{1 - \Pi^{\mathrm{n}}_{\mathrm{i}=1} \left(1 - (\mu^{\mathrm{L}}_{\mathrm{e_i}})^2\right)^{\mathrm{w_i}}}, \sqrt{1 - \Pi^{\mathrm{n}}_{\mathrm{i}=1} \left(1 - (\mu^{\mathrm{U}}_{\mathrm{e_i}})^2\right)^{\mathrm{w_i}}} \right] \right. \\ \left[\Pi^{\mathrm{n}}_{\mathrm{i}=1} (\mathrm{v}^{\mathrm{L}}_{\mathrm{e_i}})^{\mathrm{w_i}}, \Pi^{\mathrm{n}}_{\mathrm{i}=1} (\mathrm{v}^{\mathrm{U}}_{\mathrm{e_i}})^{\mathrm{w_i}}\right] \\ \sqrt{1 - \Pi^{\mathrm{n}}_{\mathrm{i}=1} \left(1 - (\mu^{\mathrm{L}}_{\mathrm{e_i}})^2\right)^{\mathrm{w_i}}}, \prod^{\mathrm{n}}_{\mathrm{i}=1} (\mathrm{v}_{\mathrm{e_i}})^{\mathrm{w_i}} \end{pmatrix} \tag{26}$$

(27) shows the CPFS *Division Operation* as following:

$$\frac{p_1}{p_2} = \left(\frac{\mu^{\mathrm{L}}_1}{\mu^{\mathrm{L}}_2}, \frac{\mu^{\mathrm{U}}_1}{\mu^{\mathrm{U}}_2}, \sqrt{\frac{v^{\mathrm{L}}_1 - v^{\mathrm{L}}_2}{1 - v^{\mathrm{L}}_2}}, \sqrt{\frac{v^{\mathrm{U}}_1 - v^{\mathrm{U}}_2}{1 - v^{\mathrm{U}}_2}}, \frac{\mu_1}{\mu_2}, \sqrt{\frac{v_1 - v_2}{1 - v_2}} \right)$$
$$\text{if } \mu^{\mathrm{L}}_1 \leq \boldsymbol{min}\left\{\mu^{\mathrm{L}}_2, \frac{\mu^{\mathrm{L}}_2 \pi^{L}_1}{\pi^{L}_2}\right\}, v^{L}_1 \geq v^{L}_2$$
$$\mu^{\mathrm{U}}_1 \leq \mathbf{m\,i\,n}\left\{\mu^{\mathrm{U}}_2, \frac{\mu^{\mathrm{U}}_2 \pi^{U}_1}{\pi^{U}_2}\right\}, v^{U}_1 \geq v^{U}_2$$
$$\mu_1 \leq \mathbf{m\,i\,n}\left\{\mu_2, \frac{\mu_2 \pi_1}{\pi_2}\right\}, v_1 \geq v_2. \tag{27}$$

(28) shows the equation of CPFS division on crisp value.

$$\frac{p}{\lambda} = \begin{pmatrix} \left\langle \left[\sqrt{1 - \left(1 - (\mu^{L}_{p})^2\right)^{\frac{1}{\lambda}}}, \sqrt{1 - \left(1 - (\mu^{U}_{p})^2\right)^{\frac{1}{\lambda}}} \right], \left[(v^{L}_{p})^{\frac{1}{\lambda}}, (v^{U}_{p})^{\frac{1}{\lambda}} \right] \right\rangle \cdot \\ \left(\sqrt{1 - (1 - \mu^2_p)^{\frac{1}{\lambda}}}, v^{\frac{1}{\lambda}}_{p} \right) \end{pmatrix} \tag{28}$$

5- Neutrosophic Fuzzy Sets (NFSs)

FDOSM must be enhanced and extended in another fuzzy environment to effectively tackle ambiguity issues because triangular fuzzy numbers (TFNs) can't handle many real-world problems [40]. Recently, neutrosophic fuzzy sets (NFSs) were proposed. NFSs enabled decision-makers handle with neural thinking knowledge[65]. Because of the neutrality of this type, it is feasible to add additional functionality to model ambiguous information [66]. Table 7 displays the value of every linguistic term using NFNS.

Table 7. - Linguistic Terms with Equivalent NFNs [40]

	NFNs		
Linguistic terms	**P**	**Σ**	**T**
No difference	0.05	0.95	0.95
Slight difference	0.25	0.75	0.75
Difference	0.50	0.50	0.50
Big difference	0.75	0.25	00.25
Huge difference	0.95	0.05	0.05

NFS is presented by [67] and defined in (29).

$$N = \{x, \rho_N(x), \sigma_N(x), \tau_N(x) \mid x \in X\} \quad (29)$$

Where X denotes a discourse universe and N denotes a simplified neutrosophic set (SNS). N in X is signified by a truth membership function. $\rho_N(x)$, function of indeterminacy-membership σN, as well as a falsity-membership function N (x), where functions ρN (x), σN (x) and τN (x) are singleton subintervals/subsets in the real standard interval [0, 1], such that ρN (x) : X → [0, 1], σN (x) : X → [0, 1] and τN (x) : X → [0, 1].

An essential arithmetic operation with NFNS is shown as the following:

Summation and Aggregation Operation:

Where SNWG (neutrosophic weighted geometric) represents an average operator.
Let a˜ j u j , p j ,
v j (j 1, 2, ... , n) be a collection of SNNs, and SNG: $Q_n \to Q$, if

$$\begin{aligned}\mathrm{SNWG}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \\ = \prod_{j=1}^{n} \tilde{a}_j^{\omega_j} \\ = \left(\prod_{j=1}^{n} \rho_j^{\omega_j}, 1 - \prod_{j=1}^{n} (1-\sigma_j)^{\omega_j}, 1 - \prod_{j=1}^{n} (1-\tau_j)^{\omega_j}\right),\end{aligned} \quad (30)$$

Where ωj is the weight of a˜ j(j = 1, 2, ... , n), ωj∈ [0, 1] and $\sum_{j/=1}^{n} \omega_j = 1$

Division operation:
For any two given SNSs X and Y, the division operation of SNSs X and Y is defined as following:

$$\frac{x}{y} = \left\{\left\langle z, \frac{\rho_x(z)}{\rho_y(z)}, \frac{\sigma_x(z)-\sigma_y(z)}{1-\sigma_y(z)}, \frac{\tau_x(z)-\tau_y(z)}{1-\tau_y(z)} \right\rangle \mid z \in Z\right\}, \quad (31)$$

Which is valid under the conditions Y ≥ X, $\rho_x(z) \neq 0$, $\sigma_y(z) \neq 1$, and $\tau_y(z) \neq 1$.

6- Interval type-2 trapezoidal (IT2T)

Because Fuzzy type 1 has constraint, such as confirming that a type-1 fuzzy set yields a crisp value for every input membership degree [68]. Furthermore, determining membership values directly is difficult [69]. One of the problems of fuzzy type-1, on the other hand, is the inability to directly model and minimize the influence of data uncertainties. As a result, fuzzy type-2 is the most extensively utilized since it can model second-order uncertainty and is computationally simple. The conversion of linguistic terms into IT2T fuzzy set is shown in Table 8.

Table 8. - Conversion of Linguistic Terms into IT2T Fuzzy Set [44]

Linguistic Terms	IT2T Fuzzy Sets
No difference	(0,0,0,0.1;1,1) (0,0,0,0.5;0.9,0.9)
Slight difference	((0,0.1,0.1,0.3;1,1) (0.05, 0.1, 0.1, 0.2; 0.9, 0.9))
Difference	((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))
Big difference	((0.7, 0.9, 0.9,1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9,0.9))
Huge difference	((0.9, 1, 1,1; 1, 1), (0.95, 1, 1,1; 0.9, 0.9))

Fuzzy type-2 is also important in determining the exact membership function [44]. A fuzzy type-2 method was presented, followed by a brief review of various definitions of type-2 fuzzy sets and interval type-2 trapezoidal fuzzy sets.

Definition 1. In the universe of discourse X, A type-2 fuzzy set $\overset{\approx}{A}$ can be represented by a type-2 membership function $\mu_{\overset{\approx}{A}}$, as shown below [70]:

$$\underset{A}{\approx} = \{(x,u), \mu_{\tilde{\tilde{A}}}(x,u) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1\}, \quad (32)$$

Here Jx denote an interval in [0, 1]. The type-2 fuzzy set $\overset{\approx}{A}$ can also be presented as following:

$$\underset{A}{\approx} = \int_{x\in X} \int_{u\in U} \mu_{\overset{\approx}{A}}(x,u)/(x,u), \quad (33)$$

Where Jx⊆ [0, 1] and ∬ denote the union overall acceptable x and u.

Definition 2. $\overset{\approx}{A}$ In the discourse universe X, the type−2 function membership represents a type−2 fuzzy set. $\mu_{\underset{A}{\approx}}$. If $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\underset{A}{\approx}$ is an interval type-2 (IT2) fuzzy set.

An IT2 fuzzy set $\overset{\approx}{A}$ is a particular type of type-2 fuzzy set, as represents in the following equation:

$$\underset{A}{\approx} = \int_{x\in X} \int_{u\in J_x} 1/(x,u),$$

Where $J_x \subseteq [0, 1]$. (34)

Definition 3. An IT2 fuzzy set's upper and lower membership functions are type-1 [71].

Moreover, every item in the fuzzy type-2 has a degree of membership in a subset of [0,1]. In addition, for every matching primary membership grade, there is a secondary membership grade of [0,1], which defines the possibility of the primary membership grade [72].

Definition 4. The addition operation between IT2T fuzzy sets $\overset{\approx}{A_1}$ and $\overset{\approx}{A_2}$ are as following:

$$\underset{A1}{\approx}{}_1 = (A_1^U, A_1^L) = \Big(\big(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(A_1^U), H_2(A_1^U)\big),$$
$$\Big(a_{11}^L, a_{12}^L, a_{\sim13}^L, a_{\sim14}^L; H_1(A_1^L), H_2(A_1^L)\Big)\Big) \text{ and}$$

$$\underset{A_2}{\approx} = (A_2^U, A_2^L) = \Big(\big(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(A_2^U), H_2(A_2^U)\big),$$
$$\big(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(A_2^L), H_2(\tilde{A}_2^L)\big)\Big) \text{ the addition as follow}$$

$$\begin{aligned}
&\overset{\approx}{A_1} \oplus \overset{\approx}{A_2} \\
&= \left(\widetilde{A_1^U}, \widetilde{A_1^L}\right) \oplus \left(\widetilde{A_2^U}, \widetilde{A_2^L}\right) = \big((a_{11}^U + a_{21}^U, a_{12}^U + \tilde{a}_{22}^U, a_{13}^U + \tilde{a}_{23}^U, a_{14}^U \\
&\quad + a_{24}^U, min\left(H_1\left(\widetilde{A_1^U}\right), H_1(A_2^U)\right), min\left(H_2(A_1^U), H_2(\tilde{A}_2^U)\right)\Big), \\
&\quad \Big(a_{11}^L + a_{21}^L, a_{12}^L + \tilde{a}_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; min\Big(H_1(\tilde{A}_1^L), H_1(A_2^L)\Big), \\
&\quad min\left(H_2(A_1^L), H_2(\tilde{A}_2^L)\right)\Big)\Big)
\end{aligned} \quad (35)$$

According to [71], the defuzzifcation steps are reported as follows:

$\overset{\approx}{A_i} = \left(\widetilde{A_i^U}, \widetilde{A_i^L}\right)$, the value $\left(\widetilde{A_i^U}\right)$ of the upper function of trapezoidal membership $\left(\widetilde{A_i^U}\right)$ of the IT2 fuzzy set $\overset{\approx}{A_i}$ is calculated as follows:

$$V(\widetilde{A_i^U}) = \frac{1}{n(n-1)}\left(\sum_{k=1}^{n} p(\widetilde{A_i^U} \geq \widetilde{A_k^U}) + \frac{n}{2} - 1\right) \quad (36)$$

The value $(\widetilde{A_i^L})$ of the lower function of trapezoidal membership $(\widetilde{A_i^L})$ of the IT2 fuzzy set $\overset{\approx}{A}_i$ is calculated as follows:

$$V(\widetilde{A_i^L}) = \frac{1}{n(n-1)}\left(\sum_{k=1}^{n} p\left(\widetilde{A_i^L} \geq \widetilde{A_k^L}\right) + \frac{n}{2} - 1\right) \quad (37)$$

Finally, the final value for $\overset{\approx}{A}_i$ is computed as follows:

$$\overset{\approx}{A}_i = \frac{V(A_i^U) + V(\widetilde{A_i^L})}{2} \quad (38)$$

7- Fermatean fuzzy set (FFS)

Senapati and Yager introduced the Fermatean fuzzy set in 2020 [73], to deal with uncertainty and ambiguity. Furthermore, the Fermatean fuzzy set is a novel type of fuzzy set. A number of researchers solved the uncertainty problem by using Fermatean fuzzy sets with MCDM, and they recommended utilizing them [74]. The Fermatean fuzzy set was formed from intuitionistic fuzzy sets and Pythagorean fuzzy sets; nevertheless, the Fermatean fuzzy set is much more flexible in dealing with uncertainty than Pythagorean fuzzy sets and intuitionistic fuzzy sets. FFS consists of three significant components (i.e., membership degree, non-membership degree, and indeterminacy) [27]. The value of convert the linguistic terms into Fermatean fuzzy set is presented in Table 9.

Table 9. - Convert the linguistic terms into Fermatean fuzzy set[27]

linguistic terms	FFs
No difference	(0.90, 0.10)
Slight difference	(0.75, 0.20)
Difference	(0.50, 0.45)
Big difference	(0.35, 0.60)
Huge difference	(0.10, 0.90)

The following are some basic definitions and operators for the Fermatean fuzzy set:

Definition 1. Let X be a discourse universe. F in X is a fermatean fuzzy set.

$$F = \{\langle x, \alpha_F(x), \beta_F(x)\rangle : x \in X\} \quad (39)$$

where αF (x): X → [0, 1] and βF (x): X → [0, 1], includes the condition $0 \leq (\alpha_F(x))^3 + (\beta_F(x))^3 \leq 1$ for all x ∈ X. The number of αF (x) expresses the membership grade, and the number of βF (x) represented the non-membership grade of the element x in the set F. The third component is the grade of indeterminacy is $\pi_F(x) =$

$$\sqrt[3]{1 - (\alpha_F(x))^3 - (\beta_F(x))^3)}$$

In Figure 3, can see visibly difference between the Intuitionistic, Pythagorean, and Fermatean fuzzy sets (FFs, IFs, and PFs) in spaces. as a result of this difference, The FFs have greater flexibility to handle the uncertainty issue.

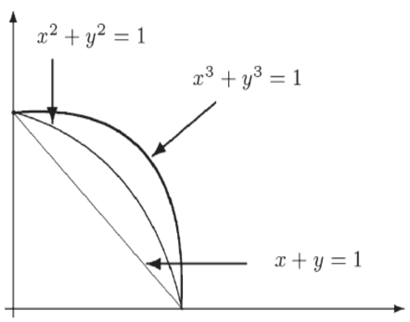

FIGURE 3. - The Differences among Fermatean Fuzzy Set, Intuitionistic Fuzzy Sets,

Definition 2. Let A˜ = (αA, βA) and B˜ = (αB, βB) two FFs, and ∂ a positive real number (∂ > 0). Then can be define the following operators for FFs [22]:

$$\tilde{A} + \tilde{B} = \left(\sqrt[3]{\alpha_A^3 + \alpha_B^3 - \alpha_A^3\alpha_B^3}, \beta_A\beta_B\right) \tag{40}$$

$$\tilde{A} \times \tilde{B} = \left(\alpha_A\alpha_B, \sqrt[3]{\beta_A^3 + \beta_B^3 - \beta_A^3\beta_B^3}\right) \tag{41}$$

$$\partial.\, \tilde{A} = \left(\sqrt[3]{1 - (1 - \alpha_A^3)^\partial}, \beta_A^\partial\right) \tag{42}$$

$$\tilde{A}^\partial = \left(\alpha_A^3, \sqrt[3]{1 - (1 - \beta_A^3)^\partial}\right) \tag{43}$$

Definition 3. Let A˜ = (αA, βA) is a FFs, and the score function S and accuracy function T for this FFs are defined as follows [38]:

$$S(\tilde{A}) = \alpha_A^3 - \beta_A^3 \tag{44}$$

$$T(\tilde{A}) = \alpha_A^3 + \beta_A^3 \tag{45}$$

The above functions can be used to comparing two FFs, A˜ = (αA, βA) and B˜ = (αB, βB). There are different conditions to compare these two FFs.

If $S(\tilde{A}) < S(\tilde{B})$, then $\tilde{A} < \tilde{B}$;
If $S(\tilde{A}) > S(\tilde{B})$, then $\tilde{A} > \tilde{B}$;
If $S(\tilde{A}) = S(\tilde{B})$, then
$T(\tilde{A}) < T(\tilde{B})$, then $\tilde{A} < \tilde{B}$
$T(\tilde{A}) > T(\tilde{B})$, then $\tilde{A} > \tilde{B}$
$T(\tilde{A}) = T(\tilde{B})$, then $\tilde{A} = \tilde{B}$.

Definition 4. The complement of FFs A˜ = (αA, βA) is defined as follows [22]:

$$\text{Com}\ (\tilde{A}) = (\beta_A, \alpha_A) \tag{46}$$

Definition 5. The score function of FFs was defined in definition 3. Suppose that A˜ = (αA, βA) is a FFs. The value of S A˜ can be in the range of –1 to 1.

According to [22] define the following function for positive score function:

$$S^p(\tilde{A}_{ij}) = 1 + S(\tilde{A}_{ij}) \tag{47}$$

8- q-rung orthopair probabilistic hesitant fuzzy numbers (q-ROPHFS)

Atanassov et al. [75] introduced the concept of the q-rung orthopair probabilistic hesitant fuzzy set (q-ROPHFS), which adds probability to hesitant fuzzy sets in order to improve the dependability of information in fuzzy sets. It supports several view points and gives each one a probability of occurring. The q-ROPHFS is a more accurate and resilient way to describe uncertainty because it combines the concepts of q-rung orthopair hesitant fuzzy set (q-ROHFS) and probability [41]. Table 10 shows linguistic and numerical scales and their corresponding q-ROPHFNs.

Table 10. - Linguistic and Numerical Scales with Corresponding q-ROPHFNs[41]

Linguistic expression	Numeric scale	ROPHFNs							
		μ1	pμ1	μ2	pμ2	ν1	pν1	ν2	pν2
No difference	1	0.9	0.4	0.95	0.6	0.15	0.2	0.1	0.8
Slight difference	2	0.7	0.5	0.75	0.5	0.35	0.3	0.3	0.7
Difference	3	0.5	0.3	0.55	0.7	0.55	0.5	0.5	0.5
Big difference	4	0.4	0.8	0.5	0.2	0.55	0.4	0.6	0.6
Huge difference	5	0.2	0.7	0.25	0.3	0.8	0.35	0.9	0.65

q-ROPHFS and q-ROPHFNs can be defined as follows:

Definition 1[72]. Let M and N be sets of q-rung membership and non-membership functions, respectively, in a universal set represented by Ω. The following represents the q-ROPHFS Q_P linked to M and N:

$$Q_p = \{\langle x, h(x), g(x) \mid\rangle \mid x \in \Omega\}$$

where $h(x) = \cup_{\mu\in\mathcal{M}}\ \{\mu(x) \mid p_\mu(x)\}$(resp. $g(x) = \cup_{\nu\in\mathcal{N}}\ \{\nu(x) \mid p_\nu(x)\}$)

The q-ROPHF Einstein arithmetic mean (q-ROPHEWAM) operator is used to aggregate the q-ROPHFNs of each choice within the fuzzy opinion matrix, as stated in (48).

q-ROPHFEWAM $((w_1,\dots,w_r),(Q_1,\dots,Q_r))$

$$= \left(\begin{array}{l} \bigcup_{\mu_{Q_i}\in\mathcal{M}_i} \left\{ \dfrac{\sqrt[q]{\prod_{i=1}^{r}\left(1+(\mu_{Q_i})^q\right)^{w_i}-\prod_{i=1}^{r}\left(1-(\mu_{Q_i})^q\right)^{w_i}}}{\sqrt[q]{\prod_{i=1}^{r}\left(1+(\mu_{Q_i})^q\right)^{w_i}+\prod_{i=1}^{r}\left(1-(\mu_{Q_i})^q\right)^{w_i}}} \right\} \mid \prod_{i=1}^{r} p_{\mu_{Q_i}}, \\ \bigcup_{v_{Q_i}\in\mathcal{N}_i} \left\{ \dfrac{\sqrt[q]{2\prod_{i=1}^{r}\left((v_{Q_i})^q\right)^{w_i}}}{\sqrt[q]{\prod_{i=1}^{r}\left(2-(v_{Q_i})^q\right)^{w_i}+\prod_{i=1}^{r}\left((v_{Q_i})^q\right)^{w_i}}} \right\} \mid \prod_{i=1}^{r} p_{v_{Q_i}} \end{array} \right), \quad (48)$$

Where $(w_i)_{1<i<r} \in [0,1]^r$ represent a vector of weights with $\sum_{i=1}^{r} w_i = 1$, and $(Q_i)_{1\le i\le r}$is an r-tuple of q-ROPHFNs.

9- 2-tuple-FDOSM

In the Fuzzy Decision by Opinion Score technique (FDOSM) and its extension, the 2-tuple fuzzy technique is utilized. According to the 2-tuple fuzzy technique, a 2-tuple is defined by two values: a real number that denotes the symbolic translation's value and a linguistic label [76]. In order to overcome the issue of information loss during the conversion of a decision matrix into an opinion decision matrix in the FDOSM, the 2-tuple-FDOSM makes use of the 2-tuple fuzzy environment. Through the incorporation of the 2-tuple environment, Due to uncertainty, the 2-tuple fuzzy technique is the best method to solve the problem of missing information in the aggregation process with the opinion matrices and the ranking of alternatives with the same ranks [45]. Table 11 shows the conversion of the linguistic terms into fuzzy numbers.

Table 11. – The Conversion of the Linguistic Terms into Fuzzy Numbers [45]

Linguistic Terms	Fuzzy Numbers
No difference	(0, 0.1, 0.3)
Slight Difference	(0.1, 0.3, 0.5)
Difference	(0.3, 0.5, 0.75)
Big Difference	(0.5, 0.75, 0.9)
Huge Difference	(0.75, 0.9, 1)

In this section, this study describes how employed the 2-tuple fuzzy approach based on the following mathematical definitions to extend the FDOSM into the 2-tuple-FDOSM.

If a symbolic method to aggregating linguistic information provides a value of β ∈ [0, g], and β∈/{0,, . . ., g}, then an approximation function (app2 (•)) is used to express the result's index in S. Let S = s0,..., sg represent a linguistic term set.

Definition 1. Let β be the outcome of a symbolic aggregation operation conducted on the index values of a number of labels evaluated in a linguistic term set S. β ∈ [0, g], where q + 1 is S's the number of elements in a set or other group. If i = round (β) and α = β − i are two numbers such that i ∈ [0, g] and α ∈ [−0.5, 0.5], then α is a symbolic translation of β. A value assessed using [−0.5, 0.5) is the symbolic equivalent of a language word, si. This validates the "variance of information" between information measurements β ∈ [0, g] following a symbolic aggregation process; the value closest to \0,..., g} indicates the index of the closest linguistic term in S (i = round (β)) .

Definition 2. The 2-tuple reflecting the corresponding information to β is computed using the following function, which represents the result of a symbolic aggregation operation [77], given a linguistic term set S = s0,..., sg and a value β ∈ [0, g]:

$$\Delta: [0,g] \rightarrow S \times [-0.5,0.5) \quad (49)$$

$$\Delta(\beta) = (s_i,\alpha), \text{ with } \begin{cases} s_i, & i = \text{round}(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5,0.5) \end{cases} \quad (50)$$

While round (•) represents a standard rounding operation, s_i is the index's closest label to "β," and α denotes the symbolic translation's value.

Definition 3. Given a set of 2-tuples x = {r1, α1), . . ., (rn, αn)}, the 2-tuple arithmetic mean $\bar{x}^e$ is calculated as

$$\bar{x}^e = \Delta\left(\sum_{i=1}^{n} \frac{1}{n}\Delta^{-1}(r_i,\alpha_i)\right) = \Delta\left(\frac{1}{n}\sum_{i=1}^{n} \beta_i\right) \quad (51)$$

The arithmetic mean for 2-tuples can be used to compute the mean of a set of linguistic values without loss any information.

10- Intuitionistic fuzzy sets (IFSs)

As an extension of Zadeh's fuzzy set, the principle of intuitionistic fuzzy sets (IFSs) can take into account membership and non-membership degrees with hesitation index at the same time [58]. As a result, the IFSs theories is extensively employed since it can reflect inexorably imperfect or not completely dependable assessments [78]. Moreover, with the use of membership definitions, affirmation, negation, and hesitation can be express well in IFSs. When it comes to provide accurate decision results in group decision-making (GDM), the consistency of IF preference relations and the expert opinions gathered from these preference relations play a crucial role [24]. Table 12 shows conversion of the linguistic terms into IFSs.

Table 12. - Conversion of the linguistic terms into IFSs [24]

linguistic terms	intuitionistic fuzzy number	
No difference	0.90	0.05
Slight difference	0.75	0.20
Difference	0.50	0.40
Big difference	0.25	0.60
Huge difference	0.10	0.80

The following is a definition of the application of intuitionistic fuzzy theory:

Definition 1. Let X be the universal set:

A set $\tilde{A} = \{x, m_{\tilde{A}}(x) \mid x \in X\}$ is known as a fuzzy set of X, where $m_{\tilde{A}}(x): X \to [0,1]$ is a membership function. for all x ∈ X, $m_{\tilde{A}}(x)$ represents the score of membership of element x in $\tilde{A}$.

A set $\tilde{A} = \{x, m_{\tilde{A}}(x), n_{\tilde{A}}(x) \mid x \in X\}$ is referred to IFS of X, where $m_{\tilde{A}}(x), n_{\tilde{A}}(x)$ are membership function and non-membership function, respectively. Thus, 0 ≤ $0 \leq m_{\tilde{A}}(x) + n_{\tilde{A}}(x) \leq 1, \forall x \in X$.

In addition, $\pi_{\tilde{A}}(x) = 1 - m_{\tilde{A}}(x) - n_{\tilde{A}}(x)$ is the hesitation score of x.

The following equation presents the aggregation operator using IFS:

$$\left(1 - \prod_{j=1}^{n}\left(1 - \mu_{\beta_{\sigma(j)}}\right), \prod_{j=1}^{n} v_{\beta_{\sigma(j)}}\right) \tag{52}$$

11- Fermatean probabilistic hesitant-fuzzy sets (FPHFSs)

FPHFS (fermatean probabilistic hesitant fuzzy sets) are a mathematical principle that is utilized in decision-making and evaluation processes. FPHFS are fuzzy set extensions that combine uncertainty and probability. They are expressed by a pair of values, one of which reflects the element's probably positive grade and the other the element's potential negative grade [13]. These grades are linked to probabilities, which indicate how likely each grade is. FPHFS make it possible to take into account evaluation information that is more complex and ambiguous while making decisions. Experts' uncertainties can be effectively addressed by FPHFSs, which can also effectively each element's likelihood of occurring [46]. Table 13 shows Linguistic expressions and their corresponding numeric scale and FPHFNs.

Table 13. - Linguistic Expressions and their Corresponding Numeric Scale and FPHFNs[46]

Linguistic expression	Numeric scale	FPHFNs							
		M1	P1	M2	P2	V1	P1	V2	P2
No difference	1	0.9	0.4	0.95	0.6	0.15	0.2	0.1	0.8
Slight difference	2	0.7	0.5	0.75	0.5	0.35	0.3	0.3	0.7
Difference	3	0.5	0.3	0.55	0.7	0.55	0.5	0.5	0.5
Big difference	4	0.4	0.8	0.5	0.2	0.55	0.4	0.6	0.6
Huge difference	5	0.2	0.7	0.25	0.3	0.8	0.35	0.9	0.65

The essential definitions and the formation of the FPHFS in order to comprehend the fundamental principle are shown as follows:

Definition 1. R is as before. FPHFS ℵ on R is expressed as:

$$\aleph = \left\{\left\langle r, \tau_{h_k}(r)/\tilde{p}_{\dot{g}}, \partial_{h_\chi}(r)/b_{\dot{g}}\right\rangle \mid r \in R\right\},$$

Where r ∈ R, $\tau_{h_\chi}(r)$ and $\delta_{h_\chi}(r)$ are sets with specific values in [0,1]. $\tau_{h_\chi}(r)/\tilde{p}_{\dot{g}} \& \partial_{h_\chi}(r)/b_{\dot{g}}$ The likely positive and negative grades of r with respect to FPHFS ℵ, respectively $\tilde{p}_{\dot{g}}$ and $b_{\dot{g}}$ expressed the probabilities of degrees.

In addition, $0 \leq \hbar_i, \varrho_{\hat{\imath}} \leq 1$ and $0 \leq \tilde{p}_i, b_j \leq 1$ with $\sum_{i=1}^{L} \tilde{p}_i 1, \sum_{\hat{\imath}=1}^{L} b_j \leq 1$

(L is a positive integer that represents the number of items in FPHFS)

where $\hbar_i \in \tau_{h_\chi}(r)$ and $\varrho_j \in O_{h_*}(r), \tilde{p}_i \in \tilde{p}_{\dot{g}}, b_j \in b_{\dot{g}}$ Moreover, it is required that $\left(max\left(\tau_{h_h}(r)\right)\right)^3 + \left(min\left(\tilde{\partial}_{h_x}(r)\right)\right)^3 \leq 1$ and $\left(min\left(\tau_{h_k}(r)\right)\right)^3 + \left(max\left(\partial_{h_k}(r)\right)\right)^3 \leq 1$.

The Fermatean probabilistic hesitant fuzzy number (FPHFN) is expressed by the pair $\left(\tau_{h_x}/\tilde{p}_{\dot{g}}, \partial_{h_x}/b_{\dot{g}}\right)$.

The set of all FPHFSs in R is represented by FPHF ˆS(R).

Definition 2. Let $\aleph 1 = \left(\tau_{h_{g_1}}/\tilde{p}_{\dot{g}1}, \partial_{h_{g_1}}/b_{\dot{g}1}\right)$ and $\aleph_2 = \left(\tau_{h_{g2}}/\tilde{p}_{\dot{g}2}, \partial_{h_{g2}}/b_{\dot{g}2}\right)$

Be FPHFNs. The fundamental operational laws are as follows:

$$\aleph_1 \cup \aleph_2 = \left\{ \bigcup_{\substack{\hbar_1 \in \tau_{h_{\dot{g}_1}(l_{\dot{g}})}, \tilde{p}_{1} \in \tilde{p}_{\dot{g}} \\ \hbar_2 \in \tau_{h_{\dot{g}_2}(l_{\dot{g}})}, \tilde{p}_{2} \in \tilde{p}_{\dot{g}_2}}} \left(max(\hbar_1/\tilde{p}_1, \hbar_2/\tilde{p}_2)\right), \bigcup_{\substack{\varrho_1 \in \hat{\partial}_{h_{\dot{g}}}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}_1} \\ \varrho_2 \in O_{h_{\dot{g}_2}}(l_{\dot{g}}), b_2 \in b^{\dot{g}_2}}} \left(min(\varrho_1/b_1, \varrho_2/b_2)\right) \right\} \tag{53}$$

$$\aleph_1 \cap \aleph_2 = \left\{ \bigcup_{\substack{\hbar_1 \in \tau_{h_{\dot{g}_1}(l_{\dot{g}})}, \tilde{p}_{1} \in \tilde{p}_{\dot{g}} \\ \hbar_2 \in \tau_{h_{\dot{g}_2}(l_{g})}, \tilde{p}_{2} \in \tilde{p}_{\dot{g}_2}}} \left(min(\hbar_1/\tilde{p}_1, \hbar_2/\tilde{p}_2)\right), \bigcup_{\substack{\varrho_1 \in \hat{\partial}_{\tilde{h}_{\dot{g}_1}}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}_1} \\ \varrho_2 \in O_{h_{\dot{g}_2}}(l_{\dot{g}}), b_2 \in b^{\dot{g}_2}}} \left(max(\varrho_1/b_1, \varrho_2/b_2)\right) \right\}. \tag{54}$$

$$\aleph_1^c = \left(\hat{\partial}_{h_x}/b_{\dot{g}}, \tau_{h_x}/\tilde{p}_{\dot{g}}\right). \tag{55}$$

Definition 3. Let $\aleph 1 = \left(\tau_{h_{g1}}/\tilde{p}_{\dot{g}_1}, \partial_{h_{g_1}}/b_{\dot{g}1}\right)$ and $\aleph_2 = \left(\tau_{h_{g2}}/\tilde{p}_{\dot{g}_2}, \partial_{h_{g2}}/b_{\dot{g}2}\right)$

Be FPHFNs and ζ > 0(∈ R); Later, their operations are described as follows:

$$\aleph_1 \oplus \aleph_2 = \left\{ \bigcup_{\substack{\hbar_1 \in \tau_{h_{\dot{g}_1}(l_{\dot{g}})}, \tilde{p}_{1} \in \tilde{p}_{\dot{g}} \\ \hbar_2 \in \tau_{h_{\dot{g}_2}(l_{\dot{g}})}, \tilde{p}_{2} \in \tilde{p}_{\dot{g}_2}}} \left(\sqrt[3]{\hbar_1^3 + \hbar_2^3 - \hbar_1^3\hbar_2^3}/\tilde{p}_1\tilde{p}_2\right), \bigcup_{\substack{\varrho_1 \in \hat{\partial}_{h_{\dot{g}_1}}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}_1} \\ \varrho_2 \in O_{h_{\dot{g}_2}}(l_{\dot{g}}), b_2 \in b^{\dot{g}_2}}} (\varrho_1\varrho_2/b_1b_2) \right\}. \tag{56}$$

$$\aleph_1 \otimes \aleph_2 = \left\{ \bigcup_{\substack{\hbar_1 \in \tau_{h_{h_1}(l_g)}, \tilde{p}_{1} \in \tilde{p}_{g} \\ h_2 \in \tau_{h_{g2}(l_{\dot{g}})}, \tilde{p}_{2} \in \tilde{p}_{\tilde{g}_2}}} \left(\frac{h_1h_2}{\tilde{p}_1\tilde{p}_2}\right), \bigcup \right.$$

$$\begin{matrix} \varrho_1 \in \hat{\partial}_{h_{\dot{g}_1}}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}_1} \\ \varrho_2 \in O_{h_{g_2}}(l_{\dot{g}}), b_2 \in b^{\dot{g}_2} \end{matrix} \left(\sqrt{\varrho_1^2 + \varrho_2^2 - \varrho_1^2\varrho_2^2}/b_1 b_2\right)\Bigg\} \quad (57)$$

$$\zeta\aleph_1 = \Big\{\cup_{\hbar_1 \in \tau_{h_{\dot{g}_1}}(l_{\dot{g}})}, \tilde{p}_{1\in\tilde{p}\dot{g}_1}\left(\sqrt[3]{1-(1-\hbar_1^3)^{\zeta}}/\tilde{p}_1\right), \\ \cup_{\varrho_1 \in \hat{\partial}_{\dot{g}_{\dot{g}_1}}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}}} \left(\varrho_1^{\zeta}/b_1\right)\Big\} . \quad (58)$$

$$\aleph_1^{\zeta} = \Big\{\cup_{h_1 \in \tau_{h_{\dot{g}_1}}(l_{\dot{g}}), \tilde{p}_{1\in\tilde{p}p_1}}\left(h_1^{\zeta}/\tilde{p}_1\right), \\ \cup_{\varrho_1 \in \hat{\partial}_{\dot{g}_{\dot{g}_1}}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}_1}} \left(\sqrt[3]{1-(1-\varrho_1^3)^{\zeta}}/b_1\right)\Big\}. \quad (59)$$

Definition 4. For any FPHFN $\aleph = \left(\tau_{h_x}/\tilde{p}_{\dot{g}}, \hat{\partial}_{h_\chi}/b_{\dot{g}}\right)$, a score function be introduced as:

$$s(\aleph) = \left(\frac{1}{M_\aleph}\sum_{\hbar_i \in \tau h_{\dot{g}}, \tilde{p}_i \in \tilde{p} h_{\dot{g}}} (\hbar_i \tilde{p}_i)\right)^3 - \left(\frac{1}{N_\aleph}\sum_{\varrho_i \in \delta h_k, b_i \in b h_{\dot{g}}} (\varrho_i b_i)\right)^3,$$

Where M$\aleph$ and N$\aleph$ express the number of components in $\tau_{h_{\dot{g}}}$ and $\hat{\partial}_{h_x}$respectively.

Definition 5. For any FPHFN $\aleph = \left(\tau_{h_x}/\tilde{p}_{\dot{g}}, \hat{\partial}_{h_x}/b_{\dot{g}}\right)$, An accuracy function is defined as follows:

$$h(\aleph) = \left(\frac{1}{M_\aleph}\sum_{h_i \in \tau h_{\dot{g}}, \tilde{p}_i \in \tilde{p} h_{\dot{g}}} (h_i \tilde{p}_i)\right)^3 + \left(\frac{1}{N_\aleph}\sum_{\varrho_i \in \delta h_k, b_i \in b h_{\dot{g}}} (\varrho_i b_i)\right)^3,$$

Where $M_\aleph$ and $N_\aleph$ express the number of components in $\tau_{h_{\dot{g}}}$ and $\hat{\partial}_{h_h}$, respectively.

Definition 6. Let $\aleph_1 = \left(\tau_{h_{g1}}/\tilde{p}_{g1}, \partial_{h_{g1}}/b_{\dot{g}1}\right)$ and $\aleph_2 = \left(\tau_{h_g}/\tilde{p}_{g2}, \partial_{h_{g2}}/b_{\dot{g}2}\right)$ be FPHFNs.

A comparison of FPHFNs can be defined using this definition as:
If s (ℵ1) > s (ℵ2), then ℵ1 >ℵ2. (60)
If s (ℵ1) = s (ℵ2), and h (ℵ1) > h (ℵ2) then ℵ1 >ℵ2 (61)

Definition 7. Let $\aleph_j = \left(\tau_{h_{g_j}}\left(\tilde{p}_{g_j}, \succ_{h_{g_j}}/b_{g_j}\right)\hat{J} = 1,2,\ldots,r\right)$ be any set of FPHFNs and the Fermatean probabilistic hesitant fuzzy average mean (FPHFAM): FPHFNr → FPHFN. Then, the FPHFAM operator can be expressed as:

$$\text{FPHFAM } (\aleph_1, \aleph_2, \ldots, \aleph_r) = 1lr\aleph_1 \oplus 1lr\aleph_2 \oplus \cdots \oplus 1lr\aleph_r.$$

Theorem 1. Let $\aleph_j = \left(\tau_{h_{g_j}}\left(\tilde{p}_{g_j}, \partial_{h_{g_j}}/b_{g_j}\right)(\hat{J} = 1,2,\ldots,r)\right.$ be any group of FPHFNs. The aggregation result achieved by applying the FPHFAM is then as follows:

FPHFAM (ℵ1, ℵ2, . . .,ℵr)

$$= \left(\begin{matrix} \bigcup_{\hbar_j \in \epsilon_{\aleph_j}, \tilde{p}_{\aleph_j} \in \tilde{p}_{\aleph_j}} \sqrt[3]{1 - \prod_{\hat{j}=1}^{r}\left(1-\left(\hbar_{\aleph_j}\right)^3\right)^{1/r}} / \Pi_{\hat{j}=1}^{r}\tilde{p}_{\aleph_j} \\ \bigcup_{\varrho_{\aleph_j} \in \partial_{\aleph_j}, b_{\aleph_j} \in b_{\aleph_j}} \prod_{\hat{j}=1}^{r}\left(\varrho_{\aleph_j}\right)^{11/r} / \Pi_{\hat{j}=1}^{r} b_{\aleph_j} \end{matrix}\right).$$

Definition 8. Let $\aleph_{\hat{j}} = \tau_{h_{g_j}}/\tilde{p}_{g_j}, \partial_{h_{g_j}}/b_{g_j} (\hat{j} = 1,2, \dots, r)$ be any set of FPHFNs and the Fermatean probabilistic hesitan fuzzy weighted average (FPHFWA): FPHFNr → FPHFN. Then, the FPHFWA operator can be represent as:

$$\text{FPHFWA}\ (\aleph_1, \aleph_2, \dots, \aleph_r) = \text{W}_1 \aleph_1 \oplus \text{W}_2 \aleph_2 \oplus \cdots \oplus \text{W}_r \aleph_r,$$

Where W = (W1, W2, ..., Wr) ^{T} are the weights of $\aleph_{\hat{j}} \in [0,1]$ with $\sum_{\hat{j}=1}^{r} \text{W}_{\hat{j}} = 1$.

Theorem 2. Let $\aleph_{\hat{j}} = \tau_{h_{g_j}}/\tilde{p}_{g_j}, \partial_{h_{g_j}}/b_{g_j} (\hat{j} = 1,2, \dots, r)$ be any set of FPHFNs. The aggregation result obtained with the FPHFWA is then as follows:

FPHFWA (ℵ1, ℵ2, ..., ℵr)

$$= \left(\begin{array}{c} \bigcup_{\hbar_{\hat{j}} \in \tau_{\aleph_{\hat{j}}}, \tilde{\aleph}_{\aleph_{\hat{j}}} \in \tilde{p}_{\aleph_{\hat{j}}}} \sqrt[3]{1 - \prod_{\hat{j}=1}^{r} \left(1 - \left(\hbar_{\aleph_{\hat{j}}}\right)^3\right)^{W_{\hat{j}}} / \Pi_{\hat{j}=1}^{r} \tilde{p}_{\aleph_{\hat{j}}}} \\ \bigcup_{\varrho_{\aleph_{\hat{j}}} \in \partial_{\aleph_{\hat{j}}}, b_{\aleph_{\hat{j}}} \in b_{\aleph_{\hat{j}}}} \prod_{\hat{j}=1}^{r} \left(\varrho_{\aleph_{\hat{j}}}\right)^{W_{\hat{j}}} / \Pi_{\hat{j}=1}^{r} b_{\aleph_{\hat{j}}} \end{array} \right)$$

12- Spherical Fuzzy Rough Set (SFRSs)

Spherical fuzzy rough set (SFRSs) is the more robust FRS fuzzy environment [40]. that has been shown to overcome the shortcomings of not only conventional fuzzy sets but also intuitionistic fuzzy sets (IFS), Pythagorean fuzzy sets, and rough sets. therefore, SFRSs integration, especially with FDOSM and FWZIC, is necessary [79]. The SFS membership is utilized to address the uncertainty and imprecision issues revealed by the crisp value of specialist preferences for every related criterion [47]. Linguistic terms and their equivalent SFSs is shown in Table 14.

Table 14. - Linguistic Terms with Equivalent SFSs [47]

Linguistic scoring scale	Numerical scoring scale	SFSs		
		μ	V	Π
Huge Difference	5	0.15	0.85	0.1
Very Difference	4	0.25	0.75	0.2
Medium No Difference	3	0.55	0.5	0.25
Very No Difference	2	0.75	0.25	0.2
Extremely No Difference	1	0.85	0.15	0.1

The spherical fuzzy set is defined in the following equations:

SFS $\tilde{A}_s$ of the universe of discourse U is provided by:

$$\tilde{A}_s = \{\langle \text{u}, \mu_{\tilde{A}_s}(u), v_{\tilde{A}_s}(u), \pi_{\tilde{A}_s}(u) | u \in \text{U}\} \quad (62)$$

Where

$\mu_{\tilde{A}_s}(u): U \to [0,1], v_{\tilde{A}_s}(u): U \to [0,1], \pi_{\tilde{A}_s}(u): U \to [0,1]$

And

$$0 \leq \mu_{\tilde{A}_s}^2(u) + v_{\tilde{A}_s}^2(u) + \pi_{\tilde{A}_s}^2(u) \leq 1 \forall_u \in \text{U} \quad (63)$$

For each $u, \mu_{\tilde{A}_s}(u), v_{\tilde{A}_s}(u)$, and $\pi_{\tilde{A}_s}(u)$ express the scores of memberships, non-membership, and hesitancy of u to $\tilde{A}_s$, respectively. $\chi_{\tilde{A}_s} = \left(1 - \mu_{\tilde{A}_s}^2(u) - v_{\tilde{A}_s}^2(u) - \pi_{\tilde{A}_s}^2(u)\right)^{1/2}$ represents the refusal score.

(64) is used to perform the aggregate of the W-SFRS-OP value:

$$\text{SFREWA}\left(g(b_1), g(b_1), \dots, g(b_n)\right) = \left\{\oplus_{i=1}^{n} w_i \underline{g}(b_i), \oplus_{i=1}^{n} w_i \bar{g}(b_i)\right\}$$

$$= \left\{ \sqrt{\frac{\prod_{j=1}^{k}\left(1+\underline{\mu}_j^2\right)^{w_j} - \prod_{j=1}^{k}\left(1-\underline{\mu}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(1+\underline{\mu}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(1-\underline{\mu}_j^2\right)^{w_j}}}, \right.$$
$$\sqrt[t]{\frac{2\prod_{j=1}^{k}\left(\underline{v}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(2-\underline{v}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(\underline{v}_j^2\right)^{w_j}}},$$
$$\left. \sqrt[t]{\frac{2\prod_{j=1}^{k}\left(\underline{\pi}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(2-\underline{\pi}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(\underline{\pi}_j^2\right)^{w_j}}} \right\},$$
$$\left\{ \sqrt{\frac{\prod_{j=1}^{k}\left(1+\bar{\mu}_j^2\right)^{w_j} - \prod_{j=1}^{k}\left(1-\bar{\mu}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(1+\bar{\mu}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(1-\bar{\mu}_j^2\right)^{w_j}}}, \right.$$
$$\sqrt[t]{\frac{2\prod_{j=1}^{k}\left(\bar{v}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(2-\bar{v}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(\bar{v}_j^2\right)^{w_j}}},$$
$$\left. \sqrt[t]{\frac{2\prod_{j=1}^{k}\left(\bar{\pi}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(2-\bar{\pi}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(\bar{\pi}_j^2\right)^{w_j}}} \right\}, \Bigg)$$

(64)

13- q-rung orthopair fuzzy rough sets (q-ROFRSs)

Q-rung orthopair Fuzzy rough sets are a notion that integrates several theories to deal with uncertainty and ambiguity with processes of decision-making [80]. It is an extension of fuzzy sets theory and rough sets theory. Fuzzy sets theory addresses with expressing vague and uncertain information, whereas rough sets theory is dealing with imprecise and ambiguous data. The q-ROFRS (q-ROF rough sets) is a hybrid smart structure composed of rough sets and q-ROFSs (q-ROFSs are a kind of fuzzy set) [81]. When compared to other fuzzy sets, q-ROFRSs provide a larger space for representing vague information. They enable experts to independently provide positive and negative scores by adjusting the q parameter. The q-ROFRS-FDOSM (fuzzy decision by the opinion score method) approach is utilized to rank alternatives based on the weights of the criteria [48]. Linguistic terms and their corresponding numerical scale and q-ROFNs is present in Table 15.

Table 15. - Linguistic Terms and their Corresponding Numerical Scale and q-ROFNs [48]

Linguistic terms	Numerical scale	q-ROFN	
		M	***V***
No difference	1	0.95	0.15
Slight difference	2	0.75	0.35
Difference	3	0.55	0.55
Big difference	4	0.35	0.75
Huge difference	5	0.15	0.95

Definition 1. presents the basic concept of q-ROFS:

Definition 1: Let F be the discourse universe. Let μ, v: F→ [0, 1] represent the non-membership degree (NMD) and membership degree (MD) respectively. MD and NMD are denoted by μ(ħ) and v(ħ), respectively, for any ħ ∈ F. qROFNs is displayed below.

$$\mathrm{N} = (\mu(\hbar), v(\hbar))$$

Where
$\mu(\hbar), v(\hbar) \in [0,1]$
$0 \leq \mu(\hbar)^q + v(\hbar)^q \leq 1$
$q \geq 1$

Definition 2. Shows the basic concepts of RS and q-ROFRS.

Definition 2: Let Z be the universe of discourse. Let μ, v: Z→[0, 1] be the MD and NMG. For any ħ ∈ Z, μ(ħ) and v(ħ) express MD and NMD, respectively. The q-rung orthopair fuzzy rough numbers (qROFRNs) as presented below:
$N = ((\underline{\mu}(\hbar), \underline{v}(\hbar)), (\bar{\mu}(\hbar), \bar{v}(\hbar)))$

Where

$\underline{\mu}(\hbar), \underline{v}(\hbar), \bar{\mu}(\hbar), \bar{v}(\hbar) \in [0,1]$
$0 \leq \underline{\mu}(\hbar)^q + \underline{v}(\hbar)^q \leq 1$
$0 \leq \bar{\mu}(\hbar)^q + \bar{v}(\hbar)^q \leq 1$

The q-ROFRS-OM of every expert is aggregate utilizing the fuzzy rough EINSTEIN weighted aggregation operator (q-ROFREWA) operator described in (65).

$$q-\mathrm{ROFREWA}\left(\Upsilon(\pounds_1), \Upsilon(\pounds_2), \ldots, \Upsilon(\pounds_n)\right) = \left(\oplus_{i=1}^{n} w_l \underline{\Upsilon}(\pounds_l), \oplus_{i=1}^{n} w_l \overline{\Upsilon}(\pounds_2)\right) =$$

$$\left[\begin{array}{l}\left(\sqrt[q]{\frac{\prod_{i=1}^{n}\left[1+\underline{\mu}_i^q\right]^{w_i} - \prod_{i=1}^{n}\left[1-\underline{\mu}_i^q\right]^{w_i}}{\prod_{i=1}^{n}\left[1+\underline{\mu}_i^q\right]^{w_i} + \prod_{i=1}^{n}\left[1-\underline{\mu}_i^q\right]^{w_i}}}, \frac{\sqrt[q]{2}\prod_{i=1}^{n}\left[\underline{v}_i^q\right]^{w_i}}{\sqrt[q]{\prod_{i=1}^{n}\left[2-\underline{v}_i^q\right]^{w_i} + \prod_{i=1}^{n}\left[\underline{v}_i^q\right]^{w_i}}}\right), \\ \left(\frac{\prod_{i=1}^{n}\left[1+\bar{\mu}_i^q\right]_i^{w_i} - \prod_{i=1}^{n}\left[1-\bar{\mu}_i^q\right]^{w_i}}{\prod_{i=1}^{n}\left[1+\bar{\mu}_i^q\right]^{w_i} + \prod_{i=1}^{n}\left[1-\bar{\mu}_i^q\right]^{w_i}}, \frac{\sqrt[q]{2}\prod_{i=1}^{n}\left[\bar{v}_i^q\right]^{w_i}}{\sqrt[q]{\prod_{i=1}^{n}\left[2-\bar{v}_i^q\right]^{w_i} + \prod_{i=1}^{n}\left[\bar{v}_i^q\right]^{w_i}}}\right)\end{array}\right] \quad (65)$$

14- Dual-Hesitant Fuzzy decision by opinion score method

Dual-Hesitant Fuzzy Sets (DHFS) are a kind of fuzzy set which combines the benefits of intuitionistic fuzzy set (IFS) and hesitant fuzzy set (HFS). DHFS enables decision makers to communicate their hesitations by correlating membership grades with numerous alternative values, rather than just two values [82]. In DHFS, membership grades and non-membership grades are represented by a set of predetermined values, allowing it to be both precise and flexible in representing real-world problems. DHFS is different from IFSs and HFSs in that it expresses a collection of possible values instead of a single number or a margin of error [15]. DHFS is utilized in the domain of multicriteria decision making (MCDM) to deal with imprecise and ambiguous information, and it has been extended to tackle similar problems in various fuzzy set environments [83]. Table 16 is shown Linguistic terms and their corresponding DHFNS.

Table16 . - Linguistic Terms and Their Corresponding DHFNS [15]

Linguistic scoring scale	DHFNS	
	$\tilde{h}$	$\tilde{g}$
No difference	0.90	0.05
Slight difference	0.75	0.20
Difference	0.50	0.50
Big difference	0.23	0.65

DHFSs are define as extensions to HFSs. A DHFS D in X, given a fixed set U, is expressed as
$\tilde{D} = \{x, \tilde{h}_{\tilde{D}}(x), \tilde{g}_{\tilde{D}}(x) \mid x \in U\}$

, in which $\tilde{h}_{\tilde{D}}(x)$ and $\tilde{g}_{\tilde{D}}(x)$ are in the range [0], [1] refer to the degrees of membership and non-membership of the element x ∊ U to the collection D, respectively, In the following conditions

$0 \leq \gamma, \eta \leq 0 \leq \gamma^+ + \eta^+ \leq 1$ for all $x \in U \gamma \in \tilde{h}_{\tilde{D}}(x), \eta \in \tilde{g}_{\tilde{D}}(x), \gamma^+ \in \tilde{h}_{\tilde{D}}^+(x) = \cup_{\gamma \in \tilde{h}_{\tilde{D}}(x)} \max\{\gamma\}, \eta^+ \in \tilde{g}_{\tilde{D}}(x) = \cup_{\eta \in \tilde{g}_{\tilde{D}}(x)} \mathrm{Max}\{\eta\}$

Each fuzzy opinion matrix can be aggregated by using:

dual hesitant fuzzy weighted averaging (DHFWA) operator, as represent in (66), or
dual hesitant fuzzy weighted geometric (DHFWG) operator, as displayed in (67)

$$\mathrm{DHFWA}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right) = \oplus_{j=1}^{n}\left(\omega_j \tilde{d}_j\right) = \cup_{\tilde{\gamma}_j \in \tilde{h}_j, \tilde{\eta}_j \in \tilde{g}_j}\left\{\left\{1-\prod_{j=1}^{n}(1-\tilde{\gamma})^{\omega_j}\right\}, \left\{\prod_{j=1}^{n}(\tilde{\eta})^{\omega_j}\right\}\right\}. \quad (66)$$

$$\mathrm{DHFWG}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right) = \oplus_{j=1}^{n}\left(\tilde{d}_j^{\omega_j}\right) = \cup_{\tilde{\gamma}_j \in \tilde{h}_j, \tilde{\eta}_j \in \tilde{g}_j}\left\{\left\{\prod_{j=1}^{n}(\tilde{\gamma})^{\omega_j}\right\}, \left\{1-\prod_{j=1}^{n}(1-\tilde{\eta})^{\omega_j}\right\}\right\}. \quad (67)$$

15- Rough Fermatean fuzzy sets (RFFSs)

Rough Fermatean fuzzy sets (RFFSs) are a kind of fuzzy set that have been defined in the literature. They are depending on the rough set concept, which is a data mining method utilized to detect hidden patterns in data [84]. Because they are capable of overcoming imprecision and uncertainty, RFFSs are considered to be more reliable than intuitionistic fuzzy sets (IFS) and Pythagorean fuzzy sets (PFS). RFFSs have been combined with robust MCDM methods such as Fuzzy-Weighted Zero-Inconsistency (FWZIC) and fuzzy decision by opinion score method (FDOSM) to generate exact solutions with decreasing uncertainty [49]. Table 17 shows the Linguistic terms and FFSNs.

Table 17. - Linguistic Terms and FFSNs [49]

Linguistic scale	Numerical scale	M	V
No difference	1	0.85	0.2
Slight difference	2	0.7	0.35
Difference	3	0.55	0.5
Big difference	4	0.35	0.7
Huge difference	5	0.2	0.85

Definition (1), for the application of FFS

Definition (1) Let N be the discourse universe. Let NMG and the membership degree (MD) be $\mu, v : N \rightarrow [0,1]$. Then, for any ħ∈N, the MD and non-membership degree (NMD) are denoted by the terms "μ(ħ)" and "v(ħ). There are two ways to present the FFN representation (see (68)):

$$\begin{aligned} N &= ((\mu(\hbar), v(\hbar))) \\ &= (\langle \mu(\hbar), v(\hbar) \rangle) \end{aligned} \tag{68}$$

where $\mu(\bar{\hbar}), v(\hbar) \in [0,1]$

$0 \leq \mu(\hbar)^3 + v(\hbar)^3 \leq 1$

Definition (2) The definition of the score function of FFS is show in (69) .

$$\mathrm{Score}(\alpha) = \left[(\mu_\gamma)^3 - (v_\gamma)^3 \right] \tag{69}$$

The scores for each criterion are arranged from lowest to highest. The lowest FFS value in the first set is an estimation of the lower space. The upper space estimate is then determined as the mean of the remaining FFS values utilizing a similar criterion as in Definition (3). This technique is followed for all FFS values in order to meet all requirements. When the highest order (final FFS) is reached, the closest approximation is taken as the FFS, and the lowest value is selected as the arithmetic mean of the remaining values, as defined in Definition (4).

Definition (3)

$$\begin{aligned} \mathrm{IFAM}(\gamma) &= \frac{1}{n}(\gamma_1 \oplus \gamma_2 \oplus \cdots \oplus \gamma_n) \\ &= \left(\left[1 - \prod_{j=1}^{n} \left(1 - \mu_{\gamma_j}^3\right)^{\frac{1}{n}} \right]^{1/3}, \prod_{j=1}^{n} v_{\gamma_j}^{\frac{1}{n}} \right) \end{aligned} \tag{70}$$

Definition (4) Let F be the universe of discourse. Let $\mu, v : F \rightarrow [0,1]$ be the MD and NMG. Then, for any $\hbar \in F$, the terms $\mu(\hbar)$ and $v(\hbar)$ refer to the MD and NMD. The FFRNs are represented as follows (see (71)):

$$\begin{aligned} N &= \left(\left[\underline{\mu}(\hbar), \underline{v}(\hbar) \right], [\bar{\mu}(\hbar), \bar{v}(\hbar)] \right) \\ &= \left(\left\langle \underline{\mu}(\hbar), \underline{v}(\hbar) \right\rangle, \langle \bar{\mu}(\hbar), \bar{v}(\hbar) \rangle \right) \end{aligned} \tag{71}$$

where $\underline{\mu}(\hbar), \underline{v}(\hbar), \bar{\mu}(\hbar), \bar{v}(\hbar) \in [0,1]$

$0 \leq \underline{\mu}(\hbar)^3 + \underline{v}(\hbar)^3 \leq 1$

$0 \leq \bar{\mu}(\hbar)^3 + \bar{v}(\hbar)^3 \leq 1$

Definition (5) The intuitionistic fuzzy rough arithmetic mean operator of dimension n is performed (see (72)):

$$\mathrm{FFRA}(\gamma) = \frac{1}{n}(\gamma_1 \oplus \gamma_2 \oplus \cdots \oplus \gamma_n)$$

$$= \left(\begin{array}{l} \left\{ \left[1 - \prod_{j=1}^{n} \left(1 - \underline{\mu}_{\Upsilon_j}^{3} \right)^{\frac{1}{n}} \right]^{3}, \prod_{j=1}^{n} \underline{v}_{\Upsilon_j}^{\frac{1}{n}} \right\}, \\ \left\{ \left[1 - \prod_{j=1}^{n} \left(1 - \bar{\mu}_{\Upsilon_j}^{3} \right)^{\frac{1}{n}} \right]^{3}, \prod_{j=1}^{n} \bar{v}_{\Upsilon_j}^{\frac{1}{n}}, \right\} \end{array} \right) \tag{72}$$

Definition (6) The FFRSs scoring function is defined as follows (see (73)):

$$\mathrm{Score}(\alpha) = \frac{\left[2 + \underline{\mu}_{\Upsilon}^{3} + \bar{\mu}_{\Upsilon}^{3} - \underline{v}_{\Upsilon}^{3} - \bar{v}_{\Upsilon}^{3} \right]}{4} \tag{73}$$

To determine the weight value for every criterion, the weight values of all criteria were combined together in a process called rescaling to determine the final values of the weight (see (74)).

$$w_j = s_j / \sum_{j=1}^{J} s_j \tag{74}$$

Where s_j is the score of every criterion's weight value.

Definition (7) The 'fuzzy rough EINSTEIN weighted aggregation (FFREWA) operator' is performed as the following:

$$\mathrm{FFREWA}\big(\Upsilon(\pounds_1), \Upsilon(\pounds_2), \ldots, \Upsilon(\pounds_n)\big)\big) = \left(\oplus_{i=1}^{n} w_l \underline{\Upsilon}(\pounds_l), \oplus_{i=1}^{n} w_l \overline{\Upsilon}(\pounds_2) \right)$$

$$= \left(\begin{array}{ll} \dfrac{\sqrt[3]{\prod_{i=1}^{n} \left[1+\underline{\mu}_i^{3}\right]^{w_i} - \prod_{i=1}^{n} \left[1-\underline{\mu}_i^{3}\right]^{w_i}}}{\sqrt[3]{\prod_{i=1}^{n} \left[1+\underline{\mu}_i^{3}\right]^{w_i} + \prod_{i=1}^{n} \left[1-\underline{\mu}_i^{3}\right]^{w_i}}}, & \dfrac{\sqrt[3]{2} \prod_{i=1}^{n} [\underline{v}_i]^{w_i}}{\sqrt[3]{\prod_{i=1}^{n} [2-\underline{v}_i^{3}]^{w_i} + \prod_{i=1}^{n} [\underline{v}_i]^{3w_i}}} \\ \dfrac{\sqrt[3]{\prod_{i=1}^{n} [1+\bar{\mu}_i^{3}]_i^{w_i} - \prod_{i=1}^{n} [1-\bar{\mu}_i^{3}]^{w_i}}}{\sqrt[3]{\prod_{i=1}^{n} [1+\bar{\mu}_i^{3}]^{w_i} - \prod_{i=1}^{n} [1-\bar{\mu}_i^{3}]^{w_i}}}, & \dfrac{\sqrt[3]{2} \prod_{i=1}^{n} [\bar{v}_i]^{w_i}}{\sqrt[3]{\prod_{i=1}^{n} [2-\bar{v}_i^{3}]^{w_i} + \prod_{i=1}^{n} [\bar{v}_i]^{3w_i}}} \end{array} \right) \tag{75}$$

16- Hexagonal Fuzzy Number (HXFN)

The concept of the Hexagonal-Fuzzy Number (HXFN) was introduced by Rajarajeshwari in 2013 [85]. is determined by six tuples W= (e, r, t, y, s, d); in which (e, r, t, y, s, and d) are real numbers. This kind was chosen for numerous reasons that distinguish this kind, including: (1) The hexagonal-fuzzy number (HXFN) preserves the essence of a fuzzy number while reducing information loss [86]. (2) In a few situations, Trapezoidal or Triangular is not relevant to handle the issue if it has six different points; in such cases, hexagonal fuzzy numbers are used. Hexagonal-fuzzy numbers (HFN) identify ambiguity more comprehensively than triangular (TFN), trapezoidal (TrFN) [85] [87]. (3) Using this fuzzy number, we examine the expert data more closely [87]. The Linguistic terms to HFN is shown in Table 18.

Table 18. - Linguistic Terms to HFN [21]

Linguistic terms	HFN
No difference	(1,2,4,6,7,9)
Slight difference	(2,4,6,7,9,11)
Difference	(4,6,7,9,11,13)
Big difference	(6,7,9,11,13,15)
Huge difference	(7,9,11,13,15,16)

[HFN] $\tilde{F}_H$ Is a Fuzzy Number denoted by $\tilde{F}_H = (\hat{j}_1, \hat{j}_2, \hat{j}_3, \hat{j}_4, \hat{j}_5, \hat{j}_6)$, where $\hat{j}_1, \hat{j}_2, \hat{j}_3, \hat{j}_4, \hat{j}_5$ and $\hat{j}_6$ are real numbers, and $\hat{j}1 \le \hat{j}2 \le \hat{j}3 \le \hat{j}4 \le \hat{j}5 \le \hat{j}6$. We can represent its membership function as following [21]:

$$\mu_{\tilde{F}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-\hat{\jmath}_1}{\hat{\jmath}_2-\hat{\jmath}_1}\right), & \text{for } \hat{\jmath}_1 \leq x \leq \hat{\jmath}_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-\hat{\jmath}_2}{\hat{\jmath}_3-\hat{\jmath}_2}\right), & \text{for } \hat{\jmath}_2 \leq x \leq \hat{\jmath}_3 \\ 1, & \text{for } \hat{\jmath}_3 \leq x \leq \hat{\jmath}_4 \\ 1-\frac{1}{2}\left(\frac{x-\hat{\jmath}_4}{\hat{\jmath}_5-\hat{\jmath}_4}\right), & \text{for } \hat{\jmath}_4 \leq x \leq \hat{\jmath}_5 \\ \frac{1}{2}\left(\frac{\hat{\jmath}_6-x}{\hat{\jmath}_6-\hat{\jmath}_5}\right), & \text{for } \hat{\jmath}_5 \leq x \leq \hat{\jmath}_6 \\ 0, & \text{otherwise} \end{cases} \quad (76)$$

Operation on Hexagonal-Fuzzy Numbers:

Assume that in order to specify the arithmetic operations between two HFN,

U = (Ù1, Ù2, Ù3, Ù4, Ù5, Ù6) and w = (ŵ1, ŵ2, ŵ3, ŵ4, ŵ5, ŵ6):

1. *Addition:* $(U+w) = (\grave{U}_1+\hat{w}_1, \grave{U}_2+\hat{w}_2, \grave{U}_3+\hat{w}_3, \grave{U}_4+\hat{w}_4, \grave{U}_5+\hat{w}_5, \grave{U}_6+\hat{w}_6)$ (77)
2. *Subtraction:* $(U-w) = (\grave{U}_1-\hat{w}_1, \grave{U}_2-\hat{w}_2, \grave{U}_3-\hat{w}_3, \grave{U}_4-\hat{w}_4, \grave{U}_5-\hat{w}_5, \grave{U}_6-\hat{w}_6)$ (78)
3. *Multiplication:* $(U\times w) = (\grave{U}_1\hat{w}_1, \grave{U}_2\hat{w}_2, \grave{U}_3\hat{w}_3, \grave{U}_4\hat{w}_4, \grave{U}_5\hat{w}_5, \grave{U}_6\hat{w}_6)$ (79)
4. *Division:* $\left(\frac{U}{w}\right) = \left(\frac{\grave{U}_1}{\hat{w}_1}, \frac{\grave{U}_2}{\hat{w}_2}, \frac{\grave{U}_3}{\hat{w}_3}, \frac{\grave{U}_4}{\hat{w}_4}, \frac{\grave{U}_5}{\hat{w}_5}, \frac{\grave{U}_6}{\hat{w}_6}\right)$ (80)

17- Z-Cloud Rough Numbers (ZCRNs)

Are a paradigm for handling uncertainty that integrates cloud model theory, Z-number theory, and rough number theory. The cloud model theory is utilized to represent and capture experts' ambiguous preferences, whereas the Z-number theory allow experts to present their fuzzy preferences and judgment reliability in a single arranged pair (A, B) where A denotes the ambiguous amount of the evaluated item and B expresses the fuzziness of A's reliability [88]. Rough number theory is used to deal with interpersonal ambiguity by characterizing groupings of items and determining their upper and lower limits using higher and lower approximations. The ZCRNs model integrated these theories to handle both individual and interpersonal uncertainty, giving a comprehensive framework to address ambiguity in data and making assessments in different fields [89]. Table 19 shows the converting the linguistic terms into ZC Likert scales.

Table 19. - Converting the linguistic Terms into ZC Likert Scales [37]

A				**B**			
Linguistic terms	**Cloud value**			**Linguistic terms**	**TFNs**		
No difference	0.000	0.673	0.101	Very small (VS)	0.100	0.200	0.300
Slight difference	3.098	0.453	0.068	Small (S)	0.300	0.400	0.500
Difference	5.000	0.278	0.041	Medium (M)	0.600	0.700	0.800
Big difference	8.262	0.579	0.086	High (H)	0.700	0.800	0.900
Huge difference	10	0.673	0.101	Very high (VH)	0.900	1.000	1.000

Z-number indicates an ordered pair of fuzzy numbers that appear as Z= (A, B). The computation complexity of Z The following equations is used to convert the Z number to ZC, as well as the ZC transform to ZCR [37]:

Step 1: Transformation the reliability $\widetilde{B}$ of the element $\widetilde{A}$ into a real number.

$$\tilde{\alpha} = \frac{\int x\varphi_B(x)dx}{\int \varphi_B(y)dx} \quad (81)$$

Where ∫ represent an integration in algebra.

Step 2: The weighted Z-number is obtained by first transforming the value of judgment reliability $\widetilde{B}$ into the fuzzy restriction $\widetilde{A}$.

$$\tilde{Z}^{\tilde{\alpha}} = \left\{\langle x, \tilde{A}^{\sim}(x)\rangle \mid \mu_{\tilde{A}}^{\tilde{z}}(x) = \tilde{\alpha}\mu_{\tilde{A}}(x), x \in X\right\} \quad (82)$$

For simplicity, the Z number is represented by the symbol $\tilde{Z}^{\tilde{\alpha}} = (\tilde{A}, \tilde{\alpha})$.

Step 3: transform the irregular cloud number to a conventional cloud value number.

$$\tilde{Z} = \left\{\langle x, \mu_{\tilde{Z}}(x)\rangle \mid \mu_{\tilde{Z}}(x) = \mu\left(\frac{x}{\sqrt{\tilde{\alpha}}}\right), x \in X\right\} \quad (83)$$

Following that, the steps below outline the basic methods for converting ZC numbers to ZC rough numbers [90].
Let $\tilde{Z}_i^{Ex} = \{\tilde{E}x_1, \tilde{E}x_2, \dots, \tilde{E}x_n\}, \tilde{Z}E = \{\tilde{E}n_1, \tilde{E}n_2, \dots, \tilde{E}n_n\}$, and $\tilde{Z}_i^{He} = \{\tilde{H}e_1, \tilde{H}e_2, \dots, \tilde{H}e_n\}$
The lower approximation Apr $(\tilde{Z}_i)$ of $\tilde{Z}_i$ can thus be identified as follows:

$$\underline{\mathrm{Apr}}(\tilde{E}x_i) = \cup \{\tilde{E}x_j \in \tilde{Z}\tilde{Z}_i^{Ex} \mid \tilde{E}x_j \leqslant \tilde{E}x_i\} \quad (84)$$

$$\underline{\mathrm{Apr}}(\tilde{E}n_i) = \cup \{\tilde{E}n_j \in \tilde{Z}\tilde{Z}_i^{En} \mid \tilde{E}n_j \leqslant \tilde{E}n_i\} \quad (85)$$

$$\underline{\mathrm{Apr}}(\tilde{H}e_i) = \cup \{\tilde{H}e_j \in \tilde{Z}_i^{He} \mid \tilde{H}e_j \leqslant \tilde{H}e_i\} \quad (86)$$

Where $(Ex_i, \tilde{E}n_i, \tilde{H}e_i)$ are elements in $(\tilde{Z}_i^{Ex}, \tilde{Z}_i^{En}, \tilde{Z}_i^{He})$ respectively; 1 i, j k The lower approximation $\underline{\mathrm{Apr}}(\tilde{E}x_i)$ of $\tilde{E}x_i$ involves all elements in $\tilde{Z}_i^{Ex}$ that have class values equal to and less than $(\tilde{E}x_i)$. Likewise with the rest.

Likewise, the upper approximation Apr $(\tilde{Z}_i)$ of $\tilde{Z}_i$ can be determined to be follows:

$$\overline{\mathrm{Apr}}(\tilde{E}x_i) = \cup \{\tilde{E}x_j \in \tilde{Z}\tilde{Z}_i^{Ex} \mid \tilde{E}x_j \geqslant \tilde{E}x_i\} \quad (87)$$

$$\overline{\mathrm{Apr}}(\tilde{E}n_i) = \cup \{\tilde{E}n_j \in \tilde{Z}_i^{En} \mid \tilde{E}n_j \geqslant \tilde{E}n_i\} \quad (88)$$

$$\overline{Apr}(\tilde{H}e_i) = \cup \{\tilde{H}e_j \in \tilde{Z}\tilde{Z}_i^{He} \mid \tilde{H}e_j \geqslant \tilde{H}e_i\} \quad (89)$$

The lower approximation $\underline{\mathrm{Apr}}(\tilde{E}x_i)$ of $\tilde{E}x_i$ contains all objects in the set $\tilde{Z}_i^{Ex}\tilde{E}x_i$. Next, the lower limit $\underline{\mathrm{Lim}}(\tilde{Z}_i)$ of $\tilde{Z}_i$ is calculated as:

$$\underline{\mathrm{Lim}}(\tilde{E}x_i) = \frac{1}{\vartheta_L^{Ex}} \sum_{j=1}^{v_L^{Ex}} \tilde{E}x_j \mid \tilde{E}x_j \in \underline{\mathrm{Apr}}(\tilde{E}x_i) \quad (90)$$

$$\underline{\mathrm{Lim}}(\tilde{E}n_i) = \sqrt{\frac{1}{\vartheta_L^{En}} \sum_{j=1}^{v_L^{En}} (\tilde{E}n_j)^2} \mid \tilde{E}n_j \in \underline{\mathrm{Apr}}(\tilde{E}n_i) \quad (91)$$

$$\underline{\mathrm{Lim}}(\tilde{E}h_i) = \sqrt{\frac{1}{\vartheta_L^{He}} \sum_{j=1}^{v_L^{He}} (\tilde{H}e_j)^2} \mid \tilde{H}e_j \in \underline{\mathrm{Apr}}(\tilde{H}e_i) \quad (92)$$

Where $\vartheta_{I.}^{Ex}, \vartheta_{I.}^{En}$, and $\vartheta_{I.}^{He}$ express the total numbers of elements in Apr $(\tilde{E}x_i)$, $\underline{\mathrm{Apr}}(\tilde{E}n_i)$, $\underline{\mathrm{Apr}}(\tilde{H}e_i)$, respectively. For convenience, $\underline{\mathrm{Lim}}(\tilde{E}x_i)$, Lim $(\tilde{E}n_i)$, and $\underline{\mathrm{Lim}}(\tilde{H}e_i)$ are represent as $\tilde{E}x_i^L, \tilde{E}n_i^L$, and $\tilde{H}e_i^L$ in subsequent contents, respectively. In briefly, the lower limit of a class ZC value is the average value of the classes included in its lower approximal. Similar to this, the upper limit Lim $(\tilde{Z}_i)$ of $\tilde{Z}_i$ is identified as:

$$\overline{\mathrm{Lim}}(\tilde{E}x_i) = \frac{1}{\vartheta_U^{Ex}} \sum_{j=1}^{\vartheta_U^{Ex}} \tilde{E}x_j \mid \tilde{E}x_j \in \overline{Apr}(\tilde{E}x_i) \quad (93)$$

$$\overline{\mathrm{Lim}}(\tilde{E}n_i) = \sqrt{\frac{1}{\vartheta_U^{En}} \sum_{j=1}^{\vartheta_U^{En}} (\tilde{E}n_j)^2} \mid \tilde{E}x_j \in \overline{Apr}(\tilde{E}n_i) \quad (94)$$

$$\overline{\mathrm{Lim}}(\tilde{H}e_i) = \sqrt{\frac{1}{\vartheta_U^{He}} \sum_{j=1}^{\vartheta_U^{He}} (\tilde{H}e_j)^2} \mid \tilde{E}x_j \in \overline{Apr}(\tilde{H}e_i) \quad (95)$$

Where $\vartheta_U^{Ex}, \vartheta_U^{En}$, and ϑ_U^{He} refer to the total number of elements in $\overline{\mathrm{Apr}}(\tilde{E}x_i)$, $\overline{\mathrm{Apr}}(\tilde{E}n_i)$, and $\overline{Apr}(\tilde{H}e_i)$, respectively, for simplicity, Lim $(\tilde{E}x_i)$, Lim $(\tilde{E}n_i)$, and Lim $(\tilde{H}e_i)$ are represent as $\tilde{E}x_i^U, \tilde{E}n_i^U$, and $\tilde{H}e_i^U$ in the subsequent contents, respectively. The upper limit of a class ZC value is the average value of the classes including in its upper approximation. Once the lower limit $\underline{\mathrm{Lim}}(\tilde{Z}_i)$ and the upper limit $(\tilde{Z}_i)$ for an arbitrary Z-cloud class $\tilde{Z}_i$ have been established, the ZCRN value ZCRN $(\tilde{Z}_i)$ of $(\tilde{Z}_i)$ can be described as following:

$$[\tilde{Z}_i] = [\tilde{Z}_i^L, \tilde{Z}_i^U][(\tilde{E}x_i^L, \tilde{E}n_i^L, \tilde{H}e_i^L), (\tilde{E}x_i^U, \tilde{E}n_i^U, \tilde{H}e_i^U)] \quad (96)$$

Suppose $[\tilde{Z}_i][\tilde{Z}_i^L, \tilde{Z}_i^U]$
$= [(\tilde{E}x_i^L, \tilde{E}n_i^L, \tilde{H}e_i^L), (\tilde{E}x_i^U, \tilde{E}n_i^U, \tilde{H}e_i^U)] (i = 1,2, \dots, n)$

are nZCRNs, and w = ($w_1, w_2, \ldots, w_n$) is the weight vector of $[\tilde{Z}_i]$, with the condition $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. where the Z-cloud rough weighted average (ZCRWA) operator is defined in (88).

$$\begin{aligned} &\text{ZCRWA}\left([\tilde{Z}_1],[\tilde{Z}_2],\ldots,[\tilde{Z}_n]\right)=\sum_{i=1}^{n} w_i[\tilde{Z}_i] \\ &=\left[\left(\sum_{i=1}^{n} w_i\tilde{E}_i^L,\sqrt{\sum_{i=1}^{n} w_i\left(\tilde{E}n_i^L\right)^2},\sqrt{\sum_{i=1}^{n} w_i\left(\tilde{H}e_i^L\right)^2}\right),\left(\sum_{i=1}^{n} w_i\tilde{E}_i^U,\sqrt{\sum_{i=1}^{n} w_i\left(\tilde{E}n_i^U\right)^2},\sqrt{\sum_{i=1}^{n} w_i\left(\tilde{H}e_i^U\right)^2}\right)\right] \\ &=\left[\text{ZCWA}\left(\tilde{Z}_1^L,\tilde{Z}_2^L,\ldots,\tilde{Z}_n^L\right),\text{ZCWA}\left(\tilde{Z}_1^U,\tilde{Z}_2^U,\ldots,\tilde{Z}_n^U\right)\right] \end{aligned} \tag{97}$$

3.2 Aggregation Operators

In many fields, aggregation Operators are significant like decision making [91]. Researchers developed multiple aggregation operators in the literature to aggregate numerical data in various situations [92, 93]. The purpose of aggregation step's is combining a set of criteria in such a way that the final aggregate output takes into consideration the complete single criterion. Because the final option of rating is inherently distilled from this collection of comprehensive scores, valuable ratings are not eliminated for failing to meet some criteria [94]. Furthermore, several aggregation operators exist to solve MCDM problems, including Geometric. Mean (GM), Bonferroni mean (BM), Arithmetic Mean (AM) and others. Aggregation operators are a crucial feature of fuzzy logic, especially when dealing with many fuzzy numbers. These operators combine information from many fuzzy numbers to a single fuzzy number, summarizing the overall sentiment of the equations used with fuzzy numbers in the table below. Table 20 contains a summary of the aggregation operators and the equations of the fuzzy types.

Table 20. - The Aggregation Operators with Equations of the Fuzzy Type

Ref.	Fuzzy type	Aggregation Operator & Equation
[20]	q-rung orthopair Fuzzy set	**q-rung orthopair Fuzzy Weighted Arithmetic Mean (q-ROFWA)** $\text{q-ROFWA}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\left\langle\left(1-\prod_{k=1}^{n}\left(1-\mu_k^q\right)^{wk}\right)^{1/q},\prod_{k=1}^{n} v_k^{w_k}\right\rangle$ q-ROFWA is an aggregation operator used to combine multiple q-rung orthopair fuzzy numbers into a single q-rung orthopair fuzzy number. It essentially calculates a weighted average of the input fuzzy numbers.
[19]	Pythagorean fuzzy set	**Arithmetic Mean** $\text{PFWA}\left(p(m_1),p(m_2),\cdots,p(m_n)\right)=\left(\sum_{i=1}^{n} w_i\mu_p(m_i),\sum_{i=1}^{n} w_i v_p(m_i)\right)$
[43]	T-spherical fuzzy set	**Arithmetic Mean** $\text{T- SWAM}(\tilde{p}_1,\tilde{p}_2,\ldots,\tilde{p}_n)=\left\{\left[1-\prod_{i=1}^{n}\left(1-\mu_{\tilde{p}_i}^2\right)^{w_i}\right]^{1/T}\right.$ $\left.\prod_{i=1}^{n} v_{\tilde{p}_i}^{w_i},\left[\prod_{i=1}^{n}\left(1-\mu_{\tilde{p}_i}^2\right)^{w_i}-\prod_{i=1}^{n}\left(1-\mu_{\tilde{p}_i}^2-s_{\tilde{p}_i}^2\right)^{w_i}\right]^{1/T}\right\}$
[25]	Cubic Pythagorean Fuzzy set	**Arithmetic Mean** $\text{CPFWA}(e_1 e_2\ldots,e_h)=\left(\begin{array}{c}\left\langle\left[\sqrt{1-\Pi_{i=1}^{n}\left(1-\left(\mu_{e_i}^L\right)^2\right)^{w_i}},\sqrt{1-\Pi_{i=1}^{n}\left(1-\left(\mu_{e_i}^U\right)^2\right)^{w_i}}\right]\right. \\ \left[\Pi_{i=1}^{n}\left(v_{e_i}^L\right)^{w_i},\Pi_{i=1}^{n}\left(v_{e_i}^U\right)^{w_i}\right] \\ \sqrt{1-\Pi_{i=1}^{n}\left(1-\left(\mu_{e_i}^L\right)^2\right)^{w_i}},\prod_{i=1}^{n}\left(v_{e_i}\right)^{w_i}\end{array}\right)$
[40]	neutrosophic fuzzy set	**Neutrosophic Weighted Geometric** $\text{SNWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\prod_{j=1}^{n}\tilde{a}_j^{\omega_j}=\left(\prod_{j=1}^{n}\rho_j^{\omega_j},1-\prod_{j=1}^{n}\left(1-\sigma_j\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-\tau_j\right)^{\omega_j}\right),$

[44]	interval type-2 trapezoidal	**Arithmetic Mean** $\overset{\approx}{A}_1 \oplus \overset{\approx}{A}_2 = \left(\widetilde{A_1^U}, \widetilde{A_1^L}\right) \oplus \left(\widetilde{A_2^U}, \widetilde{A_2^L}\right) = \Big(\big(a_{11}^U + a_{21}^U, a_{12}^U + \tilde{a}_{22}^U, a_{13}^U + \tilde{a}_{23}^U, a_{14}^U + a_{24}^U; min\left(H_1\left(\widetilde{A_1^U}\right), H_1(A_2^U)\right), min\left(H_2(A_1^U), H_2(\tilde{A}_2^U)\right)\big), \big(a_{11}^L + a_{21}^L, a_{12}^L + \tilde{a}_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; min\left(H_1(\tilde{A}_1^L), H_1(A_2^L)\right), min\left(H_2(A_1^L), H_2(\tilde{A}_2^L)\right)\big)\Big)$
[27]	Fermatean fuzzy set	**Arithmetic Mean** $\tilde{A} + \tilde{B} = \left(\sqrt[3]{\alpha_A^3 + \alpha_B^3 - \alpha_A^3\alpha_B^3}, \beta_A\beta_B\right)$
[41]	q-rung orthopair probabilistic hesitant	q-ROPHF Einstein arithmetic mean (q-ROPHEWAM) q-ROPHFEWAM ((w1, . . ., wr) , (Q1, . . . , Qr)) $= \left(\begin{array}{c} \bigcup_{\mu_{Q_i} \in \mathcal{M}_i} \left\{\dfrac{\sqrt[q]{\prod_{i=1}^r \left(1 + (\mu_{Q_i})^q\right)^{w_i} - \prod_{i=1}^r \left(1 - (\mu_{Q_i})^q\right)^{w_i}}}{\sqrt[q]{\prod_{i=1}^r \left(1 + (\mu_{Q_i})^q\right)^{w_i} + \prod_{i=1}^r \left(1 - (\mu_{Q_i})^q\right)^{w_i}}}\right\} \mid \prod_{i=1}^r p_{\mu_{Q_i}}, \\ \bigcup_{v_{Q_i} \in \mathcal{N}_i} \left\{\dfrac{\sqrt[q]{2\prod_{i=1}^r \left((v_{Q_i})^q\right)^{w_i}}}{\sqrt[q]{\prod_{i=1}^r \left(2 - (v_{Q_i})^q\right)^{w_i} + \prod_{i=1}^r \left((v_{Q_i})^q\right)^{w_i}}}\right\} \mid \prod_{i=1}^r p_{v_{Q_i}} \end{array}\right)$
[45]	2-Tuple Fuzzy	**Arithmetic Mean** $\overline{x^e} = \Delta\left(\sum_{i=1}^n \frac{1}{n}\Delta^{-1}(r_i, \alpha_i)\right) = \Delta\left(\frac{1}{n}\sum_{i=1}^m \beta_i\right)$
[24]	intuitionistic fuzzy set	**Arithmetic mean** $\left(1 - \prod_{j=1}^n \left(1 - \mu_{\beta_{\sigma(j)}}\right), \prod_{j=1}^n v_{\beta_{\sigma(j)}}\right)$
[46]	Fermatean probabilistic hesitant	**Fermatean probabilistic hesitant fuzzy weighted average (FPHFWA)** FPHFWA (ℵ1, ℵ2, . . ., ℵr) $= \left(\begin{array}{c} \bigcup_{\hbar_{\hat{j}} \in \tau_{\aleph_j}, \overline{\aleph}_{\aleph_j} \in \bar{p}_{\aleph_j}} \sqrt[3]{1 - \prod_{\hat{j}=1}^r \left(1 - \left(\hbar_{\aleph_j}\right)^3\right)^{W_{\hat{j}}}} / \Pi_{\hat{j}=1}^r \tilde{p}_{\aleph_j} \\ \bigcup_{\varrho_{\aleph_j} \in \partial_{\aleph_j}, b_{\aleph_j} \in b_{\aleph_j}} \prod_{\hat{j}=1}^r \left(\varrho_{\aleph_j}\right)^{w_{\hat{j}}} / \Pi_{\hat{j}=1}^r b_{\aleph_j} \end{array}\right)$

[47] spherical fuzzy rough

Arithmetic mean

$$\mathbf{SFREWA}\left(g(b_1), g(b_1), \dots, g(b_n)\right) = \left\{\oplus_{i=1}^{n} w_i \underline{g}(b_i), \oplus_{i=1}^{n} w_i \overline{g}(b_i)\right\}$$

$$= \left\{ \sqrt{\frac{\prod_{j=1}^{k}\left(1+\underline{\mu}_j^2\right)^{w_j} - \prod_{j=1}^{k}\left(1-\underline{\mu}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(1+\underline{\mu}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(1-\underline{\mu}_j^2\right)^{w_j}}}, \right.$$

$$\sqrt[t]{\frac{2\prod_{j=1}^{k}\left(\underline{v}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(2-\underline{v}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(\underline{v}_j^2\right)^{w_j}}},$$

$$\left. \sqrt[t]{\frac{2\prod_{j=1}^{k}\left(\underline{\pi}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(2-\underline{\pi}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(\underline{\pi}_j^2\right)^{w_j}}} \right\},$$

$$\left\{ \sqrt{\frac{\prod_{j=1}^{k}\left(1+\overline{\mu}_j^2\right)^{w_j} - \prod_{j=1}^{k}\left(1-\overline{\mu}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(1+\overline{\mu}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(1-\overline{\mu}_j^2\right)^{w_j}}}, \right.$$

$$\sqrt[t]{\frac{2\prod_{j=1}^{k}\left(\overline{v}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(2-\overline{v}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(\overline{v}_j^2\right)^{w_j}}},$$

$$\left. \sqrt[t]{\frac{2\prod_{j=1}^{k}\left(\overline{\pi}_j^2\right)^{w_j}}{\prod_{j=1}^{k}\left(2-\overline{\pi}_j^2\right)^{w_j} + \prod_{j=1}^{k}\left(\overline{\pi}_j^2\right)^{w_j}}} \right\}, \Bigg)$$

[48] q-rung orthopair fuzzy rough

fuzzy rough EINSTEIN weighted aggregation operator (q-ROFREWA)

$$\boldsymbol{q-\mathbf{ROFREWA}}\left(\Upsilon(\pounds_1), \Upsilon(\pounds_2), \dots, \Upsilon(\pounds_n)\right)\Big) = \left(\oplus_{i=1}^{n} w_l \underline{\Upsilon}(\pounds_l), \oplus_{i=1}^{n} w_l \overline{\Upsilon}(\pounds_2)\right) =$$

$$\left[\begin{array}{l} \left(\sqrt[q]{\dfrac{\prod_{i=1}^{n}\left[1+\underline{\mu}_i^q\right]^{w_i} - \prod_{i=1}^{n}\left[1-\underline{\mu}_i^q\right]^{w_i}}{\prod_{i=1}^{n}\left[1+\underline{\mu}_i^q\right]^{w_i} + \prod_{i=1}^{n}\left[1-\underline{\mu}_i^q\right]^{w_i}}}, \dfrac{\sqrt[q]{2}\prod_{i=1}^{n}\left[\underline{v}_i^q\right]^{w_i}}{\sqrt[q]{\prod_{i=1}^{n}\left[2-\underline{v}_i^q\right]^{w_i} + \prod_{i=1}^{n}\left[\underline{v}_i^q\right]^{w_i}}}\right), \\ \left(\dfrac{\prod_{i=1}^{n}\left[1+\overline{\mu}_i^q\right]_i^{w_i} - \prod_{i=1}^{n}\left[1-\overline{\mu}_i^q\right]^{w_i}}{\prod_{i=1}^{n}\left[1+\overline{\mu}_i^q\right]^{w_i} + \prod_{i=1}^{n}\left[1-\overline{\mu}_i^q\right]^{w_i}}, \dfrac{\sqrt[q]{2}\prod_{i=1}^{n}\left[\overline{v}_i^q\right]^{w_i}}{\sqrt[q]{\prod_{i=1}^{n}\left[2-\overline{v}_i^q\right]^{w_i} + \prod_{i=1}^{n}\left[\overline{v}_i^q\right]^{w_i}}}\right) \end{array}\right]$$

[15] Dual-Hesitant Fuzzy

i) dual hesitant fuzzy weighted averaging (DHFWA)

$$\mathbf{DHFWA}\left(\tilde{d}_1, \tilde{d}_2, \dots, \tilde{d}_n\right) = \oplus_{j=1}^{n}\left(\omega_j \tilde{d}_j\right)$$

$$= \bigcup_{\tilde{\gamma}_j \in \tilde{h}_j, \tilde{\eta}_j \in \tilde{g}_j} \left\{\left\{1 - \prod_{j=1}^{n}\left(1-\tilde{\gamma}\right)^{\omega_j}\right\}, \left\{\prod_{j=1}^{n}\left(\tilde{\eta}\right)^{\omega_j}\right\}\right\}.$$

ii) dual hesitant fuzzy weighted geometric (DHFWG)

$$\mathbf{DHFWG}\left(\tilde{d}_1, \tilde{d}_2, \dots, \tilde{d}_n\right) = \oplus_{j=1}^{n}\left(\tilde{d}_j^{\omega_j}\right)$$

$$= \cup_{\tilde{\gamma}_j \in \tilde{h}_j, \tilde{\eta}_j \in \tilde{g}_j} \left\{\left\{\prod_{j=1}^{n}\left(\tilde{\gamma}\right)^{\omega_j}\right\}, \left\{1 - \prod_{j=1}^{n}\left(1-\tilde{\eta}\right)^{\omega_j}\right\}\right\}.$$

[49] Rough Fermatean fuzzy

fuzzy rough EINSTEIN weighted aggregation

(FFREWA)

$$\mathrm{FFREWA}\left(\Upsilon(\pounds_1), \Upsilon(\pounds_2), \dots, \Upsilon(\pounds_n)\right)\Big) = \left(\oplus_{i=1}^{n} w_l \underline{\Upsilon}(\pounds_l), \oplus_{i=1}^{n} w_l \overline{\Upsilon}(\pounds_2)\right)$$

$$= \left(\begin{array}{ll} \dfrac{\sqrt[3]{\prod_{i=1}^{n}\left[1+\underline{\mu}_i^3\right]^{w_i} - \prod_{i=1}^{n}\left[1-\underline{\mu}_i^3\right]^{w_i}}}{\sqrt[3]{\prod_{i=1}^{n}\left[1+\underline{\mu}_i^3\right]^{w_i} + \prod_{i=1}^{n}\left[1-\underline{\mu}_i^3\right]^{w_i}}}, & \dfrac{\sqrt[3]{2}\prod_{i=1}^{n}\left[\underline{v}_i\right]^{w_i}}{\sqrt[3]{\prod_{i=1}^{n}\left[2-\underline{v}_i^3\right]^{w_i} + \prod_{i=1}^{n}\left[\underline{v}_i\right]^{3w_i}}} \\ \dfrac{\sqrt[3]{\prod_{i=1}^{n}\left[1+\overline{\mu}_i^3\right]_i^{w_i} - \prod_{i=1}^{n}\left[1-\overline{\mu}_i^3\right]^{w_i}}}{\sqrt[3]{\prod_{i=1}^{n}\left[1+\overline{\mu}_i^3\right]^{w_i} - \prod_{i=1}^{n}\left[1-\overline{\mu}_i^3\right]^{w_i}}}, & \dfrac{\sqrt[3]{2}\prod_{i=1}^{n}\left[\overline{v}_i\right]^{w_i}}{\sqrt[3]{\prod_{i=1}^{n}\left[2-\overline{v}_i^3\right]^{w_i} + \prod_{i=1}^{n}\left[\overline{v}_i\right]^{3w_i}}} \end{array}\right)$$

[21] Hexagonal Fuzzy Numbers

Arithmetic Mean

$$(\boldsymbol{U} + \boldsymbol{w}) = \left(\grave{U}_1 + \hat{w}_1, \grave{U}_2 + \hat{w}_2, \grave{U}_3 + \hat{w}_3, \grave{U}_4 + \hat{w}_4, \grave{U}_5 + \hat{w}_5, \grave{U}_6 + \hat{w}_6\right)$$

[37]	Z-Cloud Rough Numbers	**Bonferroni Mean (BM)** $\mathbf{ZCRWA}([\tilde{Z}_1],[\tilde{Z}_2],\dots,[\tilde{Z}_n]) = \sum_{i=1}^{n} w_i[\tilde{Z}_i] = \left[\left(\sum_{i=1}^{n} w_i\tilde{E}_i^L, \sqrt{\sum_{i=1}^{n} w_i(\tilde{E}n_i^L)^2}, \sqrt{\sum_{i=1}^{n} w_i(\tilde{H}e_i^L)^2}\right), \left(\sum_{i=1}^{n} w_i\tilde{E}x_i^U, \sqrt{\sum_{i=1}^{n} w_i(\tilde{E}n_i^U)^2}, \sqrt{\sum_{i=1}^{n} w_i(\tilde{H}e_i^U)^2}\right)\right] = [\mathbf{ZCWA}(\tilde{Z}_1^L,\tilde{Z}_2^L,\dots,\tilde{Z}_n^L), \mathbf{ZCWA}(\tilde{Z}_1^U,\tilde{Z}_2^U,\dots,\tilde{Z}_n^U)]$

Figure 4 shows many aggregation operators available to handle MCDM problems from 2021 to 2023, including Geometric. Mean (GM), Bonferroni Mean (BM), Arithmetic Mean (AM), and others.

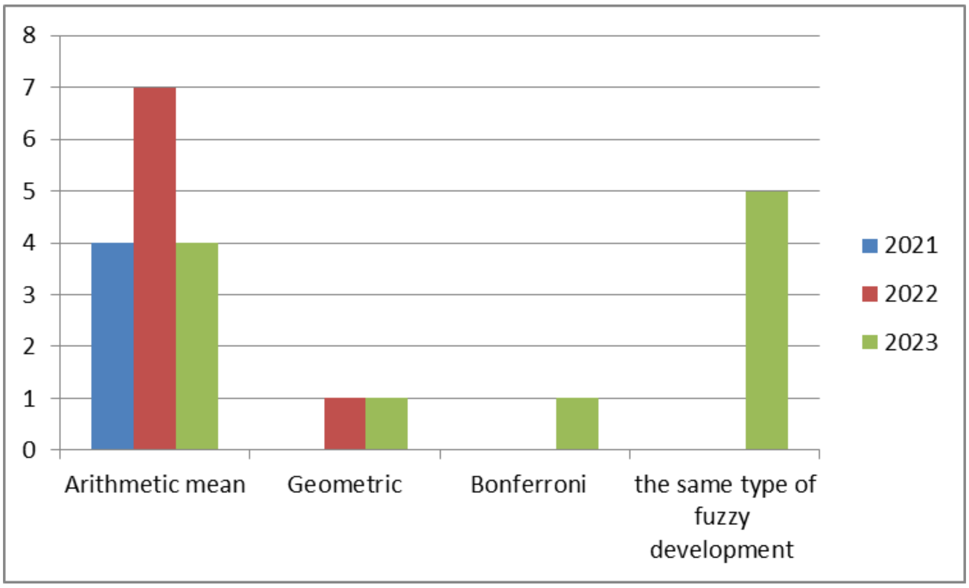

FIGURE 4. - Aggregation Operators for MCDM (2021-2023)

3.3 The Integration Method

Multiple studies have integrated FDOSM with MCDM methods or with other techniques to address a variety of complex MCDM problems in the studies [46]. Recently, several academic literatures have focused on the integration of FDOSM to facilitate decision-making under conditions of uncertainty and ambiguity issues [37]. The inconsistency and distance measurement issues can be successfully overcome by integrating a new version of FDOSM with other methods, providing a dynamic distribution mechanism for prioritizing distribution [19]. Integrating the Fuzzy Decision by Opinion Score Method (FDOSM) with other Multi-Criteria Decision-Making (MCDM) methods offers several advantages. Firstly, it enhances accuracy by capturing uncertainty and vagueness in decision-making. When integrated with other MCDM methods, FDOSM improves the accuracy of the decision-making process. Additionally, integrating FDOSM with other MCDM methods increases the robustness of the decision-making process. It enables effective handling of various types of criteria and uncertainties [27]. In addition, integrated the FDOSM with a new fuzzy sets to improve the final decision. In summary, integrating FDOSM with other MCDM methods leads to more informed, robust, and transparent decision-making processes [37]. Table 21 shows the summary of the methods that are integrated with FDOSM and are used to solve a specific problem.

Table 21.- Integrated Methods with FDOSM

Ref.	Fuzzy type	The integrated method	Year
[20, 56]	q-rung orthopair Fuzzy set	FDOSM+FWZIC	2022
[19]	Pythagorean fuzzy set	FDOSM+FWZIC	2022
[43]	T-spherical fuzzy	FDOSM+FWZIC	2021
[25]	Cubic Pythagorean Fuzzy	FDOSM+FWZIC	2021

[40]	neutrosophic fuzzy	FDOSM+FWZIC	2022
[44]	interval type-2 trapezoidal	N/A	2021
[27]	Fermatean fuzzy	N/A	2022
[41]	q-rung orthopair probabilistic hesitant	FDOSM+FWZIC + MULTIMOORA	2023
[45]	2-Tuple Fuzzy	N/A	2022
[24]	intuitionistic fuzzy set	N/A	2021
[46]	Fermatean probabilistic hesitant	FDOSM+FWZIC + multi attributive ideal-real comparative analysis (MAIRCA)	2023
[47]	spherical fuzzy rough	FDOSM+FWZIC	2023
[48]	q-rung orthopair fuzzy rough sets	FDOSM+FWZIC	2023
[15]	Dual-Hesitant Fuzzy	FDOSM+FWZIC	2023
[49]	Rough Fermatean fuzzy	FDOSM+FWZIC	2023
[21]	Hexagonal Fuzzy	N/A	2023
[37]	Z-Cloud Rough Numbers	FDOSM+FWZIC	2023
[22, 38]	Fermatean fuzzy set	FDOSM+ CRITIC	2022,2023
[32]	intuitionistic fuzzy set	FDOSM+FWZIC	2022
[42]	Pythagorean fuzzy set	N/A	2023

Figure 5 shows FDOSM integrated with other Methods to deal with various complicated MCDM problems in various studies based on the years of publication.

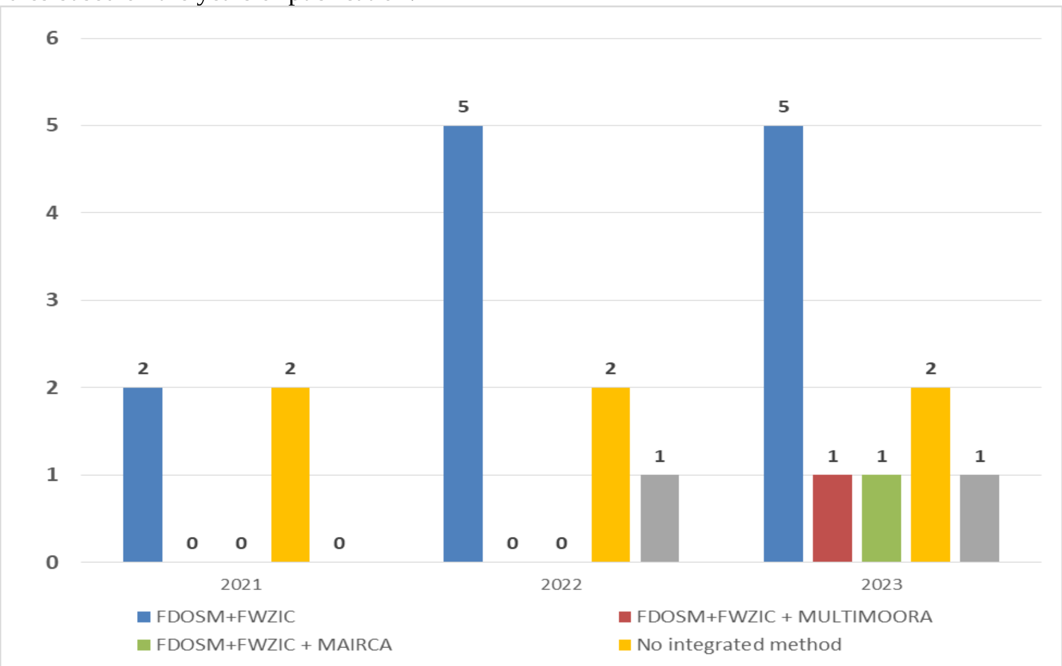

FIGURE 5. - FDOSM Integrated with Other Methods

3.4 The Defuzzification Method

After completing the aggregation process, the aggregated fuzzy values are defuzzied into crisp values according to the defuzzification equation, and the crisp values represent the score of each alternative for each expert [41]. There are several defuzzification methods used with fuzzy numbers to produce the final weighting and ranking results, It is specific in the equations mentioned in the table below . The specific equations might have slight variations depending on the type of fuzzy number being used It's always best to consult the relevant literature for the exact formulation applicable for each case [95]. Table 22 summarizes the defuzzification equations of the fuzzy types.

Table 22. - Summary of the Equation of Defuzzification for Fuzzy Type

Ref.	Fuzzy type	Defuzzification Equation
[20]	q-rung orthopair	$S_k = \mu_k^q - v_k^q$, where $q \geq 1$.
[19]	Pythagorean fuzzy set	$\alpha = (\mu_\alpha, v_\alpha)$ beaPFN,$s(\alpha) = \mu_\alpha{}^2 - v_\alpha{}^2 h(\alpha) = \mu_\alpha{}^2 + v_\alpha{}^2$
[43]	T-spherical fuzzy	$\text{Score}\ (\tilde{p}) = \mu_{\tilde{p}}{}^T - S_{\tilde{p}}{}^T$
[25]	Cubic Pythagorean Fuzzy	$S(\check{c}) = \frac{1}{2}\left[\frac{1}{2}\left[(\mu_{\check{c}}^L)^2 + (\mu_{\check{c}}^U)^2 - (v_{\check{c}}^L)^2 - (v_{\check{c}}^U)^2\right] + ((\mu_{\check{c}})^2 - (v_{\bar{c}})^2)\right].$
[40]	neutrosophic fuzzy set	$s(A) = (\rho_A + 1 - \sigma_A + 1 - \tau_A)/3.$
[44]	interval type-2 trapezoidal	$\overset{\approx}{A_i} = \frac{V(A_i^U) + \widetilde{V(A_i^L)}}{2}$
[27]	Fermatean fuzzy	$S(\tilde{A}) = \alpha_A^3 - \beta_A^3$ $S^p(\tilde{A}_{ij}) = 1 + S(\tilde{A}_{ij})$
[41]	q-rung orthopair probabilistic hesitant	$s(Q) = \frac{1}{\lvert\mathcal{M}\rvert}\sum_{\mu \in \mathcal{M}} (\mu \cdot p_\mu)^q - \frac{1}{\lvert\mathcal{N}\rvert}\sum_{v \in \mathcal{N}} (v \cdot p_v)^q$
[24]	Intuitionistic Fuzzy set	$s(\alpha) = \mu\alpha - v\alpha$
[46]	Fermatean probabilistic hesitant	$s(\aleph) = \left(\frac{1}{M_\aleph}\sum_{h_i \in \tau_{h_g}, \tilde{p}_i \in p_{h_g}} (h_i \tilde{p}_i)\right)^3 - \left(\frac{1}{N_\aleph}\sum_{\varrho_i \in \delta_{h_k}, b_i \in b_{h_g}} (\varrho_i b_i)\right)^3$,
[47]	spherical fuzzy rough	$\text{Def}(\tilde{A}_s) = \frac{1}{6}\left(4 + \underline{\mu}_{\tilde{A}_s} + \bar{\mu}_{\tilde{A}_s} - \underline{v}_{\tilde{A}_s} - \bar{v}_{\tilde{A}_s} - \underline{\pi}_{\tilde{A}_s} - \bar{\pi}_{\tilde{A}_s}\right)$
[48]	q-rung orthopair fuzzy rough	$\text{Score}(\alpha) = \frac{\left[2 + \underline{\mu}_\gamma^q + \bar{\mu}_\gamma^q - \underline{v}_\gamma^q - \bar{v}_\gamma^q\right]}{4}, q \geq 1$
[15]	Dual-Hesitant Fuzzy	$s(d_j) = \frac{1}{\#\tilde{h}}\sum_{\tilde{\gamma}_j \in \tilde{h}_j} \tilde{\gamma}_j - \frac{1}{\#\tilde{g}}\sum_{\tilde{\eta}_j \in \tilde{g}_j} \tilde{\eta}_j.$
[49]	Rough Fermatean fuzzy	$\text{Score}(\alpha) = \frac{\left[2 + \underline{\mu}_\gamma^3 + \bar{\mu}_\gamma^3 - \underline{v}_\gamma^3 - \bar{v}_\gamma^3\right]}{4}$
[21]	Hexagonal Fuzzy Number	$\frac{3h_1 + 3h_2 + 10h_3 + 10h_4 + 5h_5 + 3h_6}{34}$
[37]	Z-cloud Rough Numbers	$\widetilde{W}_J = \left(\sum_{i=1}^{m} \frac{\text{Imp}(E_{tj}/C_{tj})}{\sum_{j=1}^{n} \text{Imp}(\text{E}(\text{E}_{tj}/C_{tj})}\right)/m\right)$

3.5 Case Studies

The final compilation of articles on FDOSM features case studies, as depicted in Figure 6. This figure presents a breakdown of the case studies into various categories and subcategories. The initial category focuses on healthcare, specifically addressing COVID-19 in five studies [20, 43, 19, 27, 56]. The second category delves into sign language, explored in two studies [25, 42]. The third category encompasses communications, covering topics such as (1) Global Positioning System (GPS), discussed in one study [21]; (2) Networks, mentioned in three studies [37, 38, 45] ; and (3) Active queue management (AQM), addressed in one study [29]. Next is the category of sustainability, which encompasses transportation and is discussed in four studies [15, 46, 48, 24]. Lastly, the category of tourism is mentioned in two studies [40, 47].

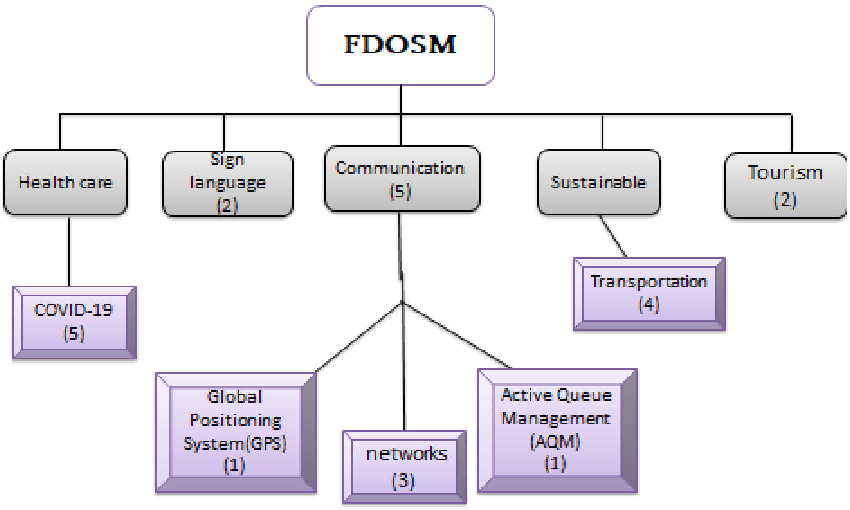

FIGURE 6. – Case Study Categories in FDOSM Research

To establish a clear correlation between the case studies and the recommendations for addressing various challenges, the mentioned case studies and their association with proposed recommendations will be briefly explained, thus strengthening the practical importance of the proposed methodology.

Firstly, the implementation of FDOSM in healthcare according to the literature [20, 43, 19, 27], plays an essential role in converting the decision matrix into an opinion matrix. This integration allows the expert opinions and preferences to be incorporated into the decision-making process. The recommendations related to this case study including Establishing the relative importance of evaluation criteria to facilitate the equitable distribution of COVID-19 vaccination doses. Prioritizing vaccine distribution equitably based on well-evaluated criteria. Conducting rigorous ranking assessments and sensitivity analysis to validate the proposed methodologies and provide reliable decision-making in different scenarios. Also, a study [56] displays the significance of integrating FDOSM and FWZIC for accurate criterion weighting and hospital ranking. This method enhances the decision-making for remote MCD patients using wearable body medical sensors and IoT technologies. The recommendations suggest adopting FDOSM and FWZIC techniques to address the issues of selecting hospitals for MCD patients. Secondly, the Implementation of FDOSM in Sign Language Recognition Systems plays an important role in research as it expands the conventional approach to address uncertainty in decision-making through the use of Cubic Pythagorean fuzzy sets (CPFS). Where the integration of CPFS with FDOSM methods is utilized to priorities alternatives in the case study of Sign Language Recognition Systems (SLRS). The recommendations proposed by FDOSM in this study involve the utilization of CP-FDOSM to rank alternatives in SLRS, based on the assigned values of each alternative. The combination of FDOSM and CPFS provides a robust for making decisions in intricate and unpredictable situations, such as evaluating SLRS performance and guaranteeing a methodical ranking and dependable outcomes [25, 42]. Thirdly, Implementation of FDOSM in communications case study, this case covering topics such as (1) Global Positioning System (GPS), This study assesses static and dynamic location modes in the GPS, using FDOSM to overcome challenges like inconsistency, ambiguity, and criteria weighting issues. This method uses fuzzy numbers to address uncertainty, a major issue in fuzzy multicriteria decision-making. Recommendations to overcome challenges in the GPS case study include using MCDM methods and adaptable GPS systems. Employing FDOSM in the GPS case study enable effective management of uncertainties and informed decisions based on expert opinions, eventually enhancing decision-making processes in GPS-related scenarios [21]. (2) In the realm of networks, there are several studies have been conducted. The case study in [45], focuses on evaluating and comparing network protocols within the context of LTE-A by employing (MCDM) techniques to address challenges such as the significance of criterion, variability in data, and multicriteria problems. The case study utilizes the 2-tuple-FDOSM methodology to overcome the information loss inherent in basic FDOSM. By extending FDOSM with the 2-tuple linguistic fuzzy method to avoid any loss of information during the conversion of the decision matrix. The case study's outcomes demonstrate the effectiveness of the 2-tuple-FDOSM in in resolving information loss issues, hence demonstrating its practicality in resolving network protocol assessment challenges encountered in real-world scenarios. (3) Active queue management (AQM), in this study [29], FDOSM tackles the theoretical challenges in multicriteria decision-making (MCDM) by introducing a new method that surpasses current methods. The objective of this method is to enhance the benchmarking process of (AQM) methods by providing a more thorough and resilient evaluation framework. The recommendations have produced is expanding the FDOSM framework by using different operators and MCDM techniques to obtain a wider range of ranking outcomes. The study highlights the significance of verifying the benchmarking results by using group decision-making contexts to guarantee the dependability and accuracy of the FDOSM-based AQM benchmarking outcomes. Fourthly, in sustainability case study, FDOSM is employed to make logical decisions by leveraging the experience of specialists (DMs) to identify the optimal solutions for each criterion in an opinion matrix. It assists in addressing issues with Multi-Criteria Decision Making (MCDM) by clearly assigning weights to criteria and depending on the specialists' opinions for decision-making. FDOSM's research yielded the following recommendations: the FDOSM approach suggests integration with the fuzzy-weighted zero-inconsistency (FWZIC) method to determine the relative importance of each criterion, because FDOSM cannot alone directly give weights to criteria. Lastly, in tourism case study, FDOSM plays a crucial role in offers a strong and reliable way for making fuzzy decisions. It assigns a weight to each attribute, which helps in the benchmarking process of intelligent e-tourism apps. FDOSM tackles the problem of uncertain and imprecise data by offering a dependable approach for making decisions in a fuzzy setting. The recommendations provided in this study are: the expansion of FDOSM within a new fuzzy environment to improve its abilities in evaluating smart e-tourism apps. Also, the use of FDOSM to standardize the variability observed in individual benchmarking results across all categories in group decision-making scenarios [40, 47].

4. RECOMMENDATIONS

Recommended suggestions and ideas for the current issues have been explained in this section, extracted from recent academic research. Essentially, these recommendations are classified into three categories: Improving Fuzzy Environments and Decision-Making Methods, Proposed Methodologies for Addressing Various Challenges, and Optimizing COVID-19 Vaccine Recipient Prioritization. The following subsections discuss in more detail these issues:

4.1 Improving Fuzzy Environments and Decision-Making Methods

Several studies [19, 24, 25, 40, 43] recommended applying other fuzzy environments, such as interval type-2hesitant. The various fuzzy sets: T-spherical fuzzy rough sets, soft hesitant fuzzy rough sets, image fuzzy rough sets, interval-valued intuitionistic fuzzy soft rough sets, or type-2 fuzzy sets are suggested to use [48]. Furthermore, [40] suggested expanding to spherical and M-polar to overcome the uncertainty. Besides, the studies [19, 25, 43] provided an intuitionistic and interval-valued extension that can be applied in the FDOSM to avoid the uncertainty limitation too. While [15, 37] proposed that the FDOSM be combined with a new fuzzy set to improve the final decision.

A linguistic scale (Likert scales) (e.g seven-, nine-, or eleven-) has been suggested by several studies for using it. In [15, 32, 41], use it to build the opinion matrices; other studies [46] have used it for building the Agri4SC OM. [48] has been used to measure the suitability of the suggested method. Moreover, studies [15, 32, 41, 46] recommend giving the experts some influence based on their knowledge that can be used in determining the criteria weights and to provide more reasonable results because all the experts were treated equally, regardless of their level of experience.

Advocating using more than one aggregation operator and defuzzification strategies to offer the ranking alternatives in FDOSM or solely with MULTIMOORA that was introduced in [32, 41, 46, 47]. One [40] advocates using a different aggregation operation for ranking alternatives in FDOSM, while another study [15] recommends using various defuzzification algorithms.

4.2 Proposed Methodologies for Addressing Various Challenges

As per the research findings, the proposed methodology can be applied in various tourism management case studies to address other Multi-Attribute Decision Analysis (MADA) issues, such as uncertainty and ambiguity [47]. A method is proposed for using different MCDM methods in this field to improve the strength and performance of Network-on-Chip (NoC)-based Multi-Processor Systems-on-Chip (MPSoC)[37]. Furthermore, the proposed method suggested by [24] can be used to benchmark any future potential energy systems in the transport industry. The intersection of construction and demolition wastes (CDW) management strategies and the driver and barrier qualities of reuse distribution can also be used to build a thorough evaluation, as the Study recommends [41]. Additionally, [15] advocates exploring fuzzy failure mode effect analysis to assign weights to pavement criteria. Lastly, [32] suggests the utilization of FDOSM to benchmark the numerous security and privacy aspects of intelligent medical systems relying on federated learning and blockchain technology.

4.3 Optimizing COVID-19 Vaccine Recipient Prioritization

In [19, 20, 43] have recommended the following: (1) presenting and processing a large dataset of COVID-19 vaccine recipients by taking into account all probabilities for each alternative and distribution criteria; (2) carrying out the proposed MCDM method on two levels: first, each vaccine recipient membership (i.e. frontline health workers, key workers, and frontline staff employees, as well as none or both children and homemakers) will be prioritized, and second, each alternative within each membership will be prioritized. Other distribution parameters and their usefulness, such as family income and nutritional habits, should be investigated.

5. Limitations

This study has some limitations, including this review is limited by the number of articles identified that specifically address the development of MCDM methods for systematic review scanning using FDOSM in a fuzzy environment. Focusing primarily on introductory articles might not provide a clear ideal view for solving MCDM problems using developed approaches with FDOSM in a fuzzy environment.

6. CONCLUSIONS

MCDM techniques continue to be adopted by researchers in many domains, with both old and unique methodologies employed to improve this topic. To date, MCDM is the best approach for offering the best answer to complicated issues in expert systems. Nonetheless, Uncertainty and ambiguity are the major MCDM issues indicated in academic literature. Given the importance of monitoring such methodological advancements, this study aims to contribute to the field by conducting an in-depth review of developing ranking-based MCDM (FDOSM) with other methods. The search has been inappropriate keywords through the IEEE Xplore (IEEE), ScienceDirect (SD), Scopus, and PubMed databases, obtaining 22 related papers for study. This systematic review addressed the main highlights: the protocol that explains how the last set of articles was chosen, an analysis of current papers in the field, and previous research efforts in the form of a discussion of the fundamentals in Fuzzy Set Numbers, Aggregation Operators, then, it highlighted on studies' recommendations for Improving Fuzzy Environments, Proposed Methodologies for Addressing Various Challenges, Optimizing Vaccine Recipient Prioritization of COVID-19. The findings from this review encompass studies that combined FDOSM with various methodologies, including extracting the development types FDOSM method, extracting aggregation operator types, Integration Methods with FDOSM (hybrid with other methods), and Case studies types that show how FDOSM approaches may help decision-makers in a variety of decisions. Despite the relatively low number of studies in this field, existing information is necessary to shed light on different techniques to develop MCDM methods with FDOSM with other approaches. For future directions, we recommend extending FDOSM into different fuzzy types (M-Polar fuzzy sets, complex neutrosophic hesitant fuzzy sets) to compare and determine whether these types can solve the uncertainty and ambiguity issues. Also, applying these developed fuzzy sets to other case studies, employing other aggregation operators, and comparing them with operators used in these studies.

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CONFLICTS OF INTEREST

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