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Fuzzy Weighted Zero Inconsistency Method (FWZIC) for Multi-Criteria Decision-Making Weighting Criteria: A Systematic Literature Review

Rasha A. Yousif[1], Mahmood M. Salih [1,*], Reem D. Ismail[1], Wesal S. Hussain[1], M.A.Ahmed [1], Z.T.Al-Qaysi [1], ML Shuwandy[1]

[1,2,3] Department of Computer, College of Computer and Mathematics, University of Tikrit, Tikrit, Iraq

*Corresponding Author: Mahmood M. Salih

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ABSTRACT: Researchers in various fields have consistently used Multi-Criteria Decision-Making (MCDM) methods for discipline improvement, employing both standard and novel approaches. Selecting a weighting mechanism for evaluation criteria is critical in MCDM problems. Recognizing the importance of remaining up to date on such developments in methodology, this study aims to review several innovative methods integrated with Fuzzy Weighted Zero Inconsistency (FWZIC). Relying on the papers taken from the significant databases: IEEE Xplore (IEEE), ScienceDirect (SD), Scopus, and PubMed from 23 August 2023 to 30 October 2023, where each method was read and analyzed for its characteristics and steps. These indexes were deemaed extensive and dependable enough to encompass the scope of our literature review. A total number of articles, n = 26 were chosen based on the criteria for inclusion and exclusion have been selected for this systematic review. By using bibliometric and content analysis, this study examined the developing ways with (FWZIC), as well as study components (sources, authors, different countries, and affiliations), areas of application, case studies, fuzzy implementations, hybrid studies (use of other weighting methods), and application tools for these methods. The results of this literature systematic review (LSR) provide an accurate depiction of each new development related to the weighting method and its utilization, such as: 1- Extracting the types of development employed in the FWZIC approach based on Fuzzy Set. 2- Extracting types of aggregation operators. 3- Analyzing integration methods with FWZIC (hybridized with other methods), and 4- Case study types showing how MCDM approaches may help decision-makers in a variety of decisions. Also, a set of recommendations has been presented to the researchers for the development of new method types, as a new direction for future work. This will provide academics and practitioners in the field of MCDM with valuable insights and significant expertise for data analysis and decision-making.".

Keywords: Fuzzy-weighted zero-inconsistency, FWZIC, Multi-criteria Decision Making, MCDM, Weighting Methods, Fuzzy Set.

1. INTRODUCTION

Overall, machine learning (ML), particularly deep learning, and decision-making (DM), play an important role in human activities because they achieve success at data analysis, prediction, enhancement, and decision support, allowing researchers to find useful information from huge and complicated data sets and make confident decisions. In today's complex and busy world where decision-makers face intricate choices with wide-ranging consequences, they are essential in numerous fields[1, 2].especially in health care [3], education [4], and environmental management, which involve significant risks and many variables [5]. Conventional decision-making approaches require adaptation to account for real-world situations' inherent subtleties and uncertainties [6]. It can be emphasized that decision-making (DM) is a primary part of human activities because it is required in all aspects of life [6, 7]. Real-life problem resolution frequently necessitates weighing several opposing viewpoints to make a well-informed choice [8]. Therefore, a variety of simple and complicated judgments with varied levels of possible impact and repercussions must be made by decision-makers (DMs) [6]. A decision is a choice made based on the facts at hand or a technique utilized to solve a specific decision problem. In both organizational and household environments, decision-makers face numerous options with constrained resources. That includes the assessment of specific decision choices, considering the preferences, expertise, and pertinent data of the decision-makers (DMs) [6, 9, 10].

Multicriteria decision-making (MCDM) is a multi-use method utilized in various professions and fields involving many criteria or objectives [3] [9], such as healthcare [11], education [4, 12], transportation [13, 14], management [15, 16], investment [5, 17], environment [18], immigration [19], and military affairs [3, 9, 20]. When compared to traditional approaches, MCDM is rapidly gaining favor due to its capacity to improve decision quality through a more explicit, rational, and efficient procedure [8]. The origins of MCDM go back to operations research; MCDM's purpose is to use various approaches to solve multi-aspect problems and to give decision-makers tools that help in better decision-making to address intricate challenges [7, 8, 21]. The motivation for using the MCDM methods depends on selecting the most eligible alternatives among a set that shares the same decision criteria to solve DM problems as a decision matrix, where these alternatives are based on chosen criteria [10, 22]. However, the MCDM problems and challenges can be identified as uncertainty and imprecision; assigning a specific preference rate to any criterion is problematic. Besides, making decisions requires using the advice of specialists and experts [21, 23, 24]. Decision-makers (experts) cannot determine weights in actual numbers since they employ linguistic phrases. As a result, it is harder to address these challenges, so numerous researchers have tackled this issue [22, 23]. Because of the ambiguity in the data of real-world problems and the difficulties in dealing with them, MCDM was developed in a fuzzy environment. To deal with uncertainty, Zadeh et al. first presented the fuzzy set [13].

The two primary approaches used in the MCDM methods could be classified into the human approach, which emphasizes the involvement of decision-makers' preferences, opinions, and expertise. The mathematical approach systematically assesses and compares different courses of action by applying formal models and quantitative tools by using uses mathematical functions, matrices, and algorithms to compare the performance of alternatives to several criteria objectively [22, 23, 25]. Both approaches provide criteria for weighting and/or ranking alternatives [24]. In response to the increasingly complicated and confusing problems faced by decision-makers across many fields, fuzzy logic was developed. Fuzzy environment has been combined into Multi-Criteria Decision Making (MCDM) to enable more comprehensive and flexible decision-making procedures [24] [11].

Many subjective MCDM weighting strategies, such as Analytic Hierarchy Process (AHP) [11] [26], Analytic Network Process (ANP) [26, 27], and Best–Worst Method (BWM) [27, 28], have been proposed with excellent success rates. However, the inconsistencies of these method that arise from pairwise comparisons and theoretical challenges (i.e. subjective, objective, or hybrid weighing methods), including the quantity and nature of comparisons, the amount of time required, and the impact of raw data change, remain unsolved [29]. Therefore, the fuzzy weighted zero inconsistency (FWZIC) method (published in 2021) was recently introduced for calculating the weight coefficients of criteria with zero consistency .This method computes the importance level in the decision-making process based on differences in expert preference per criterion [30-33]. since it successfully overcomes the inconsistency problem, which is a prevalent issue that can have significant effects on the accuracy and reliability of the decision-making process [34, 35], FWZIC is the most ideal subjective weighting method for weighting the relevant criteria. To handle ambiguity, hesitation, and uncertainty in a professional way FWZIC accomplishes zero inconsistency by computing the local and global weight coefficient values of all criteria at a particular hierarchy level separately and precisely [3, 36, 37]. FWZIC capture and reflect decision-makers' accumulated knowledge as well as their subjective opinions. This method is flexible and can be used in a variety of cases. it is beneficial for reducing inconsistency issues caused by the subjective nature of establishing the relative relevance and importance of multiple evaluation criteria utilizing a pairwise comparison approach [30, 38].In contrast to other methods that need direct comparisons across criteria, FWZIC does not require such comparisons or a large number of mathematical operations, which can be time-consuming, the multiple weighted attributes in FWZIC are independent, therefore adding or removing them require no recalculation. Furthermore, getting feedback from decision-makers (DMs) in FWZIC is straightforward, this means that decision-makers can conserve significant resources, concentrate their attention to other essential parts of the decision-making process and can have more confidence in the final decision because it is based on a precise and consistent weighting of the criteria [21, 39, 40]. This method has been widely utilized to handle complicated MADA problems in a wide range of industries, including agriculture, transportation, healthcare, and engineering [41, 42]. The FWZIC method overcomes the shortcomings of the best worst method (BWM) and the analytic hierarchy process (AHP): (i) the procedure's failure to provide decision makers with quick feedback on the consistency of pairwise comparisons, (ii) the lack of accounting for ordinary consistency, and (iii) the absence of a consistency threshold value for evaluating the reliability of results [25, 43, 44].

The aim of this study is to present a comprehensive review of one of the most recent methods in multi-criteria decision-making, the Fuzzy Weighted Zero Inconsistency Method (FWZIC), in order to find new directions, determine significant research gaps along with their corresponding solutions, and provide detailed methodologies that can serve as guidelines for future researchers. Furthermore, despite a relatively small number of studies (reviews) that dealt with the FWZIC method, it is necessary to scan and gather existing information in order to explore various techniques for developing MCDM methods that have high certainty with low ambiguity, which are presented alongside other approaches. Finally, Various ranking methods, such as MABAC, TOPSIS, and others, had been integrated with FWZIC. It is important to note that the methods stated above are considered to be among the strongest, highly reliable, and most commonly used in decision-making. In addition, we review the applications used with FWZIC such as in health care, communication services, transportation, business, and management, sustainable, sign language and other domains.

1.1 FWZIC Method

To determine the weights of the evaluation criteria, the proposed FWZIC method consists of five phases:

1.1.1 Phase 1: The Definition of Evaluation Criteria Set

This phase has two processes:

Step 1: Investigate and provide the predefined set of evaluation criteria.

Step 2: The behavior and measurement type of each of the obtained criteria, sub-criteria, and relative indicators are used to classify and group them [32].

Phase 2: Structured Expert Judgment

In this phase, a panel of experts evaluates the defined criteria from the previous step for their importance level. These experts should be specialists with relevant academic and scientific backgrounds. Following that, a nomination procedure is performed in accordance with the following steps:

Step 1: Expert identification: A person who was or is currently active in the case study's subjects and is considered to be knowledgeable by others is referred to be an expert in the FWZIC context. 'Domain' or 'substantive' experts are another term for specialists who are recognized in the literature [45, 46].

Step 2: Select an expert: A team of experts is chosen for the case study when expert identification is complete. In this step, at least four specialists are required. To find out their availability and willingness to be considered as possible experts for the panel, all experts from the previous stage are contacted by email [33, 39].

Step 3: Evaluation form development: The evaluation form is completed since it is a crucial instrument for gathering expert consensus. Before finalization, it is examined by all of the experts from the previous step for reliability and validity [33, 47].

Step 4: Defining the importance level scale: Using a 1-5 Likert scale, all of the experts chosen in the previous step determine the importance level for each criterion [48, 49].

Step 5: Converting from linguistic to numerical scale: All preference values are converted from subjective to numerical form for use in the study. Thus, each expert's priority level for each criterion on the utilized Likert scale is translated into a numerical scale [47, 48], as shown in Table 1.

Table 1. - Five-Point Likert scale and Equivalent Numerical Scale

Numerical scoring scale	Linguistic scoring scale
1	Not important
2	Slight important
3	Moderately important
4	Important
5	Very important

1.1.2 Phase 3: Expert Decision Matrix (EDM) is constructed based on the crossover of criteria and the Structured Expert Judgement (SEJ)

The EDM is built with the primary parts, which contain criteria and alternatives, as indicated in the table below. The previous phase defines the list of selected experts and each expert's choice within a particular criterion. The EDM is built in this stage. The decision criteria and alternatives are the fundamental components of the EDM, as indicated in Table 2, which show a crossover between the criteria [29, 31].

Table 2. - Fuzzy EDM

Experts	Criteria			
	C1	**C2**	**….**	**Cn**
E1	Imp(E1//C1)	Imp(E1//C2)		Imp(E1//Cn)
E2	Imp(E2//C1)	Imp(E2//C2)		Imp(E2//Cn)
E3	Imp(E3//C1)	Imp(E3//C2)		Imp(E3//Cn)
….	….	….	….	….
En	Imp (En//C1)	Imp (En//C2)		Imp (En//Cn)

1.1.3 Phase 4: Fuzzy Membership Function is Applied to the EDM Result

The fuzzy membership function and related defuzzification procedure are applied to the EDM data in this stage, where the data are modified to enhance precision and simplicity of use in subsequent analysis. However, with MCDM, the problem is ambiguous and imprecise due to it is hard to give an exact preference rate to any particular criteria. To solve the issue of imprecise and unclear issues, the fuzzy method uses fuzzy numbers rather than crisp numbers to evaluate the relative value of attributes (criteria) [49]. The most popular form of fuzzy number used in fuzzy MCDM is triangular fuzzy numbers (TFNs). TFNs are expressed as A = {a,b,c}. Because of their conceptual and computational simplicity, they are widely employed in practical applications [32, 50].

Definition formula: The membership function (x) of TFN A is given by:

$$\mu A(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ \dfrac{c-x}{c-b} & \text{if } b \le x \le x \\ 0 & \text{if } x > c \end{cases}, \text{ where } a \le b \le c \qquad (1)$$

Remark: Let $\tilde{x} = (a_1, b_1, c_1)$ and $\tilde{y} = (a_2, b_2, c_2)$ be two nonnegative TFNs and $\alpha \in \mathbb{R}_+$. The definition of the arithmetic operations according to the extension principle is as follows [32]:

Addition:

$$\tilde{x} + \tilde{y} = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \qquad (2)$$

Subtraction:

$$\tilde{x} - \tilde{y} = (a_1 - c_2, b_1 - b_2, c_1 - a_2) \qquad (3)$$

Multiplication:

$$\tilde{x} \times \tilde{y} \cong (a_1 a_2, b_1 b_2, c_1 c_2) \qquad (4)$$

Division:

$$\tilde{x} / \tilde{y} \cong (a_1/c_2, b_1/b_2, c_1/a_2) \qquad (5)$$

Division on crisp value:

$$\tilde{x} / \alpha = (a_1/\alpha, b_1/\alpha, c_1/\alpha) \qquad (6)$$

Defuzzification:

$$\frac{(a+b+c)}{3} \qquad (7)$$

The value of each linguistic term with TFN as shown in Table 3 that suggests all linguistic variables be converted into triangular fuzzy numbers, assuming that the fuzzy number is the variable for each criterion for expert K.

Table 3. The value of linguistic term with TFN

Linguistic terms	TFNs
Not important	(0.00,0.10,0.30)
Slight important	(0.10,0.30,0.50)
Moderately important	(0.30,0.50,0.75)
Important	(0.50,0.75,0.90)
Very important	(0.75,0.90,1.00)

1.1.4 Phase 5: Computation of the Final Weight Coefficient Values of the Evaluation Criteria

The final values of the weight coefficients of the evaluation criteria $(w1, w2, \dots, wn)^T$ are determined in three sub steps:

The fuzzification data ratio is calculated by using (2) and (5). TFNs used with the previous equations. The process is represented symbolically by (8) [31, 47].

$$\frac{\text{Imp } (\widetilde{E1}/C1)}{\sum_{j=1}^{n} \text{Imp } (\tilde{E}1/C_{1j})} \qquad (8)$$

1. To determine the final fuzzy values of the weight coefficients of the evaluation criteria $(\widetilde{w1}, \widetilde{w2}, \dots, \widetilde{wn})^T$,the average values are computed using (6). And (9) is used to determine the final weight value of each criterion using the Fuzzy EDM $(\widetilde{EDM})$ [31].

$$\widetilde{w_j} = \left(\sum_{i=1}^{m} \frac{\mathrm{Imp}\left(\widetilde{E_{ij}}/C_{ij}\right)}{\sum_{j=1}^{n} \mathrm{Imp}\left(\widetilde{E_{ij}}/C_{ij}\right)} \right)/m \right), \text{for } i = 1,2,3,..m \text{ and } j = 1,2,3,..n = 1,2,3,..n \quad (9)$$

2. Defuzzification is used to determine the final weight. Finally, defuzzification methods are used to determine the crisp weight value using (7) Prior to computing the final values of the weight coefficients, the weight of importance of each criterion should be allocated based on the total of all criteria's weights for the rescaling purpose used in this step [29, 32].

The first three phases are the same regardless of the fuzzy environment used, but the last two need different mathematical procedures based on the fuzzy environment [29, 47]. The first version of FWZIC, which was extended in a triangular fuzzy environment, could not effectively handle the ambiguity, uncertainty, and vagueness of information caused by expert uncertainty sufficiently [51, 52]. In such case, the experts have issues explaining an obvious preference for relevant alternatives based on a multiple attribute, especially when relying on incorrect, inaccurate, or insufficient information [53]. Dealing with ambiguous and uncertain information in real-life situations has always been complex. To address the complicated issues and conflict inherent in real-world tasks, various method has been developed, such problems with decision-making can be handled effectively using fuzzy sets (FSs) [40]. Therefore, to handle ambiguous, imprecise problems, FWZIC has been extended under various fuzzy environments, including neutrosophic fuzzy sets, Pythagorean fuzzy sets (PFSs), cubic Pythagorean fuzzy sets, interval type 2 trapezoidal-fuzzy sets, dual-hesitant fuzzy sets, q-rung orthopair fuzzy sets (q-ROFs), T-spherical fuzzy sets, and Pythagorean m-polar fuzzy sets [36, 47]. Despite these remarkable achievements, the issue of, unreliable, imprecise, and incomplete information still needs to be resolved [31, 39].

This study aims to provide a comprehensive view of FWZIC method using a Systematic Literature Review (LSR), which helped identify and classify existing approaches, discuss their benefits, challenges, and limitations, and then highlight the literature recommendations. In addition, this study is meant to help scholars understand and advance the MCDM field and offer decision-makers a toolset for addressing complex decision problems in a fuzzy environment.

1.2 Paper Organization

The order parts of the current paper are structured as follows: Section 2 explains the methodology applied for this systematic Literature review METHOD (SLR), emphasizing the database research, search protocols, Study Selection, and Inclusion and Exclusion Criteria. Discussion in section 3 showed the study results and their corresponding consequences in subsections: Fuzzy Set Number, Aggregation Operators, The Integration Method, and Case Study. A summary of recommendations mentioned in the final set of articles is presented in Section 4. Finally, the conclusion outlines using existing critical information to investigate alternative options for improving MCDM methods through integrating FWZIC with other techniques.

2. SYSTEMATIC LITERATURE REVIEW METHOD (SLR)

Due to the logical and holistic Systematic Literature Review (SLR), popular recently among experts and researchers, this study has employed it to completely grasp the research topic and provide adequate data for subsequent investigations [31]. SLR is a well-structured approach capable of refining research synthesis by identifying pertinent publications depending on pre-identified parameters instead of standard review procedures. As well as it's a cutting-edge technique that can be used in many different study fields and scientific specialties. It involves primary strict steps starting with identifying the scope, developing the search mechanism, study selection, extraction, and information synthesis [54, 55]. Only studies that used the FWZIC development methods were included in this review. IEEE Xplore (IEEE), ScienceDirect (SD), Scopus, and PubMed were used to search for relevant papers. These databases comprehensively cover scientific and technological research conducted. They provide clear and accurate insights for further analysis and investigation necessary for researchers in their field of specialization and the extent of its development and integration with other disciplines. In addition, the search was restricted to articles published in (2021-2023) to ensure that the review focused on the most recent and up-to-date research on integrated FWZIC with other MCDM methods. Inclusion criteria were designed to focus on specific topics and studies, thereby narrowing down the scope of the review.

Finally, MCDM methods have been adopted in this research to study the developing techniques of FWZIC methods in fuzzy environment utilization.

Study Selection

This study selection procedure follows a standard systematic review methodology. The study selection process consisted of the following steps: 1- Search in four digital databases were searched for relevant studies using a defined 'Query'. 2- Initial Filtering to find possibly relevant studies; the titles and abstracts of the collected studies were examined via specific keywords. 3- Full Filtering, considered the second filter, focused on the studies that only applied existing FWZIC developing under a fuzzy environment. The full texts of potentially relevant articles were examined to see if they met the inclusion criteria. That made a significant contribution to the field of MCDM development by screening the state-of-the-art FWZIC method extensions.

2.1 Search Strategy

Via using the specific pre-defined keywords to build the suitable query: "(MCDM OR 'Multi-criteria decision-making') AND (FWZIC OR 'Fuzzy weighted zero-inconsistency') AND (Fuzzy) AND ('development method' OR 'developing')" has begun on 23 August 2023 and ended on 30 October 2023. The four digital reliable databases: IEEE Xplore (IEEE), ScienceDirect (SD), Scopus, and PubMed were selected for a wide-ranging, English-language citation search of articles published from 2021 to 2023 because they include a large number of articles that achieve pre-identified relevant topics covering scientific and technical perspectives on the topic of development FWZIC with other methods in a Fuzzy environment. The study's limitations include relying on a limited number of articles describing MCDM development methodologies for the systematic review analysis. This limitation makes it difficult to provide a complete view of addressing MCDM difficulties with advanced methods, including FWZIC, in a fuzzy environment. Additionally, three papers on related topics were also unavailable for download.

2.2 Study Selection

The study selection process has been divided into three steps. Initially, all the studies (76 in total) were gathered, and any duplicate articles (5 in number) were removed. In the second step, the titles and abstracts of the extracted articles were reviewed n=10, using specific inclusion and exclusion criteria, leading to the selection of pertinent studies for the final round. In the third step, each paper that met our inclusion criteria underwent a thorough full-text reading. This allowed us to gather valuable information and create a comprehensive table summarizing the topic's details for this review. As a result, the final relevant articles for this review comprises 26 studies, while 35 articles that did not meet our inclusion criteria were excluded, as shown in Figure 1.

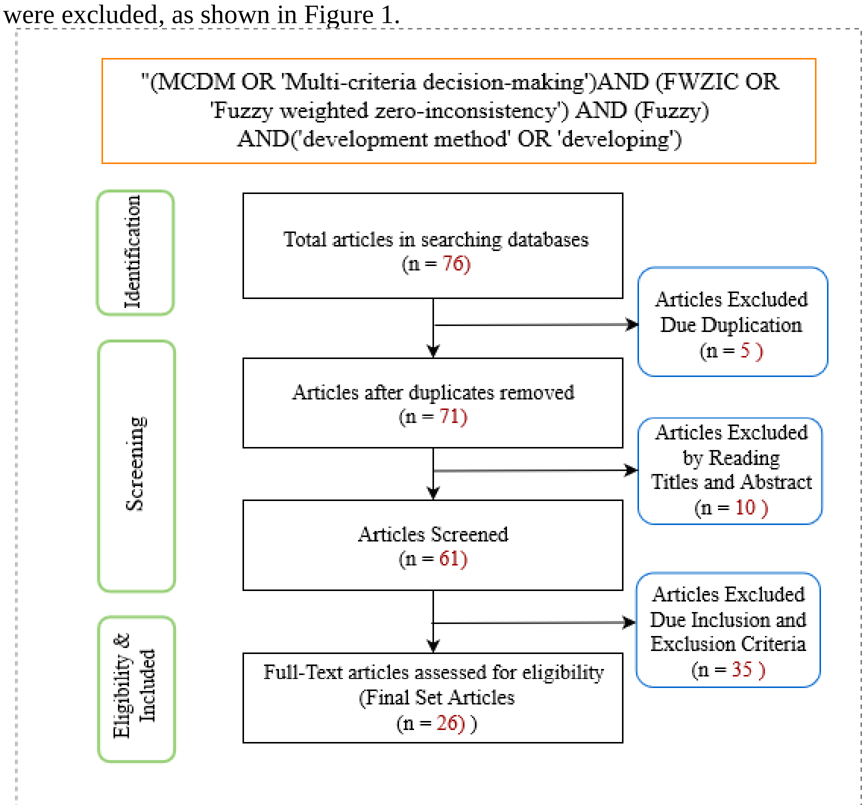

FIGURE 1. - Flowchart of the Search Query and Inclusion Criteria with Filtering Process

2.3 Inclusion and Exclusion Criteria

The selection criteria applied in this (LSR) were of paramount importance to ensure that the review focused on high-quality related studies. The following criteria were considered when determining the inclusion criteria of papers:

- *Language:* Articles written in English language only.

- *Database:* IEEE Xplore (IEEE), ScienceDirect (SD), Scopus, and PubMed.
- *Topic:* Articles are focused on a specific part of MCDM.
- *Direct Application:* Articles are used any developed methods with FWZIC.
- *Paper Types:* Article type such as: Reviews, Research Articles, and Conference Papers.
- *Subject Areas:* Computer Science.
- *Access Type:* Open access.
- *Published Years:* 2021-2023.

Non-English language studies were excluded from the analysis, with depending on four reliable databases only. Also, articles that failed to meet the inclusion criteria, including those not employing FWZIC and development methods in a fuzzy environment, were excluded. Articles not utilizing the FWZIC method or combining FWZIC with any MCDM methods for development purposes were also excluded. Lastly, papers such as Reviews, Research Articles, and Conference Papers, subject areas, access type, Article type, published Years: Computer Science, Open access, and 2021-2023, respectively.

3. DISCUSSION

This section contains the discussions for the accomplished state-of-the-art studies of FWZIC constructed with MCDM techniques in a fuzzy environment. Practically, the primary search query result showed (76) relevant articles, but the final set became (26) after two filtering processes. The final collection of articles was extensively researched in order to cope with all of the technical and scientific methods of the present study topic. Concerns such as Fuzzy Set Number, Aggregation Operators, and The Integration Method have been identified and classified in this study's literature, as shown below:

3.1 Fuzzy Set Number

Zadeh presented fuzzy set theory for the first time in 1965 [56]. To deal with the ambiguity and imprecision inherent in human judgment, FS is proposed to use language concepts and degrees of membership in decision-making methods [57]. where many terms have uncertain meanings. A characterizing or discriminating function can be used to determine which individuals from a universal set X are members or non-members of a crisp set. Every element in a predefined crisp set A is assigned the value A(x) using the function [58, 59].

$$\mu_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases} \qquad (10)$$

Hence $\mu_A(x) \in \{0,1\}$ The function $\mu_A(x)$ takes only the values 1 or 0.

A fuzzy set R is describing:

$$R = \{(x, \mu R(x))/x \in A, \mu R(x) \in [0,1]\} \qquad (11)$$

Where μR(x) is a membership function; μ R(x) calculated the grade at which each element of A belongs to the fuzzy set R [60].

The development types are employed in FWZIC approach based on the Fuzzy Set investigated in this study:

1- Probabilistic Hesitant Fuzzy Set-Fuzzy Weighted Zero-Inconsistency (P-H-FWZIC)

The hesitant fuzzy set (HFS) [61] is an interesting addition to the regular fuzzy set that improves MCDM by effectively handling uncertainty [62] did an in-depth review of HFS. Clearly, the review demonstrates that (i) HFS is a more generic and flexible preference structure with an opportunity to reduce uncertainty; (ii) HFS also facilitates expert preference elicitation; (iii) it gradually revealed the serious loss of information; and (iv) the chance of each element's occurrence is disregarded. [63] has proposed the probabilistic hesitant fuzzy set (P-HFS) in 2014, which incorporated the probability to the HFS. This novel research might successfully solve the shortcomings of HFSs. Furthermore, P-HFS not only allows for several viewpoints but also gives an occurrence probability to each perspective, improving the reliability of the data.

The Benefit:

- One of the primary advantages of P-H-FWZIC is that no inconsistencies were found in the computed weights.
- give experts a broader variety of options, improve precision in evaluating alternatives, and deal with ambiguity, uncertainty, and vagueness of data more successfully and efficiently.

The conversion is carried out using probabilistic hesitant fuzzy numbers (P-HFNs) (Table 4), which replace the EDM's crisp values (Numeric Scale).

Table 4. - Linguistic expressions with Corresponding Numeric Scale and P-HFNs

Linguistic expressions	Numeric scale	P-HFNs			
		M1	M2	P1	P2
Very Important (VI)	1	0.9	0.95	0.4	0.6
Important (I)	2	0.7	0.75	0.5	0.4
Average (Av)	3	0.5	0.55	0.47	0.5
Low Important (LI)	4	0.3	0.35	0.7	0.3
Very Low Important (VLI)	5	0.1	0.2	0.8	0.2

Definition (1) [64] :Let F be a fixed set. The P-HFS on F can be represented as follows:

$$H_P = \left\{\bar{h}(\gamma_i \mid p_i) \mid \gamma_I, I\right\} \quad (12)$$

Where $h(\gamma_i \mid p_i)$ is a collection of certain components $\gamma_i \mid p_i$ denoting the probabilities in hesitant fuzzy data for the set HP, $\gamma_i \in F, 0 \le \gamma_i \le 1, i = 1,2,\ldots,h$,where h is the number of possible elements in $\bar{h}(\gamma_i \mid p_i), p_i \in [0,1]$, pi 2 ½0; 1 is the hesitant probability of γ_1 and $\sum p_i = 1$. For convenience, $h(\gamma_i \mid p_i)$represents the P-HFNs, and HP represents the set of all P-HFS.

The probabilistic hesitant fuzzy weighted average (PHFWA) operator is used to aggregate the P-HFNs for each criterion among the three experts in the P-HFS−EDM [65] shown in (13).

$$PHFWA\,(h_1(p), h_2(p), \cdots, h_n(p)) = \bigoplus_{i=1} \omega_i h_i(p) =$$

$$\cup_{\gamma_{1_1}\in h_1,\gamma_{2_1}\in h_2,\cdots,h_{h_1}\in h_n} \left\{\left[1-\prod_{i=1}^{n}\left(1-\gamma_{i_1}\right)^{\omega_i}\right]\left(\prod_{i=1}^{n} \mathbf{P}_{i_1} / \prod_{i=1}^{n}\left(\sum_{l=1}^{|h_i(p)|} \mathbf{p}_{i_1}\right)\right)\right\} \quad (13)$$

Then, the resultant fuzzy weight values are defuzzied and transformed to crisp weight values using (14).

$$s(\bar{h}) = \sum_{i=1}^{h} \gamma_i p_i \quad (14)$$

Finally, the aggregate of the weights assigned to the main criterion and each sub-level must equal 1. If this criterion is not fulfilled, the values are rescaled according to (15).

$$w_j = s_j / \sum_{j=1}^{J} s_j \quad (15)$$

Where $s_j$represents the weight value for each criterion.

2- Spherical FWZIC (S-FWZIC)

- SFSs employ the nonlinear distance between a degree of membership, nonmembership, and hesitation, and their total may be larger than one, but their square sum must be between 0 and 1 [66].
- SFSs improve the decision-making process's intelligence (similar to human decision-making), resulting in high accuracy when evaluating alternatives. As a result, SFSs are extensively employed because they have the ability to give decision-makers with more options for dealing with ambiguity, hesitation, and uncertainty than other methods [67]. The linguistic terms are converted into equivalent numerical scoring scales, as given in Table 5

Table 5. - Linguistic Terms, Numerical Scoring Scale and their Corresponding Spherical Fuzzy Numbers [3]

Linguistic terms	Numerical scoring scale	(μ, v, π)		
Very low Importance (VLI)	1	0.15	0.85	0.1
Low Important (LI)	2	0.25	0.75	0.2
Medium importance (MI)	3	0.55	0.5	0.25
Important (I)	4	0.75	0.25	0.2
Very Important (VI)	5	0.85	0.15	0.1

- SFS $\tilde{A}_s$of the discourse universe U is written as follows:

$$\tilde{A}_s = \{u, \mu_{\tilde{A}_s}(u), v_{\tilde{A}_s}(u), \pi_{\tilde{A}_s}(u)) \mid u \in U\}$$

where $\mu_{\tilde{A}_s}(u): U \to [0,1], v_{\tilde{A}_s}(u): U \to [0,1], \pi_{\tilde{A}_s}(u): U \to [0,1]$ (16)

And

$$0 \leq \mu_{\tilde{A}_s}^2(u) + v_{\tilde{A}_s}^2(u) + \pi_{\tilde{A}_s}^2(u) \leq 1 \forall_u \in U. \quad (17)$$

For each $u, \mu_{\tilde{A}_s}(u), v_{\tilde{A}_s}(u)$ and $\pi_{\tilde{A}_s}(u)$ represent the degrees of membership, non-membership and hesitancy of u to A˜s, respectively, $\chi_{\tilde{A}_s} = \left(1 - \mu_{\tilde{A}_s}^2(u) - v_{\tilde{A}_s}^2(u) - \pi_{\tilde{A}_s}^2(u)\right)^{1/2}$ shows the level of rejection.

The following definitions show the SFS operations that are employed [68].

Let SFSs be

$\tilde{A}_s = \left(\mu_{\tilde{A}_s}, v_{\tilde{A}_s}, \pi_{\tilde{A}_s}\right)$ and $\tilde{B}_s = \left(\mu_{\tilde{B}_s}, v_{\tilde{B}_s}, \pi_{\tilde{B}_s}\right)$.

Multiplication by a scalar: for λ ≥ 0

$$\lambda \cdot \tilde{A}_s = \left\{ \begin{matrix} \left(1 - \left(1 - \mu_{\tilde{A}_s}^2\right)^{\lambda}\right)^{\frac{1}{2}}, v_{\tilde{A}_s}^{\lambda} \\ \left(\left(1 - \mu_{\tilde{A}_s}^2\right)^{\lambda} - \left(1 - \mu_{\tilde{A}_s}^2 - \pi_{\tilde{A}_s}^2\right)^{\lambda}\right)^{1/2} \end{matrix} \right\} \quad (18)$$

Division:

$$\text{if } \frac{\mu_{\tilde{B}_s}^2}{\mu_{\tilde{A}_s}^2} \geq \frac{1 - \pi_{\tilde{B}_s}^2 1 + \pi_{\tilde{A}_s}^2}{1 - \pi_{\tilde{A}_s}^2 1 + \pi_{\tilde{B}_s}^2} \geq 1$$

$$\frac{\tilde{A}_s}{\tilde{B}_s} = \left(\begin{matrix} \left(\frac{\left(\mu_{\tilde{A}_s}^2\left(2 - \mu_{\tilde{B}_s}^2\right)\right.}{1 - \left(1 - \mu_{\tilde{A}_s}^2\right) \cdot \left(1 - \mu_{\tilde{B}_s}^2\right)}\right)^{\frac{1}{2}}, \frac{\left(v_{\tilde{A}_s}^2 - v_{\tilde{B}_s}^2\right.}{\left(1 - v_{\tilde{A}_s}^2 \cdot v_{\tilde{B}_s}^{\frac{1}{2}}\right)^{\frac{1}{2}}} \\ \frac{\left(\pi_{\tilde{A}_s}^2 \cdot \pi_{\tilde{B}_s}^2\right)^{\frac{1}{2}}}{\left(1 - \pi_{\tilde{A}_s}^2 \cdot \pi_{\tilde{B}_s}^2\right)^{\frac{1}{2}}} \end{matrix} \right) \quad (19)$$

The spherical weighted arithmetic mean (SWAM) for SFS has been determined by the same set of authors in regard to w= $(w_1, w_2 \ldots, w_n)$; $w_i \in [01]$; $\sum_{i=1}^{n} w_i = 1$

$$\text{SWAM}_w\left(\tilde{A}_{S1}, \ldots, \tilde{A}_{Sn}\right) = w_1\tilde{A}_{S1} + w_1\tilde{A}_{S1} + \cdots + w_n\tilde{A}_{Sn}$$
$$= \left\{\left[1 - \prod_{i=1}^{n}\left(1 - \mu_{\tilde{A}_{Si}}^2\right)^{w_i}\right]^{\frac{1}{2}}, \prod_{i=1}^{n} v_{\tilde{A}_{Si}}^{w_i} \left[\prod_{i=1}^{n}\left(1 - \mu_{\tilde{A}_{Si}}^2\right)^{w_i} - \prod_{i=1}^{n}\left(1 - \mu_{\tilde{A}_{Si}}^2 - \pi_{\tilde{A}_{Si}}^2\right)^{w_i}\right]^{\frac{1}{2}}\right\}. \quad (20)$$

The defuzzied (crisp) value of the SFSs is defined as follows [66]:

$$\text{Def}\left(\tilde{A}_s\right) = \left(\mu_{\tilde{A}_s} - \pi_{\tilde{A}_s}\right)^2 - \left(v_{\tilde{A}_s} - \pi_{\tilde{A}_s}\right)^2. \quad (21)$$

- Compute the ratio of the fuzzified data. The real process's symbolic form is given as

$$\text{SAM}\left(\tilde{A}_{S1}, \ldots, \tilde{A}_{Sn}\right) = \tilde{A}_{S1} + \tilde{A}_{S1} + \cdots + \tilde{A}_{Sn}$$
$$= \left\{\left[1 - \prod_{i=1}^{n}\left(1 - \mu_{\tilde{A}_{Si}}^2\right)\right]^{1/2}, \prod_{i=1}^{n} v_{\tilde{A}_{Si}}, \left[\prod_{i=1}^{n}\left(1 - \mu_{\tilde{A}_{Si}}^2\right) - \prod_{i=1}^{n}\left(1 - \mu_{\tilde{A}_{Si}}^2 - \pi_{\tilde{A}_{Si}}^2\right)\right]^{1/2}\right\} \quad (22)$$

To acquire the final weight coefficient values$(\widetilde{w1}, \widetilde{w2}, \ldots, \widetilde{wn})^T$, the mean values are determined (18) is corrected through using the inverse of the constant as shown in (23). Then, using (22) and (23), calculate each value of the SFS EDM. The symbolic illustration of this step's actual procedure is provided as:

$$\tilde{A}_s\lambda = \begin{Bmatrix} \left(1-\left(1-\mu_{\tilde{A}_s}^2\right)^{\frac{1}{\lambda}}\right)^{\frac{1}{2}}, v_{\tilde{A}_s}^{\frac{1}{\lambda}}, \\ \left(\left(1-\mu_{\tilde{A}_s}^2\right)^{1/\lambda} - \left(1-\mu_{\tilde{A}_s}^2 - \pi_{\tilde{A}_s}^2\right)^{1/\lambda}\right)^{1/2} \end{Bmatrix} \quad (23)$$

3- q-rung orthopair fuzzy-weighted zero-inconsistency (q-ROFWZIC)

q-ROFSs is an effective technique for dealing with uncertainty in decision making, information measures, knowledge measures, distance measures, and aggregation information with the condition $\mu^q+v^q\leq1$, $q \geq 1$ Obviously, q-ROFSs have a larger scope for conveying ambiguous data than IFSs and PFSs. Furthermore, adjusting the q value, q-ROFSs allow experts can issue positive and negative marks separately by setting the q parameter [69]. According to Table 6, all linguistic variables are transformed into qROFS. The fuzzy number is supposed to be the variable for each Expert K criteria. In other words, Expert K must request that the importance level of the criteria for evaluation be identified within variables examined on a linguistic scale. Table 6 shows the Linguistic terms with equivalent q-ROFS

Table 6. - Linguistic terms and their Equivalent q-ROFS [43]

Linguistic scale	q-ROFS
Very Low Important (VLI)	(0.20,0.90)
Low Important (LI)	(0.40,0.60)
Average (Av)	(0.65,0.50)
Important (Im)	(0.80,0.45)
Very Important (VI)	(0.90,0.20)

Yager [69] has created a new fuzzy idea called the q-rung orthopair fuzzy set (q-ROFS) to address the shortcomings of existing fuzzy sets (i.e. IFSs and PFSs). The restriction of other fuzzy sets is eliminated in q-ROFSs, and the total of the q powers of membership and non-membership grades are real values between [0, 1]. Thus, the DMs are free to choose any grade for and anyplace freely ($\mu \in[0,1]$" and " $v\in[0,1]$) [70].

The q-ROFS restriction outperforms the others because it allows for greater freedom and flexibility under unknown situations and allows DMs to freely choose membership and non-membership degrees [32]. Since its introduction, a significant number of experts have extensively studied and applied it to deal with tough and complex fuzzy topics from a variety of perspectives. Because they offer a broader range of fuzzy information, q-ROFSs are the most adaptive and appropriate FS for dealing with vagueness and ambiguity [40].

In [71] proposed the unique concept of the rough set (RS) theory. RS theory is an expanded version of common set theory that deals with imprecise, ambiguous data, [72] recently presented q-ROF rough sets (q-ROFRSs), which it is a hybrid intelligent structure with RSs and q-ROFSs. q-ROFRSs are an improved classification approach that has received academic interest in dealing with ambiguous, partial data.

Figure 2 concludes the relationship amongst IFSs, PFSs and q-ROFS [73].

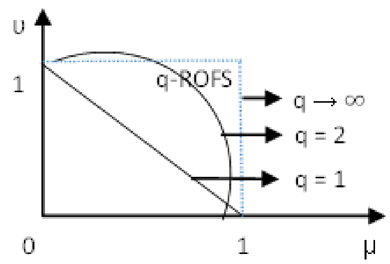

FIGURE 2. - Concept Relationship between IFSs , PFSs and q-ROFS [69]

The q-ROFS is an objective with the form [74] that is defined by (24) and (25).

$$P = \{\langle m, (\mu_d(m), v_d(m))\rangle \mid m \in m\} \quad (24)$$

Where μ_d: M → [0, 1] is the membership function, while v_d: M → [0, 1] is non-membership function of element m ∈ M to p, and It must satisfy the constraint provided in (24).

$$0 < (\mu_d(m))^q + (v_d(m))^q \leq 1, \text{ where } q \geq 1 \quad (25)$$

The degree of hesitancy is presented in (26) as following:

$$\pi_m(m) = \sqrt[q]{(\mu_d(m))^q + (v_d(m))^q - (\mu_d(m))^q \cdot (v_d(m))^q}. \tag{26}$$

Aggregation:

(27) illustrates the q-rung orthopair fuzzy arithmetic mean (q-ROFA) aggregation procedure used:

$$\text{q} - \text{ROFA}\,(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left\langle \left(1 - \prod_{k=1}^{n} \left(1 - \mu_k^q\right)\right)^{\frac{1}{q}}, \prod_{k=1}^{n} v_k \right\rangle \tag{27}$$

The q-ROFS division operation is shown in (28) as follows:

$$p_1 \oslash p_2 = \left(\frac{\mu_1}{\mu_2}, \sqrt[q]{\frac{v_1^q - v_2^q}{1 - v_2^q}}\right), \text{ if } \mu_1 \leq min\left\{\mu_2, \frac{\mu_2 \pi_1}{\pi_2}\right\}, v_1 \geq v_2. \tag{28}$$

(29) shows the equation of q-ROFS division on a crisp value. Each value of linguistic term with q-ROFS shown in Table 6.

$$p/\lambda = \left(\sqrt[q]{1 - \left(1 - (\mu_p)^q\right)^{\frac{1}{t}}}, (v_p)^{\frac{1}{\lambda}}\right), \lambda > 0 \tag{29}$$

According to Table 6, all linguistic variables are transformed into qROFS. The fuzzy number is supposed to be the variable for each Expert K criteria. In other words, Expert K must request that the importance level of the criteria for evaluation be identified within variables examined on a linguistic scale.

For the purpose of determining the final weight, defuzzification is used. For scoring each criterion, (30) is employed as the defuzzification technique.

$$S_k = \mu_k^q - v_k^q, \text{ where q} \geq 1 \tag{30}$$

4- Pythagorean fuzzy-weighted zero inconsistency (PFWZIC)

The Pythagorean fuzzy environment can manage the membership degrees of expert preferences more effectively by lowering vagueness and imprecision and improving the accuracy of final decision making, which takes into consideration the variations between membership and non-membership degrees.

A study [75] introduced the idea of the Pythagorean fuzzy number (PFN) as a new evaluation format defined by membership and non-member situation, the sum of which is below or equal to 1 to overcome uncertainty issues and record much more useful information under imprecise and ambiguous conditions [76].

PFN has come out as a useful method for capturing the fuzziness and uncertainty in MCDM issues [77] [78]. Regarding their uniqueness, the PFS must meet the requirement that the squared sum of the degrees of membership and non-membership must be equal or below than one. Because the PFN membership space is larger than the membership space of other types of fuzzy numbers membership space, PFS is more general [77].

The PFNs can be introduced in objective form [79] and are defined by (31) and (32).

$$P = \{m, (\mu_p(m), v_p(m)) \mid m \in M\} \tag{31}$$

Where $u_d, M \rightarrow [0,1]$ is the membership function, and $v_d: M \rightarrow [0,1]$ is a non-membership function of element $m \in M$ to p that must fulfil the restriction shown in (32).

$$0 < (\mu_p(m))^2 + (v_p(m))^2 \leq 1, \tag{32}$$

The degree of hesitancy is given by [80]:

$$\boldsymbol{\pi_p(m)} = \sqrt{1 - (\boldsymbol{\mu_p(m)})^2 + (\boldsymbol{v_p(m)})^2} \tag{33}$$

Aggregation Operations

The following equations represent the applied arithmetic operation of utilizing PFN. (34) defines the PFN summation and aggregation processes [81].

$$\text{PFAG}(\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_n) = \left(\sqrt{1 - \prod_{j=1}^{n} \left(1 - (\mu_j)^2\right)} = \prod_{j=1}^{n} (v_j)\right) \tag{33}$$

(35) shows the PFN division operation [80].

$$p(m_i) \oslash p(m_j) = \left(\frac{\mu_P(m_i)}{\mu_P(m_j)}, \sqrt{\frac{v_P^2(m_i) - v_P^2(m_j)}{1 - v_P^2(m_j)}}\right)$$
$$\text{if } v_p(m_i) \geq v_p(m_j), \mu_p(m_i) \leq min\left\{\mu_p(m_j), \frac{\mu_P(m_j)\pi_P(m_i)}{\pi_p(m_i)}\right\} \quad (35)$$

The formulation of the PFN division on a crisp value is seen in (36) [82].

$$p/\lambda = \left(\sqrt{1 - \left(1 - (\mu_p)^2\right)^{\frac{1}{2}}}, (v_p)^{\frac{1}{2}}\right), \lambda > 0 \quad (36)$$

The scoring (defuzzied (crisp) PFN value) is described as follows:

$$\alpha = (\mu_\alpha, v_\alpha) \text{ beaPFN}, s(\alpha) = \mu_\alpha{}^2 - v_\alpha{}^2 h(\alpha) = \mu_\alpha{}^2 + v_\alpha{}^2 \quad (37)$$

Where a is the score and the accuracy degree. For two PFNs $\alpha 1 = (\mu_{\alpha 1}, v_{\alpha 1}), \alpha 2 = (\mu_{\alpha 2}, v_{\alpha 2})$; the following holds true:

(1) If (1) If $s(\alpha 1) > s(\alpha 2)$, then $\alpha 1$ is bigger than $\alpha 2$, denoted by $\alpha 1 \succ \alpha 2$;

(2) If $s(\alpha 1) = s(\alpha 2)$, then: (a) If $h(\alpha 1) > h(\alpha 2)$ then $\alpha 1$ is bigger than $\alpha 2$, denoted by $\alpha 1 \succ \alpha 2$

(3) (b) If $h(\alpha 1) = h(\alpha 2)$ then $\alpha 1$ is equal to $\alpha 2$, denoted by $\alpha 1 = \alpha 2$.

Table 7 indicates that given that the fuzzy number is the variable for each criterion for Expert K, all linguistic variables may be turned into PFN. In other words, Expert K might be asked to determine the amount of relevance of the criteria within the variables assessed using the linguistic scale.

Table 7. - Linguistic Terms with Equivalent PFNs [25]

Linguistic scale	PFNs
Very Low Important (VLI)	(0.20,0.90)
Low Important (LI)	(0.40,0.60)
Average (Av)	(0.65,0.50)
Important (Im)	(0.80,0.45)
Very Important (VI)	(0.90,0.20)

5- Pythagorean probabilistic hesitant fuzzy sets and fuzzy weighted zero inconsistency (PPH–FWZIC)

Data hesitancy and imprecision have prevented specialists and decision-makers from achieving accurate decisions, Torra [61] presented the notion of hesitancy with FS (HFS) in 2010 to address the hesitation issue. Qian et al. [83] enhanced the idea of HFSs by extend them with IFSs. A further group of researchers [84] combined HFSs with PFS (PHFS) to describe hesitation with both FS and IFS. however, Although the extensions can be used to manage ambiguity problem effectively, they are unable to deal with circumstances in which decision-makers refuse to make decisions. to deal with such circumstances. Batool et al. [85] enhanced the PHFS and introduced the term Pythagorean probabilistic hesitant fuzzy sets (PPHFFSs). The degrees of positive and negative hesitant adhesions characterize - PPHFS equally, with the condition that the square sum of these degrees be less than or equal to 1. Every degree of negative hesitant adhesion has a preference over the others. As a result, the PPHFS idea was presented to preserve probabilistic data - extending FWZIC to the PPHF environment can efficiently overcome the uncertainty and inaccuracy issues.

Definition (1) For a set denoted by R, the PPHFS $\aleph$ in R can be described as

$$\aleph = \left\{\left\langle r, \tau_{h_x}(r)/\tilde{p}_{\hat{g}}, \partial_{h_x}(r)/b_{\hat{g}}\right\rangle \mid r \in R\right\}$$

To make the PPHFS-EDM, apply the PPHFS to the EDM. As indicated in Table 8 shows the crisp numbers (Numeric Scale) in the EDM are replaced by Pythagorean probabilistic hesitant fuzzy numbers (PPHFNs).

Table 8. - Linguistic Terms with Corresponding Numeric Scale and PPHFNs [21]

Linguistic expressions	Numeric scale	PPH FNs							
		M1	P1	M2	P2	V1	P1	V2	P2
Very Important (VI)	1	0.9	0.2	0.95	0.8	0.1	0.9	0.2	0.1
Important (I)	2	0.8	0.5	0.85	0.5	0.35	0.6	0.45	0.4
Average (Av)	3	0.65	0.5	0.7	0.5	0.4	0.55	0.5	0.45
Low Important (LI)	4	0.4	0.8	0.5	0.2	0.55	0.4	0.6	0.6
Very Low Important (VLI)	5	0.2	0.7	0.25	0.3	0.8	0.35	0.9	0.65

For all $r \in R$, $\tau_{h_x}(r)$ and $\partial_{h_x}(r)$ are sets of some values in [0,1], where $\tau_{h_x}(r)/p_{\mathrm{g}}^{\sim}$ and $\partial_{h_x}(r)/b_{\mathrm{g}}$ represent the degrees of positive and negative membership of r to PPHFS ℵ, respectively. $p_{\mathrm{g}}^{\sim}$ and b_{g} represent the degrees of possibilities.

In addition, $0 \leq \hbar_i, \varrho_{\hat{\imath}} \leq 1$ and $0 \leq p_{\hat{\imath}}^{\sim}, b_{\hat{\imath}} \leq 1$ with $\sum_{i=1}^{l} p_{\hat{\imath}}^{\sim} \leq 1$, $\sum_{\hat{\imath}=1}^{l} b_{\hat{\imath}} \leq 1$ (L is a positive integer describing the element numbers in PPHFS), and $h_i \in \tau_{h_x}(r)$, $\rangle_j \in \partial_{h_x}(r)$, $p_{\hat{\imath}}^{\sim} \in p_{\mathrm{g}}^{\sim}$, $b_{\hat{\imath}} \in b_{\dot{\mathrm{g}}}$. This scheme requires $(max(\tau_{h_x}(r)))^2 + (min(\gamma_{h_x}(r)))^2 \leq 1$ and $(min(\tau_{h_x}(r)))^2 + (max(\delta_{h_x}(r)))^2 \leq 1$. For ease of presentation, the PPHFN is represented by the pair $\tau_{h_x}/p_{\mathrm{g}}^{\sim}, \partial_{h_x}/b_{\mathrm{g}}$.

- The Pythagorean probabilistic hesitant fuzzy weighted average (PPHFWA) operator in (37) is utilized to aggregate the PPHFNs for each criterion among the PPHFS-EDMs of the three decision-makers.

Definition (2). [86] Let $\aleph_j = \left(\tau_{h_{g_j}}/\tilde{p}_{\dot{g}_j}, \partial_{h_{g_j}}/b_{\dot{g}_j}\right)(\hat{j} = 1,2,\ldots,r)$ be any combination of PPHFWA and PPHFNs. The operator for PPHFWA can also be written as.

$$\text{PPHFWA}\,(\aleph_1, \aleph_2, \cdots, \aleph_r) = \mathbb{K}_1\aleph_1 \oplus \mathbb{K}_2\aleph_2 \oplus \cdots\cdots \oplus \mathbb{K}_r\aleph_r \tag{38}$$

The PPHFWA can provide the following aggregate result:

$$\text{PPHFWA}\,(\aleph_1, \aleph_2, \cdots, \aleph_r) = \bigcup_{\mathrm{h}_j \in \tau_{\aleph_j}, \tilde{p}_{\aleph_j} \in \tilde{p}_{\aleph_j}} \sqrt{1 - \Pi_{j=1}^{r}\left(1 - \left(\hbar_{\aleph_j}\right)^2\right)^{K_j}} / \Pi_{j=1}^{r}\tilde{p}_{\aleph_j}, \bigcup_{\varrho_{\aleph_j} \in \delta_{\aleph_j}, b\aleph_j \in b\aleph_j} \Pi_{j=1}^{r}\left(\varrho_{\kappa_j}\right)^{\mathbb{K}_j} / \Pi_{j=1}^{r} b_{\kappa_j}$$

Where $\mathbb{K} = (\mathbb{K}_1, \mathbb{K}_2, \cdots, \mathbb{K}_r)^T$ denotes the weights of $\aleph_j \in [0,1]$ with $\sum_{j=1}^{r} \mathbb{K}_{\hat{j}} = 1$.

- The fuzzy weights are defuzzied with the PPHFS scoring function and turned into crisp weights using in (39).

Definition (3). For any PPHFN $\text{PPHFN}\aleph = \left(\tau_{h_{g_j}}/\tilde{p}_{\dot{g}_j}, \partial_{h_{g_j}}/b_{\dot{g}_j}\right)$ the score function is defined as.

$$s(\aleph) = \left(\frac{1}{M_\aleph}\sum_{h_i \in \tau_{h_g}, p^- \in \tilde{p}_{h_{\tilde{g}}}} (\hbar_i \cdot \tilde{p}_i)\right)^2 - \left(\frac{1}{N_\aleph}\sum_{\varrho_i \in \upsilon_{h_k}, b_i \in b_{h_{\tilde{g}}}} (\varrho_i \cdot b_i)\right)^2 \tag{39}$$

-assign the total weights of the essential criterion and each of the levels of the criteria equal 1. If these criteria are not satisfied, (40) is used to rescale the weights.

$$\mathrm{w_j} = s(\aleph) / \sum_{\mathrm{j}=1}^{\mathrm{J}} s(\aleph) \tag{40}$$

Where $s(\aleph)$ refers to the weight for each criterion.

6- T-spherical FWZIC (T-SFWZIC)

The T-SFSs structure is broader and more general, with no constraints on their constants, and it can manage uncertainty in data to capture information with a higher degree of freedom [87]. In the T-SFSs, if the power on restrictions grows to T, where T is any positive integer, we may give any value of our choice in the interval [0,1] to membership, non-membership, and hesitancy degrees. In this situation, the total of membership, non-membership, and hesitation degrees should not be greater than one. T is determined by the decision makers involved. This choice of T brings special attention to T-SFSs, causing its space to be noted for different values of T. Furthermore, the T-SFSs structure could represent people's decision-making consciousness and accurately define the decision information by a parameter which can flexibly modify the scope of information expressing [88].

The benefit:

- Many MCDM issues can be solved using this technique, and it is better capable of processing and presenting unfamiliar information in unknown situations. As a result, T-SFSs environment was utilized to offer an appropriate and robustness for problems in order to continue keeping up with the current condition in tackling the ambiguity and vagueness issues.
- The objective for such formulations was to execute THIS technique with no constraints on its constants and achieve a higher level of freedom in dealing with data uncertainty.

The T-SFS is an objective with the form and as described in (41) and (42).

$$P = \{\langle m, (\mu_d(m), v_d(m), s_d(m)) \rangle \mid m \in M\}, \tag{41}$$

Where $u_d, M \to [0,1]$ is the membership function, and $v_d: M \to [0,1]$ is a non-membership function of element $m \in M$, and $s_d: M \to [0,1]$.

$$0 < (\mu_d(m))^T + (v_d(m))^T + (s_d(m))^T \leq 1, \tag{42}$$

Where T ≥ 1

The degree of hesitancy is presented in (43) [89]

$$\pi_m(m) = \sqrt[T]{1 - (\mu_d(m))^T + (v_d(m))^T + (s_d(m))^T} \tag{43}$$

Aggregation Operations:

The following equations were used in the applied arithmetic operation utilizing T-SFS. (44) shows T-SFS summing and aggregation procedures.

$$\mathrm{T-SAM}\,(\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n) = \Big\{\big[1 - \textstyle\prod_{i=1}^{n} \left(1 - \mu_{\tilde{p}_i}^2\right)\big]^{1/T}, \prod_{i=1}^{n} v_{\tilde{p}_i}, \big[\prod_{i=1}^{n} \left(1 - \mu_{\tilde{p}_i}^2\right) - \prod_{i=1}^{n} \left(1 - \mu_{\tilde{p}_i}^2 - s_{\tilde{p}_i}^2\right)\big]^{1/T}\Big\}. \tag{44}$$

In (43) and (45) were used to execute the division operation (45), on the other hand, was adapted from [90], which is employed in the spherical fuzzy set. To fulfill the T-SFS structure, the square inside this operation was transformed to the power t in this study.

$$p_1 \oslash p_2 = \left(\left(\frac{\mu_{p1}^T(2-\mu_{p2}^T)}{1-(1-\mu_{p1}^T)\cdot(1-\mu_{p2}^T)}\right)^{\frac{1}{T}}, \frac{(v_{p1}^T - v_{p2}^T)^{\frac{1}{T}}}{(1-v_{p1}^T \cdot v_{p2}^T)^{\frac{1}{T}}}, \frac{(s_{p1}^T - s_{p2}^T)^{\frac{1}{T}}}{(1-s_{p1}^T \cdot s_{p2}^T)^{\frac{1}{T}}}\right) \quad \text{if } \frac{\mu_{p2}^T}{\mu_{p1}^T} \geq \frac{1-s_{p2}^T}{1-s_{p1}^T}\frac{1+s_{p1}^T}{1+s_{p2}^T} \geq 1 \tag{45}$$

(46) shows the equation of T-SFS division on crisp value [83].

$$\tilde{P} \oslash \lambda = \left\{\left(1 - \left(1 - \mu_{\tilde{p}}^T\right)^{1/\lambda}\right)^{1/T}, v_{\tilde{p}}^{1/\lambda}, \quad s_{\tilde{p}}^{1/\lambda}\right\} \quad \text{where } \lambda > 0. \tag{46}$$

(47), the defuzzied (crisp) value of a T-SFS fuzzy number is defined as follows [91].

$$\mathrm{Score}\,(\tilde{p}) = \mu_{\tilde{p}}{}^T - s_{\tilde{p}}{}^T \tag{47}$$

According to Table 9, all linguistic variables are translated into T-SFS, assuming that the fuzzy number is the variable for each Expert K criteria. In other words, Expert K was asked to rank the relevance of vaccine distribution criteria using a linguistic scale.

Table 9. - Linguistic Terms with Equivalent T-SFS [45]

linguistic scale	T-SFS
Very Low Important (VLI)	(0.15,0.85,0.1)
Low Important (LI)	(0.25,0.75,0.2)
Average (Av)	(0.55,0.5,0.25)
Important (Im)	(0.75,0.25,0.2)
Very Important (VI)	(0.85,0.15,0.1)

7- Interval type 2 Trapezoidal - Fuzzy Weighted with Zero Inconsistency (IT2TR-FWZIC)

Development phase

- Based on a type 1 fuzzy set which has a limitation—for example, a type 1 fuzzy set has been verified so that the membership grading is a crisp value for each input [92] added that determining membership values directly is difficult. One of fuzzy type 1's drawbacks is its failure to directly model and reduce the impact of data uncertainty [93]. The use of fuzzy type 2 is mainly motivated by its ability to model second order uncertainty and is computationally simple [94]. Furthermore, the fuzzy type 2 is important in specifying the correct membership function. Table 10 shows Linguistic terms and their equivalent IT2TR.

Table 10. - Linguistic terms and their equivalent IT2TR [95]

Linguistic Terms	IT2TR Fuzzy Sets
Very Low Important (VLI)	[(0,0,0,0.1;1,1), (0,0,0,0.5 ;0.9,0.9)]
Low Important (LI)	[(0,0.1,0.1,0.3;1, 1), (0.5, 0.1, 0.1, 0.2; 0.9, 0.9)]
Average (Av)	[(0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9)]
Important (Im)	[(0.7, 0.9, 0.9,1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9, 0.9)]
Very Important (VI)	[(0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 0.9, 0.9)]

- Fuzzy type-2 is the most commonly used since it can model second-order uncertainty and is computationally easy. Furthermore, fuzzy type-2 can be used to define the correct membership function.st reem

- IT2TRFWZIC is capable of resolving inconsistencies and achieving high precision.

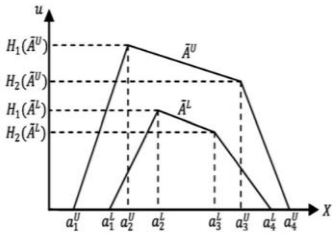

FIGURE 3. - Type 2 Trapezoidal Membership Followed by the Arithmetic Operations and Defuzzification Equations [91]

Figure 3 shows the type 2 trapezoidal membership, which is followed by the arithmetic operations and defuzzification equations.

Let h_A^L and h_A^U denote the heights of A^L and A^U, respectively, where $0 \le h_A^L \le h_A^L \le 1$.

$$H^L(x) = \begin{cases} \frac{h_A^L(x-a_1^L)}{a_2^L-a_1^L} & a_1^L < x < a_2^L, \\ h_A^L & a_2^L \le x \le a_3^L, \\ \frac{h_A^L(a_4^L-x)}{a_4^L-a_3^L} & a_3^L < x < a_4^L, \\ 0 \text{ otherwise} \end{cases} \quad (48)$$

$$H^U(x) = \begin{cases} \frac{h_A^U(x - a_1^U)}{a_2^U - a_1^U} & a_1^U < x < a_2^U, \\ h_A^U & a_2^U \le x \le a_3^U, \\ \frac{h_A^U(a_4^U - x)}{a_4^U - a_3^U} & a_3^U < x < a_4^U, \\ 0 \text{ otherwise} \end{cases} \quad (49)$$

The arithmetic operations are represented in the following definitions [96]:

Addition:

$$\tilde{A} \oplus \tilde{B} = \left(a_1^T + b_i^T; min\left(H_1(\tilde{A}^T), H_1(\tilde{B}^T)\right), min\left(H_2(\tilde{A}^T), H_2(\tilde{B}^T)\right): T \in \{U, L\}, i = 1,2,3,4\right). \quad (50)$$

Subtraction

$$\tilde{\tilde{A}} \ominus \tilde{\tilde{B}} = \left(a_1^T - b_{5-15}^T; min\left(H_1(\tilde{A}^T), H_1(\tilde{B}^T)\right), min\left(H_2(\tilde{A}^T), H_2(\tilde{B}^T)\right): T \in \{U, L\}, i = 1,2,3,4\right).$$

$$\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} = \left(X_i^T; min\left(H_1(\tilde{A}^T), H_1(\tilde{B}^T)\right), min\left(H_2(\tilde{A}^T), H_2(\tilde{B}^T)\right): T \in U, L, i = 1,2,3,4\right) \quad (51)$$

Multiplication

$$X_i^T = \begin{cases} min(a_i^T b_i^T, a_i^T b_{5-i}^T, a_{5-i}^T b_i^T, a_{5-i}^T b_{5-i}^T) \text{ if } i = 1,2 \\ max(a_i^T b_i^T, a_i^T b_{5-i}^T, a_{5-i}^T b_i^T, a_{5-i}^T b_{5-i}^T) \text{ if } i = 3,4 \end{cases} \text{ and } T \in \{U, L\} \quad (52)$$

Division

$$\tilde{A} \oslash \tilde{B} = \left(Y_i^T; min\left(H_1(\tilde{A}^T), H_1(\tilde{B}^T)\right), min(H_2(\tilde{A}^T), H_2(\tilde{B}^T): T \in \{U, L\} i = 1,2,3,4,)\right)$$

Where

$$Y_i^T = \begin{cases} min\left(\frac{a_i^T}{b_i^T}, \frac{a_i^T}{b_{5-i}^T}, \frac{a_{5-i}^T}{b_i^T}, \frac{a_{5-i}^T}{b_{5-i}^T}\right) \text{ if } i = 1,2 \\ max\left(\frac{a_i^T}{b_i^T}, \frac{a_i^T}{b_{5-i}^T}, \frac{a_{5-i}^T}{b_i^T}, \frac{a_{5-i}^T}{b_{5-i}^T}\right) \text{ if } i = 3,4 \end{cases} \quad b_j^T \ne 0, j = 1,2,3,4 \text{ and } T \in \{U, L\}. \quad (53)$$

The defuzzied (crisp) value of a trapezoidal interval type 2 fuzzy number is defined as follows [97]:

$$\text{Def}\left(\tilde{\tilde{A}}\right) = \frac{1}{2}\left(\sum_{T \in \{U,L\}} \frac{a_1^T + \left(1 + H_1(\tilde{A}^T)\right) a_2^T + \left(1 + H_2(\tilde{A}^T)\right) a_3^T + a_4^T}{4 + H_1(\tilde{A}^T) + H_2(\tilde{A}^T)}\right) \quad (54)$$

8- Cubic Pythagorean fuzzy-weighted zero-inconsistency (CP-FWZIC)

CPFS is one of the most effective strategies for dealing with uncertainty issue, particularly in complicated and tough situations. CPFS was created for representing vagueness or ill-defined information through the use of interval valued Pythagorean fuzzy sets (IVPFSs) and PFSs [98]. The preceding benefits make CPFS a strong tool, and CPFS includes complicated mathematical expressions that employ both PFS and IVPFS together. As a result, a similar kind information of might be showed for different situations under CPFS. Because of the benefits of CPFS in handling so many MCDM challenges, particularly those that occur in complex situations with imprecise data and ambiguity, as well as if the expert's judgments regarding alternatives are ambiguous in relation to the criteria stated [99]. Table 11 shows Linguistic Terms and their equivalent CPFNs.

Table 11. - Linguistic Terms with Equivalent CPFNs [100]

Numerical scale	Linguistic scale	CPFNs
1	Extremely bad (EB)	(0,0.1,0.9,1,0.1,0.9)
2	Huge bad (HB)	(0.1,0.2,0.8,0.9,0.2,0.8)
3	Very bad (VB)	(0.3,0.4,0.75,0.8,0.4,0.75)
4	Medium bad (MB)	(0.35,0.45,0.7,0.75,0.45,0.7)
5	Bad (B)	(0.4,0.5,0.6,0.7,0.5,0.6)
6	Good (G)	(0.5,0.6,0.5,0.6,0.6,0.5)
7	Medium good (MG)	(0.6,0.7,0.35,0.5,0.7,0.35)
8	Very good (VG)	(0.7,0.8,0.25,0.35,0.8,0.25)
9	Huge good (HG)	(0.8,0.9,0.2,0.25,0.9,0.2)
10	Extremely good (EG)	(0.9,1,0,0.1,1,0)

The benefits

- The benefits of Cubic Pythagorean fuzzy sets, one of the most comprehensive fuzzy environments lately proposed to solve the problem of uncertainty.
- In complicated and tough situations where the fuzziness of the expert's judgments occurs over alternatives with regard to the criteria, CPFS is a highly robust tool.

The equations to define CPFS from are defined as (55) and (56) [101].

$$P = (\tilde{p}_{\tilde{c}}, p_{\tilde{c}}) = \left(\langle[\mu^L_{\tilde{c}}, v^L_{c}], [\mu^U_{\tilde{c}}, v^U_{\tilde{c}}]\rangle; (\mu_{\tilde{c}}, v_{\tilde{c}})\right), \quad (55)$$

Where $\mu^L_{\tilde{c}}, \mu^U_{\tilde{c}}: M \to [0,1]$ is the lower and upper of membership function, while $v^L_{\tilde{c}}, v^U_{\tilde{c}}: M \to [0,1]$ is the lower and upper of the non-membership function of element $m \in M$ to p, and it must fulfil the restriction seen in (54).

$$0 < \mu^L_{\tilde{c}}(m)^2 + v^L_{c}(m)^2 \leq 1, 0 < \mu^U_{\tilde{c}}(m)^2 + v^U_{c}(m)^2 \leq 1$$
$$\text{and } 0 < \mu_{\tilde{c}}(m)^2 + v_{c}(m)^2 \leq 1. \quad (56)$$

The degree of hesitancy is presented in (57) as follows: Let

$$\pi_{\tilde{c}}(m) = [\pi^L_{\tilde{c}}(m), \pi^U_{\tilde{c}}(m)]; \varphi_{\tilde{c}}(m).$$
$$\varphi_{\tilde{c}}(m) = \sqrt{1 - \mu^2_{\tilde{c}}(x) - v^2_{\tilde{c}}(x)}$$
$$\pi^L_{\tilde{c}}(x) = \sqrt{1 - \left(\mu^U_{\tilde{c}}(x)\right)^2 - \left(v^U_{\tilde{c}}(x)\right)^2}$$
$$\pi^U_{\tilde{c}}(x) = \sqrt{1 - \left(\mu^L_{\tilde{c}}(x)\right)^2 - \left(v^L_{\tilde{c}}(x)\right)^2} \quad (57)$$

Aggregation Operation

In (58), the cubic Pythagorean fuzzy average mean (CPFA) aggregation procedure is shown as follows:

$$\mathrm{CPFA}(e_1 e_2 \dots, e_n) = \begin{pmatrix} \left\langle \left[\sqrt{1 - \Pi^n_{i=1}\left(1 - \left(\mu^L_{e_i}\right)^2\right)}, \sqrt{1 - \Pi^n_{i=1}\left(1 - \left(\mu^U_{e_i}\right)^2\right)} \right] \right. \\ [\Pi^n_{i=1}(v^L_e), \Pi^n_{i=1}(v^U_e)] \\ \left(\sqrt{1 - \Pi^n_{i=1}\left(1 - \left(\mu_{e_i}\right)^2\right)}, \Pi^n_{i=1}(v_{e_e}) \right) \end{pmatrix}. \quad (58)$$

(59) shows the CPFS division operation as follows:

$$\frac{p_1}{p_2} = \left(\frac{\mu_1^L}{\mu_2^L}, \frac{\mu_1^U}{\mu_2^U}, \sqrt{\frac{v_1^L - v_2^L}{1 - v_2^L}}, \sqrt{\frac{v_1^U - v_2^U}{1 - v_2^U}}, \frac{\mu_1}{\mu_2}, \sqrt{\frac{v_1 - v_2}{1 - v_2}}\right)$$

$$\text{if } \mu_1^L \leq min\left\{\mu_2^L, \frac{\mu_2^L \pi_1^L}{\pi_2^L}\right\}, v_1^L \geq v_2^L$$

$$\mu_1^U \leq \mathbf{m\,i\,n}\left\{\mu_2^U, \frac{\mu_2^U \pi_1^U}{\pi_2^U}\right\}, v_1^U \geq v_2^U$$

$$\mu_1 \leq \mathbf{m\,i\,n}\left\{\mu_2, \frac{\mu_2 \pi_1}{\pi_2}\right\}, v_1 \geq v_2. \tag{59}$$

(60) shows the equation of CPFS division on crisp value, as shown in Table11.

$$\frac{p}{\lambda} = \left(\frac{\left\langle\left[\sqrt{1-\left(1-(\mu_p^L)^2\right)^{\frac{1}{\lambda}}}, \sqrt{1-\left(1-(\mu_p^U)^2\right)^{\frac{1}{2}}}\right], \left[(v_p^L)^{\frac{1}{2}}, (v_p^U)^{\frac{1}{\lambda}}\right]\right\rangle}{\left(\sqrt{1-\left(1-\mu_p^2\right)^{\frac{1}{\lambda}}, v_p^{\frac{1}{2}}}\right)}\right) \tag{60}$$

According to Table 11, all linguistic variables are translated into CPFNs, assuming that the fuzzy number is the variable for each Expert K criteria. In other words, Expert K must request that the importance level of the assessment criterion be identified within the variables measured with a linguistic scale.

Fuzzification ratio has been used and determined with CPFNs using (57, 58 and 59).
Defuzzification takes place when determining the criteria's final weight. Equation (61) is utilized as the defuzzification method to score every criteria. To determine the final weight values, the sum of the weights of all the rescaling criteria is also applied at this stage.

$$S(\tilde{c}) = \frac{1}{2}\left[\frac{1}{2}[(\mu_{\tilde{c}}^L)^2 + (\mu_{\tilde{c}}^U)^2 - (v_{\tilde{c}}^L)^2 - (v_{\tilde{c}}^U)^2] + ((\mu_{\tilde{c}})^2 - (v_{\tilde{c}})^2)\right] \tag{61}$$

9- Neutrosophic FWZIC (NS-FWZIC)

Neutrosophic fuzzy sets (NFSs) have been offered [102], where the word "neutrosophy" denotes the knowledge of neutral thoughts. Decision-makers were able to work with the knowledge of neural thinking through NFS [103]. This type's neutrality allows for the addition of new capabilities to model ambiguous information [104]. Neutrosophy is a new subfield of philosophy that examines the nature, origin, and scope of neutralities as well as how they interact with different ideational spectra [105]. NFSs are praised for having the ability to handle vague, inaccurate, and insufficient information [106]. Because of these characteristics, a lot of studies used NFSs. Neutronosophic fuzzy sets.

- For further study, the opinion matrices of all decision matrices (DMs) are converted from linguistic to numerical scale, as shown in Table 12.

Table 12. - Linguistic Terms with Equivalent Numerical And Neutrosophic Fuzzy Numbers (Nfns) [107]

Linguistic scoring scale	Numerical scoring scale	NFNs		
		ρ	σ	τ
Very Low Important (VLI)	1	0.95	0.05	0.05
Low Important (LI)	2	0.75	0.25	0.25
Average (Av)	3	0.50	0.50	0.50
Important (Im)	4	0.25	0.75	0.75
Very Important (VI)	5	0.05	0.95	0.95

The NFS is presented by [102] and defined in (62).

$$N = \{x, \rho\_N(x), \sigma\_N(x), \tau\_N(x) | x \in X\} \tag{62}$$

Where N is a simplified neutrosophic set (SNS) and X is a universe of discourse. In X, N is represented by truthmembership functions: $\rho\_N(x)$, an indeterminacy-membership function $\sigma N(x)$ and a falsity-membership function $\tau N(x)$, where functions $\rho N(x)$, $\sigma N(x)$ and $\tau N(x)$ are singleton subintervals/subsets in the real standard interval [0, 1], such that $\rho N(x) : X \to [0, 1]$, $\sigma N(x) : X \to [0, 1]$ and $\tau N(x) : X \to [0, 1]$.

The following part specifies the basic arithmetic operations with NFNS:

Summation and aggregation operation [108]:

Let $\tilde{a}_j = \langle \rho_j, \sigma_j, \tau_j \rangle (j = 1,2, \dots, n)$ be a collection of SNSs, and SNG: $Q_n \to Q$ if

$$\mathrm{SNG}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_j = \left(\prod_{j=1}^{n} \rho_j, 1 - \prod_{j=1}^{n} (1 - \sigma_j), 1 - \prod_{j=1}^{n} (1 - \tau_j)\right) \quad (63)$$

SNG is identified using a simplified neutrosophic geometric average operator.

Division Operation:

The division operation of SNSs A and B for any two given SNSs A and B is defined as follows:

$$\frac{A}{B} = \left\{\left\langle x, \frac{\rho_A(x)}{\rho_B(x)}, \frac{\sigma_A(x) - \sigma_B(x)}{1 - \sigma_B(x)}, \frac{\tau_A(x) - \tau_B(x)}{1 - \tau_B(x)} \right\rangle \mid x \in X\right\}, \quad (64)$$

which is valid under the conditions B ≥ A, $\rho_B(x) \neq 0$, $\sigma_B(x) \neq 1$, and $\tau_B(x) \neq 1$.

The SNS division equation on crisp value is displayed in (65).

$$\frac{N}{\lambda} = \left\{\left\langle x, 1 - (1 - \rho_N(x))^{\frac{1}{\lambda}}, \sigma_N^{\frac{1}{\lambda}}(x), \tau_N^{\frac{1}{\lambda}}(x) \right\rangle \mid x \in X\right\}, \lambda > 0 \quad (65)$$

Aggregation Operation

Equation (66) is used to aggregate the results of (65), where SNWG (neutrosophic weighted geometric) means an average operator [108].

$\tilde{a}_j = u_j, p_j v_j (j = 1,2, \dots, n)$ be a collection of SNNs, and SNG: $Q_n \to Q$, if

$$\mathrm{SNWG}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_j^{\omega_j} = \left(\prod_{j=1}^{n} \rho_j^{\omega_j}, 1 - \prod_{j=1}^{n} (1 - \sigma_j)^{\omega_j}, 1 - \prod_{j=1}^{n} (1 - \tau_j)^{\omega_j}\right) \quad (66)$$

Where ωj is the weight of $\tilde{a}_j (j = 1,2, \dots, n)$, $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$

Scoring or the defuzzied (crisp) value of SNSs is defined as follows [120]:

$$s(A) = (\rho_A + 1 - \sigma_A + 1 - \tau_A)/3 \quad (67)$$

(67) is used to get the final weight after defuzzification.

10- Fermatean Probabilistic Hesitant-Fuzzy Weighted Zero Inconsistency (FPH–FWZIC)

A hesitant FS (HFS) [61] is an interesting complement to the regular FS that enhances MCDM by dealing with expert uncertainty effectively [62] completed research that offered an overview of the state of the art and the future prospects of HFSs. This study shows that (i) an HFS can enhance expert preference elicitation and (ii) an HFS is a common and more flexible preference structure with a capability to minimize uncertainty [109]. later merged HFSs with intuitionistic FSs to generate new FS called intuitionistic hesitant FSs. Later, [85] enhanced the probabilistic hesitant FS and created the naming scheme of Pythagorean probabilistic hesitant FSs. The degrees of positive and negative hesitant adhesions describe Pythagorean probabilistic hesitant FSs equally, with the condition that the square sum of these degrees must be less than or equal to 1. every degree of negative hesitant adhesion had a preference beyond the other ones [21]. FPHFSs, on the other hand, are limited in that the cube aggregate of positive and negative hesitant grades has to be below or equal to one. As a result, FPHFSs will be able to effectively deal with expert uncertainty and analyze the occurrence probability of each element. Because the FFS is an extended version of the Pythagorean FS, we introduce the unique idea of Fermatean probabilistic hesitant FSs (FPHFSs).
The developed evaluation form is used to gather linguistic terms that describe expert preferences, as shown in Table 13.

Table 13. - Linguistic Expressions with Corresponding Numeric Scale and FPHFNs [110]

Linguistic expressions (For FPH-FWZIC)	Numeric scale	FPHFNs							
		M1	P1	M2	P2	V1	P1	V2	P2
Very Important (VI)	1	0.9	0.4	0.95	0.6	0.15	0.2	0.1	0.8
Important (I)	2	0.7	0.5	0.75	0.5	0.35	0.3	0.3	0.7
Average (Av)	3	0.5	0.3	0.55	0.7	0.55	0.5	0.5	0.5
Low Important (LI)	4	0.4	0.8	0.5	0.2	0.55	0.4	0.6	0.6
Very Low Important (VLI)	5	0.2	0.7	0.25	0.3	0.8	0.35	0.9	0.65

Definition 1. R is as before. FPHFS ℵ on R is expressed as:

$$\aleph = \{\langle r, \tau_{h_k}(r)/\tilde{p}_{\dot{g}}, \partial_{h_x}(r)/b_{\dot{g}}\rangle \mid r \in R\},$$

Where $r \in R, \tau_{h_\gamma}(r)$ and $\delta_{h_x}(r)$ are sets of certain values in [0,1]. $\tau_{h_x}(r)/\tilde{p}_{\dot{g}} \& \partial_{h_x}\frac{\hat{r}}{b_g}$ specify the probable positive and negative grades of r with respect to FPHFS ℵ, respectively. $\tilde{p}_{\dot{g}}$ and $b_{\dot{g}}$ represent the probabilities of grades. Additionally, $0 \le ћ_i, \varrho_j \le 1$ and $0 \le \tilde{p}_i, b_j \le 1$ with $\sum_{i=1}^{L} \tilde{p}_i 1, \sum_{j=1}^{L} b_j \le 1$ (L is a positive integer used to represent the number of items in FPHFS), where $ћ_i \in \tau_{h_x}(r)$ and $\varrho_j \in O_{h_*}(r), \tilde{p}_i \in \tilde{p}_{\dot{g}}, bj \in b_{\dot{g}}$, furthermore it is required that

$$\left(max\left(\tau_{h_x}(r)\right)\right)^3 + \left(min\left(\breve{\partial}_{h_x}(r)\right)\right)^3 \le 1$$

and $\left(min(\tau_{h_k}(r))\right)^3 + \left(max(\tilde{\partial}_{h_k}(r))\right)^3 \le 1.$

The pair $(\tau_{h_x}/\tilde{p}_{\dot{g}}, \partial_{h_x}/b_{\dot{g}})$ represents the Fermatean probabilistic hesitant fuzzy number (FPHFN). FPHF ˆS(R) represents the group of all FPHFSs in R.

Definition 2. Let $\aleph_1 = (\tau_{h_1 1}/\tilde{p}_{\dot{g}_1}, \partial_{h_{g1}}/b_{\dot{g}1})$ and ℵ2 = $(\tau_{h_{g2}}/\tilde{p}_{g2}, \partial_{h_{g2}}/b_{\dot{g}2})$ be FPHFNs. The basic operational laws are:

$$\aleph_1 \cup \aleph_2 = \left\{ \bigcup_{\substack{ћ_1 \in \tau_{h_{g_1}(l_g)}, \tilde{p}_{1 \in \tilde{p}_g} \\ ћ_2 \in \tau_{h_{g_2}(l_g)}, \tilde{p}_{2 \in \tilde{p}_{g_2}}}} \left(max(ћ_1/\tilde{p}_1, ћ_2/\tilde{p}_2)\right), \bigcup_{\substack{\varrho_1 \in \hat{\partial}_{h_{\dot{g}_1}}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}_1} \\ \varrho_2 \in O_{h_{\dot{g}_2}}(l_{\dot{g}}), b_2 \in b^{\dot{g}_2}}} (min(\varrho_1/b_1, \varrho_2/b_2)) \right\}. \quad (68)$$

$$\aleph_1 \cap \aleph_2 = \left\{ \bigcup_{\substack{ћ_1 \in \tau_{h_{g_1}(l_g)}, \tilde{p}_{1 \in \tilde{p}_g} \\ ћ_2 \in \tau_{h_{g_2}(l_g)}, \tilde{p}_{2 \in \tilde{p}_{g_2}}}} \left(min(ћ_1/\tilde{p}_1, ћ_2/\tilde{p}_2)\right), \bigcup_{\substack{\hat{\partial}_1 \in \hat{\partial}_{g_1}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}_1} \\ \varrho_2 \in O_{h_{g_2}}(l_{\dot{g}}), b_2 \in b^{\dot{g}_2}}} (max(\varrho_1/b_1, \varrho_2/b_2)) \right\} \quad (69)$$

$$\aleph_1^c = (\hat{\partial}_{h_x}/b_{\dot{g}}, \tau_{h_x}/\tilde{p}_{\dot{g}}) \quad (70)$$

Definition 3. Let $\aleph 1 = (\tau_{h_{g1}}/\tilde{p}_{g1}, \partial_{h_{g1}}/b_{\dot{g}1})$ and $\aleph 2 = \aleph_2 = (\tau_{h_{g2}}/\tilde{p}_{g2}\ \partial_{h_{g2}}/b_{\dot{g}2})$ be FPHFNs and $\zeta > 0 (\in \mathbb{R})$; then, their operations are introduced as:

$$\aleph_1 \oplus \aleph_2 = \left\{ \bigcup_{\substack{\hbar_1 \in \tau_{h_{\dot{g}_1}(l_g)}, \tilde{p}_1 \epsilon \tilde{p}_g \\ \hbar_2 \in \tau_{h_{\dot{g}_2}(l_{\dot{g}})}, \tilde{p}_{2\epsilon \tilde{p}_{\dot{g}_2}}}} \left(\sqrt[3]{\hbar_1^3 + \hbar_2^3 - \hbar_1^3 \hbar_2^3} / \tilde{p}_1 \tilde{p}_2 \right) \right.$$
$$\left. \bigcup_{\substack{\varrho_1 \in \hat{\partial}_{h_{\dot{g}_1}}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}_1} \\ \varrho_2 \in O_{h_{\dot{g}_2}}(l_{\dot{g}}), b_2 \in b^{\dot{g}_2}}} (\varrho_1 \varrho_2 / b_1 b_2) \right\}. \quad (71)$$

$$\aleph_1 \otimes \aleph_2 = \left\{ \bigcup_{\substack{\hbar_1 \in \tau_{h_{h_1}(l_g)}, \tilde{p}_{1\epsilon \tilde{p}_g} \\ h_2 \in \tau_{h_{g_2}(l_{\dot{g}})}, \tilde{p}_{2\epsilon \tilde{p}_{\tilde{g}_2}}}} (h_1 h_2 / \tilde{p}_1 \tilde{p}_2), \right.$$
$$\left. \bigcup_{\substack{\varrho_1 \in \hat{\partial}_{h_{\dot{g}_1}}(l_{\dot{g}}), b_1 \in \underline{b}_{\dot{g}_1} \\ \varrho_2 \in O_{h_{g_2}}(l_{\dot{g}}), b_2 \in b^{\dot{g}_2}}} \left(\sqrt{\varrho_1^2 + \varrho_2^2 - \varrho_1^2 \varrho_2^2} / b_1 b_2 \right) \right\} \quad (72)$$

$$\zeta \aleph_1 = \left\{ \bigcup_{\hbar_1 \in \tau_{h_{\dot{g}_1}(l_g)}, \tilde{p}_{1\epsilon \tilde{p}\dot{g}_1}} \left(\sqrt[3]{1 - (1 - \hbar_1^3)^{\zeta}} / \tilde{p}_1 \right) \right.$$
$$\left. \bigcup_{\varrho_1 \in \hat{\partial}_{\dot{g}_1}(l_g), b_1 \in \underline{b}_{\dot{g}_1}} (\varrho_1^{\zeta} / b_1) \right\}. \quad (73)$$

$$\aleph_1^{\zeta} = \left\{ \bigcup_{h_1 \in \tau_{h_{g_1}(l_g)}, \tilde{p}_{1\epsilon \tilde{p}\dot{g}_1}} (h_1^{\zeta} / \tilde{p}_1) \right.$$
$$\left. \bigcup_{\varrho_1 \in \hat{\partial}_{h_{g_1}}(l_g), b_1 \in \underline{b}_{g_1}} \left(\sqrt[3]{1 - (1 - \varrho_1^3)^{\zeta}} / b_1 \right) \right\}. \quad (74)$$

Defuzzication is the process by which the computed fuzzy weights of the evaluation criteria are defuzzified and converted into crisp weights using the score function specified in this Definition.

Definition 4. For any FPHFN $\aleph\ (\tau_{h_x}/\tilde{p}_{\dot{g}}, \hat{\partial}_{h_x}/b_{\dot{g}})$, a score function be described as:

$$s(\aleph) = \left(\frac{1}{M_{\aleph}} \sum_{h_i \in \tau_{h_g}, \tilde{p}_i \in \tilde{p}_{h_g}} (\hbar_i \tilde{p}_i) \right)^3 - \left(\frac{1}{N_{\aleph}} \sum_{\varrho_i \in \delta_{h_k}, b_i \in b_{h_g}} (\varrho_i b_i) \right)^3,$$

Where M$\aleph$ and N$\aleph$ represent the number of components in τ_{h_g} and $\hat{\partial}_{h_x}$, respectively.

Definition 5. For any FPHFN $\aleph = (\tau_{h_x}/\tilde{p}_{\dot{g}}, \hat{\partial}_{h_x}/b_{\dot{g}})$, an accuracy function is described as:

$$h(\aleph) = \left(\frac{1}{M_\aleph} \sum_{h_i \in \tau_{h_g}, p_i \in \tilde{p}_{h_g}} (\hbar_i \tilde{p}_i) \right)^3 + \left(\frac{1}{N_\aleph} \sum_{\varrho_i \in \delta_{h_k}, b_i \in b_{h_g}} (\varrho_i b_i) \right)^3,$$

Where M$\aleph$ and N$\aleph$ represent the number of components in τ_{h_g} and $\hat{\partial}_{h_\varkappa}$, respectively.

Definition 6. Let $\aleph 1 = \left(\tau_{h_{g1}}/\tilde{p}_{g1}, \partial_{h_{g1}}/b_{\dot{g}1}\right)$ and $\aleph 2 = \left(\tau_{h_{g2}}/\tilde{p}_{g2}\ \partial_{h_{g2}}/b_{\dot{g}2}\right)$ be FPHFNs. With the use of this definition, a comparison of FPHFNs can be defined as:

If $s(\aleph_1) > s(\aleph_2)$, then $\aleph_1 > \aleph_2$ (75)

If $s(\aleph_1) = s(\aleph_2)$, and $h(\aleph_1) > h(\aleph_2)$ then $\aleph_1 > \aleph_2$. (76)

Aggregation Operation:

Definition 7. Let $\aleph_{\hat{j}} = \left(\tau_{h_{g_j}}\left(\tilde{p}_{g_j}, \succ_{h_{g_j}}/b_{g_j}\right) \hat{J} = 1,2,\dots,r\right)$ be any group of FPHFNs and the Fermatean probabilistic hesitant fuzzy average mean (FPHFAM): FPHFNr → FPHFN. Then, the FPHFAM operator can be expressed as:
FPHFAM $(\aleph_1, \aleph_2, \dots, \aleph_r) = 1lr\aleph_1 \oplus 1lr\aleph_2 \oplus \cdots \oplus 1lr\aleph_r$.

Theorem 1. Let $\aleph_{\hat{j}} = \left(\tau_{h_{g_j}}\left(\tilde{p}_{g_j}, \partial_{h_{g_j}}/b_{g_j}\right) (\hat{J} = 1,2,\dots,r\right)$ be any group of FPHFNs. Then, the aggregation result obtained by using the FPHFAM can be obtained as follows: FPHFAM ($\aleph$1, $\aleph$2, . . ., $\aleph$r)

$$= \left(\begin{matrix} \bigcup_{\hbar_j \in \tau_{\aleph_j}, \tilde{p}_{\aleph_j} \in \tilde{p}_{\aleph_j}} \sqrt[3]{1 - \prod_{j=1}^{r} \left(1 - (\hbar_{\aleph_j})^3\right)^{3/r}} / \Pi_{\hat{j}=1}^{r} \tilde{p}_{\aleph_j} \\ \bigcup_{\varrho_{\aleph_j} \in \partial_{\aleph_j}, b_{\aleph_j} \in b_{\aleph_j}} \prod_{\hat{j}=1}^{r} (\varrho_{\aleph_j})^{1/r} / \Pi_{\hat{j}=1}^{r} b_{\aleph_j} \end{matrix} \right).$$

Definition 8. Let $\aleph_{\hat{j}} = \tau_{h_{g_j}}/\tilde{p}_{g_j}, \partial_{h_{g_j}}/b_{g_j}(\hat{J} = 1,2,\dots,r)$ be any group of FPHFNs and the Fermatean probabilistic hesitant fuzzy weighted average (FPHFWA): FPHFNr → FPHFN. Then, the FPHFWA operator can be expressed as:

FPHFWA $(\aleph_1, \aleph_2, \dots, \aleph_r) = W_1\aleph_1 \oplus W_2\aleph_2 \oplus \cdots \oplus W_r\aleph_r$

Where $W = (W_1, W_2, \dots, W_r)^T$

where W = (W1, W2, ..., Wr) T are the weights of $\aleph_{\hat{j}}$ ∈ [0, 1] with $\sum_{\hat{j}=1}^{r} W_{\hat{j}} = 1$

Theorem 2. Let $\aleph_{\hat{j}} = \tau_{h_{g_j}}/\tilde{p}_{g_j}, \partial_{h_{g_j}}/b_{g_j}(\hat{J} = 1,2,\dots,r)$ be any group of FPHFNs. Then, the aggregation result obtained

using the FPHFWA can be obtained as follows: FPHFWA ($\aleph$1, $\aleph$2, . . ., $\aleph$r)

$$= \left(\begin{matrix} \bigcup_{\hbar_j \in \tau_{\aleph_j}, \tilde{p}_{\aleph_j} \in \tilde{p}_{\aleph_j}} \sqrt[3]{1 - \prod_{\hat{j}=1}^{r} \left(1 - (\hbar_{\aleph_j})^3\right)^{W_{\hat{j}}}} / \Pi_{\hat{j}=1}^{r} \tilde{p}_{\aleph_j} \\ \bigcup_{\varrho_{\aleph_j} \in \partial_{\aleph_j}, b_{\aleph_j} \in b_{\aleph_j}} \prod_{\hat{j}=1}^{r} (\varrho_{\aleph_j})^{w_{\hat{j}}} / \prod_{\hat{j}=1}^{r} b_{\aleph_j} \end{matrix} \right)$$

11- Interval-Valued Pythagorean Fuzzy Rough Set (IVPFRS–FWZIC)

FS and rough set (RS) concepts are utilized to deal with precision and certainty problems. One of the most interesting topics of study for scholars is FS theory, which Zadeh [111] examined. Interval valued FSs were later introduced by [112] as a generalization of FS. Closed intervals [0,1] are the focus of interval valued FSs. [113] presented intuitionistic FSs that are capable of taking into account both the nonmembership grade (NMG) and the membership grade (MG). On the other hand, intuitionistic FSs are constrained by the requirement that the total of MG and NMG not above one. As a result, [114] suggested the Pythagorean FS (PFS) to get over the limitation of intuitionistic FSs. The fundamental difference between intuitionistic FSs and the PFS is because the sum of squares of

MG and NMG in the PFS is a real value that ranges from 0 to 1. When the MG and NMG in the PFS cannot be indicated as accurate real values, the PFS is unable to characterize ambiguous data properly. Yet grade ranges are available. In order to express more complicated ambiguity information [115], extended the PFS to the interval valued PFS (IVPFS) and developed a decision mechanism for MADM issues. In 1982, [71] has proposed the RS hypothesis. Many researches have been conducted in recent years to apply RS theory to real-world situations [116] introduced fuzzy rough sets (FRSs) and rough fuzzy sets (RFSs) by combining the ideas of FS and RS in a useful way [117] established the use of interval-valued FRSs [118]. Recently presented IVPFRS by combining IVPFS with Pawlak's RS theory. This FRSs can be utilized to tackle the FWZIC's inaccuracy and ambiguity in its information. Table 14 shows the Linguistic Measures of Importance.

Table 14. - Linguistic Measures of Importance [53]

Linguistic importance	Measurement of Numeric scale	IVPFNs			
		μ^l	μ^u	v^l	v^u
Very Important (VI)	1	0.85	0.9	0.05	0.1
Important (Im)	2	0.7	0.75	0.2	0.25
Average (Av)	3	0.5	0.55	0.4	0.45
Low Important (LI)	4	0.2	0.25	0.65	0.7
Very Low Important (VLI)	5	0.05	0.1	0.8	0.85

Definition 1. Presents the description of IVPFS and IVPFNs.
Definition 1. Let Int ([0,1]) symbolize all closed subintervals of [0,1], and X represent the universe of discourse. IVPFS $\tilde{A}_s$ in X is denoted by

$$\tilde{A}_s = \{< x, \mu_{\tilde{A}_s}(x), v_{\tilde{A}_s}(x) > | \ x \in X\} \quad (77)$$

Where $\mu_{\tilde{A}_s}$: X → Int([0, 1]) (x ∈ X → $\mu_{\tilde{A}_s}$ (x) ⊆ [0, 1]) and $v_{\tilde{A}_s}$: X → Int([0, 1]) (x ∈ X → $v_{\tilde{A}_s}$ (x) ⊆ [0, 1]) refer to the MG and NMG of element x ∈ X and set $\tilde{A}_s$, respectively and for every $x \in X, 0 \le sup\{(\mu_{P\tilde{A}_s}(x))^2\} + sup\{(v_{\tilde{A}_s}(x))^2\} \le 1$. In addition, for each x ∈ X, $\mu_{\tilde{A}_s}$ (x) and $v_{\tilde{A}_s}$ (x) are closed intervals with the lower and upper bounds are represented by $\mu^l_{\tilde{A}_s}(x), \mu^u_{\tilde{A}_s}(x), v^l_{\tilde{A}_s}(x), v^u_{\tilde{A}_s}(x)$, respectively. Therefore, $\tilde{A}_s$ can also be expressed in the following manner:

$$\tilde{A}_s = \{< x, [\mu^l_{\tilde{A}_s}(x), \mu^u_{\tilde{A}_s}(x)], [v^l_{\tilde{A}_s}(x), v^u_{\tilde{A}_s}(x)] > | \ x \in X\},$$

Which is subject to the condition $0 \le (\mu^u_{\tilde{A}_s}(x))^2 + (v^u_{\tilde{A}_s}(x))^2 \le 1$ the IVPFN is represented by

$$\tilde{A}_s = ([\mu^l_{\tilde{A}_s}, \mu^u_{\tilde{A}_s}] \ [v^l_{\tilde{A}_s}, v^u_{\tilde{A}_s}])$$

The resulting scoring function has been applied to every IVPFN in the IVPFS-EDM using (78).

$$\text{Def}\ (\tilde{A}_s) = \frac{1}{2}\left(\mu^l_{\tilde{A}Si} + \mu^u_{\tilde{A}Si} - v^l_{\tilde{A}_s}(u) - v^u_{\tilde{A}_s}(u)\right) \quad (78)$$

To transform IVPFNs to IVPFRNs, the attribute scores are organized in an ascending sequence.

Definition 2. Presents the description of IVPFRS.
Definition 2. Let Int ([0,1]) symbolize all close subintervals of [0,1], and X represent the universe of discourse. IVPFRS $\tilde{A}_s$ in X is represented by:

$$\tilde{A}_s = \{< x, \langle \underline{\mu_{\tilde{A}_s}}(x), \bar{\mu}_{\tilde{A}_s}(x)\rangle, \langle \underline{v_{\tilde{A}_s}}(x), \bar{v}_{\tilde{A}_s}(x)\rangle > | \ x \in X\} \quad (79)$$

where $\underline{\mu_{\tilde{A}_s}}, \bar{\mu}_{\tilde{A}_s}: X \to$ Int ([0,1]) $(x \in X \to \underline{\mu_{\tilde{A}_s}}(x), \bar{\mu}_{\tilde{A}_s}(x) \subseteq$

[0,1]) and $\underline{v_{\tilde{A}_s}}, \bar{v}_{\tilde{A}_s}: X \to$ Int ([0,1]) $(x \in X \to \bar{v}_{\tilde{A}_s}(x), \tilde{v}_s(x) \subseteq$

[0, 1]) and $\underline{v_{\tilde{A}_s}}, \overline{\tilde{A}_{ss}} : X \to$ **Int([0, 1])** $(x \in X \to \underline{v_{\tilde{A}_s}}(x), \overline{v_{\tilde{A}_s}}(x) \subseteq$

[0, 1]) refer to the MG and NMG of element x ∈ X and set $\tilde{A}_s$, respectively, and for every

$$x \in X, 0 \leq sup\left\{\left(\underline{\mu_{\tilde{A}_s}}(x)\right)^2\right\} + sup\left\{\left(\underline{v_{\tilde{A}_s}}(x)\right)^2\right\} \leq 1 \text{ and } 0 \leq sup\left\{\left(\overline{\mu_{\tilde{A}_s}}(x)\right)^2\right\} + sup\left\{\left(\overline{v_{\tilde{A}_s}}(x)\right)^2\right\} \leq 1.$$

In addition, for each $x \in X, \underline{\mu_{\tilde{A}_s}}(x), \overline{\mu_{\tilde{A}_s}}(x)$ and $\underline{v_{\tilde{A}_s}}(x), \overline{v_{\tilde{A}_s}}(x)$ are closed intervals with the lower and upper bounds represented by:

$$\underline{\mu^l_{\tilde{A}_s}}(x), \underline{\mu^u_{\tilde{A}_s}}(x), \overline{\mu^l_{\tilde{A}_s}}(x), \overline{\mu^u_{\tilde{A}_s}}(x), \underline{v^l_{\tilde{A}_s}}(x), \underline{v^u_{\tilde{A}_s}}(x), \overline{v^l_{\tilde{A}_s}}(x), \overline{v^u_{\tilde{A}_s}}(x),$$

respectively. Therefore, A˜ s can be stated in the following ways:

$$\tilde{A}_s = \left\{ < x, \left[\left[\underline{\mu^l_{\tilde{A}_s}}(x), \underline{\mu^u_{\tilde{A}_s}}(x)\right], \left[\underline{v^l_{\tilde{A}_s}}(x), \underline{v^u_{\tilde{A}_s}}(x)\right]\right], \left[\left[\overline{\mu^l_{\tilde{A}_s}}(x), \overline{\mu^u_{\tilde{A}_s}}(x)\right], \left[\overline{v^l_{\tilde{A}_s}}(x), \overline{v^u_{\tilde{A}_s}}(x)\right]\right] > \mid x \in X \right\}. \quad (80)$$

The lowest IVPFNs are used to figure out the lower space of the initial set of IVPFRNs, while the mean of the rest of the IVPFNs for the same attribute is used to approximation the upper space. This technique is carried out for every IVPFN and attribute. Once the biggest order (the final IVPFNs) arrives, the upper approximate is chosen as the IVPFN itself, whereas the lowest value is chosen as the arithmetic mean of the remainder values utilizing the interval-valued Pythagorean weighted arithmetic mean (IVPWAM), as stated in (79).

$$\text{IVPWAM}_w\left(\tilde{A}_{S1}, \ldots, \tilde{A}_{Sn}\right) = w_1\tilde{A}_{S1} + w_1\tilde{A}_{S1} + \cdots + w_n\tilde{A}_{Sn}$$
$$= \left\{\left[\sqrt{\left(1 - \prod_{i=1}^n \left(1 - \left(\mu^l_{\tilde{A}_{Si}}\right)^2\right)^{1/n}\right)} \sqrt{\left(1 - \prod_{i=1}^n \left(1 - \left(\mu^u_{\tilde{A}_{Si}}\right)^2\right)^{1/n}\right)}\right], \left[\prod_{i=1}^n v^{1/\prod_{i=1}^n}_{\tilde{A}^{l/n}_{Si}} v^u_{\tilde{A}^{u1/n}_{Si}}\right]\right\} \quad (81)$$
$$w = (w_1, w_2, \ldots, w_n); w_i \in [0,1]$$
$$\sum_{i=1}^n w_i = 1$$

w_i= 1/n

-The final fuzzy weight of the sustainable performance attributes is determined by adding the IVPFRNs within the IVPFRS–EDM from the three experts utilizing (82), which introduce the interval valued Pythagorean fuzzy rough weighted aggregation (IVPFRWA) operator, where $w_i = 1/n$.

$$\text{IVPFRWA}\left(g(b_1), g(b_1), \ldots, g(b_n)\right) = \left\{\oplus_{i=1}^n w_i \underline{g}(b_i), \oplus_{i=1}^n w_i \bar{g}(b_i)\right\}$$
$$\left\{\begin{array}{l}\left\{\left[\sqrt{\left(1 - \prod_{i=1}^n \left(1 - \left(\underline{\mu}^l_{\tilde{A}_{Si}}\right)^2\right)^{\frac{1}{n}}\right)}, \left(\sqrt{1 - \prod_{i=1}^n \left(1 - \left(\underline{\mu}^u_{\tilde{A}_{Si}}\right)^2\right)^{\frac{1}{n}}}\right)\right], \left[\prod_{i=1}^n \left(\underline{v}^l_{\tilde{A}_{Si}}\right)^{\frac{1}{n}}, \prod_{i=1}^n \left(\underline{v}^u \tilde{A}_{Si}\right)^{\frac{1}{n}},\right]\right\} \\ \left\{\left[\sqrt{\left(1 - \prod_{i=1}^n \left(1 - \left(\bar{\mu}^l_{\tilde{A}_{Si}}\right)^2\right)^{1/n}\right)}, \sqrt{\left(1 - \prod_{i=1}^n \left(1 - \left(\bar{\mu}^u_{\tilde{A}_{Si}}\right)^2\right)^{1/n}\right)}\right], \left[\prod_{i=1}^n \left(\bar{v}^l_{\tilde{A}_{Si}}\right)^{1/n}, \prod_{i=1}^n \left(\bar{v}^u \tilde{A}_{Si}\right)^{1/n}\right]\right\}\end{array}\right. \quad (82)$$

- The final crisp weight of the performance attributes is calculated using the IVPFRS scoring function, as indicated in (83).

$$\text{Def}(\tilde{A}_s) = \frac{1}{4}(2 + \underline{s} + \bar{s}) \quad (83)$$

Where

$$\underline{s} = \frac{1}{2}\left(\left(\underline{\mu}^l_{\tilde{A}_{Si}}\right)^2 + \left(\underline{\mu}^u_{\tilde{A}_{Si}}\right)^2 - \left(\underline{v}^l_{\tilde{A}_s}\right)^2 - \left(\underline{v}^u\tilde{A}_s\right)^2\right),$$

$$\bar{s} = \frac{1}{2}\left(\left(\bar{\mu}^{l}_{\tilde{A}_{Si}}\right)^2 + \left(\bar{\mu}^{u}_{\tilde{A}_{Si}}\right)^2 - \left(\bar{v}^{l}_{\tilde{A}_s}\right)^2 - \left(\bar{v}^{u}_{\tilde{A}_s}\right)^2\right).$$

- The qualities' cumulative weights of the attributes have been assigned to one. If that requirement is not fulfilled, the weights are rescaled according to (84).

$$w_j = \frac{s(\aleph)}{\sum_{j=1}^{J} s(\aleph)} \qquad (84)$$

12- Neutrosophic cubic sets (NCS–FWZIC) method

[119] NCSs enable experts to completely express their preferences in the decision-making process by allowing them to use a larger space. NCSs integrate professional opinions on parameters using a cubic value rather than a single or interval value. As a result, we are motivated for developing a fuzzy technique within the NCS context to overcome the mentioned FWZIC method problem.

The NCS-EDM is produced by replacing the numerical scale in EDM with the NCS fuzzy numbers (NCN) shown in Table 15.

Table 15: Linguistic, numeric, and fuzzy measurements [42]

Linguistic expressions	Numeric scale	NCN								
		T	T^L	T^U	I	I^L	I^U	F	F^L	F^U
Very Important (VI)	1	0.9	0.7	0.9	0.2	0.1	0.2	0.2	0.1	0.2
Important (Im)	2	0.8	0.6	0.8	0.3	0.2	0.3	0.4	0.2	0.4
Average (Av)	3	0.5	0.4	0.5	0.5	0.4	0.5	0.5	0.4	0.5
Low Important (LI)	4	0.4	0.3	0.4	0.6	0.5	0.6	0.7	0.5	0.7
Very Low Important (VLI)	5	0.2	0.1	0.2	0.8	0.6	0.8	0.9	0.7	0.9

Definition (1) [119] provides descriptions for NCS and NCN.

Definition (1). Suppose X is a universal set. A NCS S in X is written as follows:

$S = x, \langle T(x), I(x), F(x)\rangle, \lambda T(x), \lambda I(x), \lambda F(x) \mid x \in X$

where $\langle T(x), I(x), F(x)\rangle$ is an interval NCS; $T(x) = [T^L(x), T^U(x) \subseteq [0,1]$ is a truth-membership function in X $I(x) = I^L(x), I^U(x) \subseteq [0,1]$ is an indeterminacy function in X; $F(x) = F^L(x), F^U(x) \subseteq [0,1]$ is a falsity membership function in X; and $\lambda T(x), \lambda I(x), \lambda F(x)$ is a single-valued NCS and $\lambda T(x), \lambda I(x), \lambda F(x) \in [0,1]$ are grades of truth, indeterminacy, and falsity in X, respectively. A basic element $\{x, \langle T(x), I(x), F(x)\rangle, \langle \lambda T(x), \lambda I(x), \lambda F(x)\rangle\}$ in an NCS S is expressed by $s = (\langle T, \bar{I}, \bar{F}\rangle, \langle \lambda T, \lambda \bar{I}, \lambda F\rangle)$

Which is called an NCN, where T, I, F ⊆ [0, 1] and $\lambda T, \lambda I, \lambda F \in [0,1]$ satisfying $0 \le T^U + I^U + F^U \le 3$ and $0 \le \lambda T + \lambda I + \lambda F \le 3$

The NCN inside the three NCS-EDM of the three specialists is aggregated using the NCN weighted arithmetic averaging (NCNWAA) operator shown in Definition (2).

Definition (2). [120]. Let $s_j = (\langle TT_j, I_j, F_j\rangle, \langle \lambda T_j, \lambda I_j, \lambda F_j\rangle)$ for $j = 1,2,$ …, n be a set of NCNs and NCNWAA is expressed as follows:

$$NCNWAAA(s1, s2, \cdots, sn) = \left(\left[1 - \prod_{j=1}^{n}\left(1 - T_j^L\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - T_j^U\right)^{w_j}\right]\right.$$

$$\left.\left[\prod_{j=1}^{n}\left(I_j^L\right)^{w_j}, \prod_{j=1}^{n}\left(I_j^U\right)^{w_j}\right]\left[\prod_{i=1}^{n}\left(F_j^L\right)^{w_j}, \prod_{i=1}^{n}\left(F_j^U\right)^{w_j}\right], 1 - \prod_{j=1}^{n}\left(1 - \lambda_{Tj}\right)^{w_j}, \prod_{j=1}^{n}\left(\lambda_{Ij}\right)^{w_j}, \prod_{j=1}^{n}\left(\lambda_{Fj}\right)^{w_j}\right) \qquad (85)$$

where $w = (w_1, w_2 \dots, w_n)$; $w_i \in [0,1]$, $\sum_{i=1}^{n} w_i = 1$, and $w_i = 1/n$

Using the NCS scoring function described in Definition (3), the aggregated NCNs are defuzzied and turned into crisp numbers.

Definition (3)[121]. Let $s = (\langle T, I, F\rangle, \langle \lambda T, \lambda I, \lambda F\rangle)$, be any NCN, and then, its score function may be stated as follows:

$$P(s) = [(4 + T^L - I^L - F^L + T^U - I^U - F^U)/6 + (2 + \lambda T - \lambda I - \lambda F)/3]/2 \qquad (86)$$

where P(s) ∈ [0, 1].

Finally, if the sum of these values is less than one, the resultant weight values are rescaled using (87).

$$w_j = \frac{S(\tilde{\alpha})}{\sum_{j=1}^{J} S(\tilde{\alpha})} \quad (87)$$

Where $S(\tilde{\alpha})$ represents the weight of each attribute.

13- q-rung orthopair probabilistic hesitant fuzzy set q-ROPHFS–FWZIC method

The q-ROFSs are the most flexible and appropriate FS for managing vagueness and uncertainty, due to their ability to handle a greater range of fuzzy information [122].

To address the issue of hesitation, proposed the idea of hesitant FS. [56] made an extensive review of hesitant FS and came at the following conclusions: (i) Because it is a more flexible and generic preference structure, hesitant FS can reduce uncertainty; (ii) hesitant FS helps the preference elicitation of DMs; (iii) hesitant FS gradually exposes the significant loss of information; as well (iv) the chance of occurring for every element is ignored. To add probabilities to the hesitant FS, [123] created probabilistic hesitant FS. Probabilistic hesitant FS not just accommodates multiple viewpoints but also assigns a probability of occurrence to each point of view thus, increasing the information's reliability [113]. Given q-ROFSs' dominance over older FSs. [124] presented the idea and operating rules of q-rung orthopair hesitant FS (q-ROHFFS). [125] enhanced the q-ROHFS by incorporating probability and introduced the q-rung orthopair probabilistic hesitant FS (q-ROPHFS).

By expanding FWZIC to the q-ROPHFS environment, problems of ambiguity and uncertainty, as well as the hesitancy of specialists, can be correctly handled. The preferences of the DMs on the attributes are represented by five linguistic scales, as shown in Table 16, and were obtained using the designed form.

Table 16: Linguistic and numerical scales and their corresponding q-ROPHFNs [40]

Linguistic expressions For q-ROPHFS-FWZIC	Numarical scale	q-ROPHFNs							
		μ_1	p_{μ_1}	μ_2	p_{μ_2}	v_1	p_{v_1}	v_2	p_{v_2}
Very important (VI)	1	0.9	0.4	0.95	0.6	0.15	0.2	0.1	0.8
Important (I)	2	0.7	0.5	0.75	0.5	0.35	0.3	0.3	0.7
Average (Av)	3	0.5	0.3	0.55	0.7	0.55	0.5	0.5	0.5
Low important (LI)	4	0.4	0.8	0.5	0.2	0.55	0.4	0.6	0.6
Very low important (VLI)	5	0.2	0.7	0.25	0.3	0.8	0.35	0.9	0.65

The following are the definitions of q-ROPHFS and q-ROPHFNs [125]:

Definition 1: Let M and N be the sets of q-rung membership and non-membership functions in a universal set labeled bysed by:connected with M and N can be expres Q_PROPHFS -. The qΩ

$Q_p = \{\langle x, h(x), g(x) \mid\rangle \mid x \in \Omega\}$

where $h(x) = \cup_{\mu\in\mathcal{M}} \{\mu(x) \mid p_\mu(x)\}$(resp. $g(x) = \cup_{v\in\mathcal{N}} \{v(x) \mid p_v(x)\}$) is a collection of pairs in [0, 1] × [0, 1]. The first item in each pair indicates a potential qth rung membership (or nonmembership) degree, signified by a positive (or negative) degree. The second item is the probability that the degree of membership (or non-membership) will occur.

The following properties are satisfied: $0 \le p_\mu, p_v \le 1$ for all $\mu \in \mathcal{M}$ and $v \in \mathcal{N}$ with $\oplus_{\mu\in\mathcal{M}} p_\mu \le 1$ and $\oplus_{v\in\mathcal{N}} p_v \le 1$, and $\forall x \in \Omega$ we have $(max\{\mu(x): \mu \in \mathcal{M}\})^q + (min\{v(x): v \in \mathcal{N}\})^q \le 1$and $(min\{\mu(x): \mu \in \mathcal{M}\})^q + (max\{v(x): v \in \mathcal{N}\})^q \le 1$. Mean

While, q-ROPHFN is represented by the tuple

$Q = \langle h_Q, g_Q \rangle$ or $Q = \left\langle \{\mu_Q \mid p_{\mu_Q}\}_{\mu_Q\in\mathcal{M}}, \{v_Q \mid p_{v_Q}\}_{v_Q\in\mathcal{N}} \right\rangle$ for ease of presentation.

Initially, the q-ROPHF arithmetic mean (q-ROPHFAM) operator (88) [61] is employed to aggregating the q-ROPHFNs for every attribute throughout the three DMs' q-ROPHFS-EDMs.

Let $Q_i = \left\langle \{\mu_{Q_i} \mid p_{\mu_{Q_i}}\}_{\mu_{Q_i}\in\mathcal{M}_i}, \{v_{Q_i} \mid p_{v_{Q_i}}\}_{v_{Q_i}\in\mathcal{N}_i} \right\rangle$ for $i = 1, \dots, r$ be q-ROPHFSNs, q-ROPHFAM (Q1,..., Qr) defines the q-ROPHFAM of each of these fuzzy numbers.

$$= \left(\cup_{\mu_{Q_i}\in\mathcal{M}_i, v_{Q_i}\in\mathcal{N}_i} \left\langle \left\{ \sqrt[q]{1 - \prod_{i=1}^{r} \left(1 - (\mu_{Q_i})^q\right)^{\frac{1}{r}}} \mid \prod_{i=1}^{r} p_{\mu_{Q_i}} \right\} \left\{ \prod_{i=1}^{r} (v_{Q_i})^{\frac{1}{r}} \mid \prod_{i=1}^{r} p_{v_{Q_i}} \right\} \right\rangle \right) \quad (88)$$

The obtained weights are then fuzzy values that have to be defuzzified via the q-ROPHFS scoring function and transformed to crisp weights using (89).

$$s(Q) = \frac{1}{|\mathcal{M}|}\sum_{\mu\in\mathcal{M}}\left(\mu\cdot p_\mu\right)^q - \frac{1}{|\mathcal{N}|}\sum_{\nu\in\mathcal{N}}\left(\nu\cdot p_\nu\right)^q \quad (89)$$

Where |.| denotes the cardinality of a set.

Finally, the total of the attribute weights must equal one. If this requirement is not fulfilled, (90) is used to rescale the weights for J attributes.

$$w_j = \frac{s(Q_j)}{\sum_{j=1}^{J} s(Q_j)}. \quad (90)$$

Hence, $w_j^* = \frac{s\left(q-\text{ROPHFAM}(Q_{1j},\ldots,Q_{Kj})\right)}{\sum_{j=1}^{J} q-\text{ROPHFAM}(Q_{1j},\ldots,Q_{Kj})}$, where Q_{ij} is the q-ROPHFN

of D_{ij}, represents the q-ROPHFS–FWZIC weight of the jth attribute, for $1 \le j \le J$.

14- Spherical Fuzzy Rough-Weighted Zero-Inconsistency (SFR-WZIC)

The rough set (RS) theory, which was developed in 1982, inspire the spherical fuzzy rough environment [126] [127]. These mathematical techniques have been commended for being capable to deal with ambiguous [128], inconsistent, and insufficient data and information [71]. To handle information with continuing attributes and detect inconsistencies in the data [129], fuzzy rough set (FRS) can be coupled with RSs.

Spherical fuzzy rough set (SFRSs) is a more robust FRS fuzzy environment [107] that has been proved to overcome not just the drawbacks of traditional fuzzy sets, but also intuitionistic fuzzy sets (IFS), Pythagorean fuzzy sets, and rough sets. As a result, SFRSs integration is required, particularly with FDOSM and FWZIC [129].

Table 17: The numerical scales and the equivalent SFSs values [130]

Linguistic terms	Numerical scoring scale	SFSs		
		μ	v	π
Very Low Importance (VLI)	1	0.15	0.85	0.1
Low Importance (LI)	2	0.25	0.75	0.2
Medium Importance (MI)	3	0.55	0.5	0.25
Important (I)	4	0.75	0.25	0.2
Very Important (VI)	5	0.85	0.15	0.1

The SFS membership is used to solve the uncertainty and imprecision problems demonstrated by the crisp value of specialist preferences for each associated criterion. The following (91) and (92) provide and describe the concepts of spherical fuzzy set membership and non-membership in [129]: SFS $\tilde{A}_s$ of the discourse universe U is provided by:

$$\tilde{A}_s = \{\langle u, \mu_{\tilde{A}_s}(u), v_{\tilde{A}_s}(u), \pi_{\tilde{A}_s}(u) | u \in U\} \quad (91)$$

Where

$\mu_{\tilde{A}_s}(u): U \to [0,1], v_{\tilde{A}_s}(u): U \to [0,1], \pi_{\tilde{A}_s}(u): U \to [0,1]$

And

$$0 \le \mu_{\tilde{A}_s}^2(u) + v_{\tilde{A}_s}^2(u) + \pi_{\tilde{A}_s}^2(u) \le 1 \forall_u \in U \quad (92)$$

For each u, $\mu_{\tilde{A}_s}(u)$, $v_{\tilde{A}_s}(u)$, and $\pi_{\tilde{A}_s}(u)$ represent the degrees of membership, non-membership and hesitancy of u to $\tilde{A}_s$, respectively. $\chi_{\tilde{A}_s} = \left(1 - \mu_{\tilde{A}_s}^2(u) - v_{\tilde{A}_s}^2(u) - \pi_{\tilde{A}_s}^2(u)\right)^{1/2}$ represents the refusal degree.

Experts Preference Transformation: The crisp values of the EDM are changed to the SFS-EDM in this stage. Table 17 shows the relationship of each crisp number to its corresponding SFS number. The fuzzy set with the lowest membership represents the smallest level, while the set with the greatest membership represents the largest level. Despite its many advantages of the SFS fuzzy set it is cannot deal with information that is incomplete, due to this the SFS to SFRS transition is require. To apply this change in a fuzzy environment, first compute the score value of the SFS-EDM dataset using (93), and then determine the closeness of upper and lower regions.

$$\text{Def}(\tilde{A}_s) = \left(\mu_{\tilde{A}_s} - \pi_{\tilde{A}_s}\right)^2 - \left(v_{\tilde{A}_s} - \pi_{\tilde{A}_s}\right)^2 \quad (93)$$

The arithmetic mean (SWAM), (94).

$$\text{SWAM}_w(\tilde{A}_{S1},\ldots,\tilde{A}_{Sn}) = w_1\tilde{A}_{S1} + w_1\tilde{A}_{S1} + \cdots + w_n\tilde{A}_{Sn}$$

$$= \left\{\left[1 - \Pi_{i=1}^{n}\left(1-\mu_{\tilde{A}_{Si}}^{2}\right)^{1/n}\right]^{1/2}, \Pi_{i=1}^{n} v_{\tilde{A}_{Si}}^{w_i}, \left[\Pi_{i=1}^{n}\left(1-\mu_{\tilde{A}_{Si}}^{2}\right)^{1/n} - \Pi_{i=1}^{n}\left(1-\mu_{\tilde{A}_{Si}}^{2}-\pi_{\tilde{A}_{Si}}^{2}\right)^{1/n}\right]^{1/2}\right\} \quad (94)$$

The experts' opinion is used for calculating the final SFRS fuzzy weight using SFRWA, as shown in (95).
Let w_i = 1/n

$$\text{SFRWA}(g(b_1), g(b_1), \ldots, g(b_n)) = \left\{\oplus_{i=1}^{n} w_i \underline{g}(b_i), \oplus_{i=1}^{n} w_i \bar{g}(b_i)\right\}$$

$$= \left\{\sqrt{1-\Pi_{i=1}^{n}\left(1-\underline{\mu}_i^{2}\right)^{1/n}}, \Pi_{i=1}^{n} \underline{v}_i^{1/n}, \Pi_{i=1}^{n} \underline{\pi}_i^{1/n}\right\}, \left\{\sqrt{1-\Pi_{i=1}^{n}\left(1-\bar{\mu}_i^{2}\right)^{1/n}}, \Pi_{i=1}^{n} \bar{v}_i^{1/n}, \Pi_{i=1}^{n} \bar{\pi}_i^{1/n}\right\} \quad (95)$$

Defuzzification computation is used to determine the final weight for each criterion. Using the defuzzied (Crisp) process description for SFRSs outlined in (96) [129].

$$\text{Def}(\tilde{A}_s) = \frac{1}{6}\left(4 + \underline{\mu}_{\tilde{A}_s} + \bar{\mu}_{\tilde{A}_s} - \underline{v}_{\tilde{A}_s} - \bar{v}_{\tilde{A}_s} - \underline{\pi}_{\tilde{A}_s} - \bar{\pi}_{\tilde{A}_s}\right) \quad (96)$$

The rescale procedure is then used to create a distribution of weight values out of one utilizing (97).

$$Wi=Sii=1KSi \quad (97)$$

15- dual hesitant fuzzy weighted zero inconsistency (DH-FWZIC)

In [131], has proposed dual hesitant fuzzy sets (DHFS), which combine the benefits of intuitionist and hesitant principles. HFSs allow the membership grade to be combined with more than two alternative values, allowing DMs to convey their hesitancy [131]. DHFS, like IFSs, offers membership (GM) and nonmembership (GNM) grades. These grades, however, aren't expressed by just one number but by a several of predetermined numbers. This feature accurately and flexibly represents real-world challenges.
The experts then used a five-point Likert scale to determine the importance/significance degree of each criterion. The numerical equivalents of the linguistic terms are shown in Table 18.

Table 18: Linguistic Terms with Corresponding Numerical Scoring Scale and DHFNS [132]

Linguistic scoring scale	Numerical scoring scale	DHFNs	
		$\tilde{h}$	$\tilde{g}$
Very Low Importance (VLI)	1	0.10	0.85
Low Importance (LI)	2	0.23	0.65
Medium Importance (MI)	3	0.50	0.50
High Importance (HI)	4	0.75	0.20
Very High Importance (VHI)	5	0.90	0.05

DHFS membership function application
DHFSs [133] is defined as extensions to HFSs. A DHFS D in X is represented by given a fixed set U as $\tilde{D} = \{x, \tilde{h}_{\tilde{D}}(x), \tilde{g}_{\tilde{D}}(x) \mid x \in U\}$, in which $\tilde{h}_{\tilde{D}}(x)$ and $\tilde{g}_{\tilde{D}}(x)$ are in the range [0], [1] denoting the degrees of membership and non member ship of the element x ∈ U to set D, respectively, under the conditions: $0 \le \gamma, \eta \le 0 \le \gamma^{+} + \eta^{+} \le 1$ for all $x \in U \gamma \in \tilde{h}_{\tilde{D}}(x), \eta \in \tilde{g}_{\tilde{D}}(x), \gamma^{+} \in \tilde{h}_{\tilde{D}}^{+}(x) = \cup_{\gamma \in \tilde{h}_{\tilde{D}}(x)} \; max\{\gamma\}, \eta^{+} \in \tilde{g}_{\tilde{D}}(x) = \cup_{\eta \in \tilde{g}_{\tilde{D}}(x)} \; \text{Max}\{\eta\}$

The DHFS arithmetic operations listed below are taken from [134]. (98) is used in the DHFS aggregation operation (DHFA).

$$\text{DHFA}(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n) = \oplus_{j=1}^{n}(\tilde{d}_j)$$

$$= \bigcup_{\tilde{\gamma}_j \in \tilde{h}_j, \tilde{\eta}_j \in \tilde{g}_j} \left\{\left\{1 - \prod_{j=1}^{n}(1-\tilde{\gamma})\right\}, \left\{\prod_{j=1}^{n}(\tilde{\eta})\right\}\right\} \quad (98)$$

For the DHFS division operation, (99) is used

$d_1 \varnothing d_2 =$

$$\begin{cases} \bigcup_{\tilde{\gamma}_1\in\tilde{h}_1,\tilde{\gamma}_2\in\tilde{h}_2,\tilde{\eta}_1\in\tilde{g}_1,\tilde{\eta}_2\in\tilde{g}_2} \left\langle \frac{\tilde{\gamma}_1}{\tilde{\gamma}_2}, \frac{\tilde{\eta}_1-\tilde{\eta}_2}{1-\tilde{\eta}_2} \right\rangle, & \text{otherwise}.\ 0 \le \frac{\tilde{\gamma}_1}{\tilde{\gamma}_2} \le \frac{1-\tilde{\eta}_1}{1-\tilde{\eta}_2} \le 1 \\ \langle 1,0\rangle \end{cases} \quad (99)$$

Using (100), the DHFNs can then be defuzzified and turned to crisp values:

$$S(d_j) = \frac{1}{\#\tilde{h}} \sum_{\tilde{\gamma}_j\in\tilde{h}_j} \tilde{\gamma}_j - \frac{1}{\#\tilde{g}} \sum_{\tilde{\eta}_j\in\tilde{g}_j} \tilde{\eta}_j \quad (100)$$

16- 2-tuple linguistic Pythagorean fuzzy-weighted zero-inconsistency (2 TLP-FWZIC)
It is still necessary for developing an extension that combines the entire benefits of the 2-tuple linguistic model (2 TLM) with fuzzy set applications. The 2 TLM is a model based on mathematics that represents linguistic terms and ideas using two numerical values [135]. This method has various advantages, one of which is being able to record imprecision and uncertainty, which can result in more precise analysis and decision-making. The 2 TLM is also very adaptable, with the ability to express a wide range of linguistic concepts and phrases, such as fuzzy sets and rules of linguistics. In addition, by allowing for more accuracy and realistic representations of real-world cases, it can assist overcome the limits of standard fuzzy sets [136]. Overall, the 2-tuple linguistic model is a flexible instrument that may be used in a number of fuzzy MCDM processes. The 2TLPFS is a new, robust fuzzy set that combines the advantages of the 2 TLMs with Pythagorean fuzzy sets (PFSs) to deal with challenging MCDM issues.
The experts then used a five-point Likert scale to determine the importance/significance degree of each criterion. The numerical equivalents of the linguistic terms are shown in Table 19.

Table 19: Linguistic Variables for Evaluating the Criteria [38]

Linguistic Variable	Numerical-based Score	2TLPFSs
Very Important (VI)	5	[(l_5,0),(l_1,0)]
Important (Im)	4	[(l_4,0), l_2,0)]
Average (Av)	3	[(l_3,0),(l_3,0)]
Low Important (LI)	2	[(l_2,0),(l_4,0)]
Very Low Important (VLI)	1	[(l_1,0),(l_5,0)]

The 2TLPFSs, as well as their fundamental principles and operations, will be explained below.
Definition 1. A linguistic term set (LTS) denoted as $\boldsymbol{L} = \{l_0, l_1, \dots, l_K\}$ is an odd cardinality set, where K is an even integer. Each term in the set represents a potential linguistic term for a linguistic variable, e.g. e.g. $\boldsymbol{L} = \{l_0 = \text{ bad}, l_1 = \text{ fair}, l_2 = \text{ good}\}$
A symbolic method is used to aggregate the indices of various labels in L , the result is $\beta \in [0, K]$ and $\beta \notin \{0,1,\dots,K\}$.
Let the integer value $k = \text{round}(\beta)$, and $k \in \{0,1,\dots,K\}$, then the value κ = β − k that satisfies κ ∈ [− 0.5, 0.5) is called a symbolic translating. From the preceding, a symbolic translation is described as follows.

Definition 2. A symbolic translation (κ) of an LT is the " variations in information" between the outcome of the symbolic aggregate (β) and an index of the most similar linguistic word in L to β, with a value in the semi-closed range [0.5, 0.5].

Definition 3. The linguistic information is described by the 2-tuple $(l_k, \kappa), l_k \in \boldsymbol{L}$ and $\kappa \in [-0.5, 0.5)$, where l_k represents the information's linguistic label center, and κ shows the numerical value of the conversion to the closest index (k) from the actual outcome (β) in an LTS (L).

Definition 4. For an LTS $\boldsymbol{L} = \{l_0, l_1, \dots, l_K\}$ the 2-tuple showing the information similar to the outcome of the symbolic aggregate β ∈ [0, K] is produced Utilizing the mapping,
$\Delta: [0, K] \to \boldsymbol{L} \times [-0.5, 0.5)$
$\Delta(\beta) = (l_k, \kappa)$, with $\begin{cases} l_k, k = \text{round}(\beta), \\ \kappa = \beta - k, \kappa \in [-0.5, 0.5). \end{cases}$

Definition 5. Consider an LTS $\boldsymbol{L} = \{l_0, l_1, \dots, l_K\}$ and a 2-tuple (l_k, κ), there exists an inverse function Δ^{-1} that returns the 2-tuple to its actual value β ∈ [0,K]:
$\Delta^{-1}: \boldsymbol{L} \times [-0.5, 0.5) \to [0, K]$
$\Delta^{-1}(l_k, \kappa) = \kappa + k = \beta$

Definition 6. The following rules are used to compare the 2 TL information $A = (l_{k1},\kappa_1)$ and $B = (l_{k2},\kappa_2)$:

• if k1 < k2, then A < B.
• if k1 = k2, then.

$A = B$, if $\kappa_1 = \kappa_2$,.
$A < B$, if $\kappa_1 < \kappa_2$,.
$A > B$, if $\kappa_1 > \kappa_2$

Definition 7. A PFS across universal set X is a collection of ordered pairings that have the form

$\tilde{P} = \{\langle x, (\Theta_{\tilde{P}}(x), \Phi_{\tilde{P}}(x))\rangle \mid x \in X\}$

where $\Theta_{\tilde{P}}(x): X \to [0,1]$ and $\Phi_{\tilde{P}}(x): X \to [0,1]$ Define the membership degree and non-membership degree of an element x in $\tilde{P}$, while keeping the condition

$0 \le (\Theta_{\tilde{P}}(x))^2 + (\Phi_{\tilde{P}}(x))^2 \le 1$, for all $x \in X$

The degree of hesitation of x to ˜F, denoted as $\pi_{\tilde{P}}(x)$, is relevant to these two degrees by

$\pi_{\tilde{P}}(x) = \sqrt[2]{1 - (\Theta_{\tilde{P}}(x))^2 - (\Phi_{\tilde{P}}(x))^2}$

Definition 8. A 2TLPFS has the form

$\tilde{P} = \{\langle x, (l_u(x), \mu(x)), (l_v(x), v(x))\rangle \mid x \in X\}$

where $0 \le \Delta^{-1}(l_u(x), \mu(x)) \le \mathrm{K}, 0 \le \Delta^{-1}(l_v(x), v(x)) \le \mathrm{K}$,

Where the 2-tuple (l_u, μ) is an abbreviation for the linguistic membership grade, the 2-tuple (l_v, v) stands for the non-membership grade, and $l_u, l_v \in \boldsymbol{L} = \{l_0, l_1, \dots, l_{\mathrm{K}}\}$ and $\mu, v \in [-0.5, 0.5)$. The set satisfies the condition $0 \le (\Delta^{-1}(l_u(x), \mu(x)))^2 + (\Delta^{-1}(l_v(x), v(x)))^2 \le \mathrm{K}^2$

For the 2TLPFSs $\{\tilde{P}_1, \tilde{P}_2, \dots, \tilde{P}_n\}, \tilde{P}_i = [(l_{u_i}, \mu_i), (l_{v_i}, v_i)]$, the score function, the multiplication by a scalar, and the aggregation operators that will be employed in the execution of the 2 TLP-FWZIC and 2 TLPFMABAC is given as follows:

The score function is calculated by (101):

$$S(\tilde{P}) = \Delta\left\{\mathrm{K}\left(\left(\frac{\Delta^{-1}(l_u,\mu)}{\mathrm{K}}\right)^2 - \left(\frac{\Delta^{-1}(l_v,v)}{\mathrm{K}}\right)^2\right)\right\}, \Delta^{-1}(S(\tilde{P})) \in [0, \mathrm{K}]. \qquad (101)$$

Multiplication of a 2TLPFS by a constant ω > 0

$$\omega \odot \tilde{P} = \left\{\Delta\left(\mathrm{K}\sqrt[2]{1 - \left(1 - \left(\frac{\Delta^{-1}(l_u,\mu)}{\mathrm{K}}\right)^2\right)^{\omega}}\right), \Delta\left(\mathrm{K}\left(\frac{\Delta^{-1}(l_v,v)}{\mathrm{K}}\right)^{\omega}\right)\right\} \qquad (102)$$

Given a weighting vector $w = [w1, w2, \dots, wn]$ whose elements $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, the aggregation operators are defined as given in (103) and (104).

The 2 TLPF weighting averaging operator

$$2\,\mathrm{TLPFSWA}(\tilde{P}_1, \tilde{P}_2, \dots, \tilde{P}_n) = \left\{\Delta\left(\mathrm{K}\sqrt[2]{1 - \prod_{i=1}^{n}\left(1 - \left(\frac{\Delta^{-1}(l_{u_i}, \mu_i)}{\mathrm{K}}\right)^2\right)^{\omega_i}}\right), \Delta\left(\mathrm{K}\prod_{i=1}^{n}\left(\frac{\Delta^{-1}(l_{v_i}, v_i)}{\mathrm{K}}\right)^{\omega_i}\right)\right\}. \qquad (103)$$

The 2 TLPF weighting geometric operator:

$$2\,\mathrm{TLPFSWG}(\tilde{P}_1, \tilde{P}_2, \dots, \tilde{P}_n) = \left\{\Delta\left(\mathrm{K}\prod_{i=1}^{n}\left(\frac{\Delta^{-1}(l_{u_i}, \mu_i)}{\mathrm{K}}\right)^{\omega_i}\right), \Delta\left(\mathrm{K}\sqrt[2]{1 - \prod_{i=1}^{n}\left(1 - \left(\frac{\Delta^{-1}(l_{v_i}, v_i)}{\mathrm{K}}\right)^2\right)^{\omega_i}}\right)\right\} \qquad (104)$$

The distance between two 2TLPFSs $\tilde{P}_1 = [(l_{u_1}, \mu_1), (l_{v_1}, v_1)]$ and $\tilde{P}_2 = [(l_{u_2}, \mu_2), (l_{v_2}, v_2)]$ is measured by the following distance equations:

The Hamming distance:

$$d_H(\tilde{P}_1, \tilde{P}_2) = \left\{\frac{1}{2K}\left(\left|\Delta^{-1}(l_{u_1}, \mu_1) - \Delta^{-1}(l_{u_2}, \mu_2)\right| + \left|\Delta^{-1}(l_{v_1}, v_1) - \Delta^{-1}(l_{v_2}, v_2)\right|\right)\right\}. \quad (105)$$

The Euclidean distance:

$$d_E(\tilde{P}_1, \tilde{P}_2) = \left\{\frac{1}{2K}\left(\left|\left(\Delta^{-1}(l_{u_1}, \mu_1)\right)^2 - \left(\Delta^{-1}(l_{u_2}, \mu_2)\right)^2\right| + \left|\left(\Delta^{-1}(l_{v_1}, v_1)\right)^2 - \left(\Delta^{-1}(l_{v_2}, v_2)\right)^2\right|\right)\right\}. \quad (106)$$

The LTS $\boldsymbol{L} = \{l_0 l_1, l_2, l_3 l_4, l_5, l_6\}, K = 6$ will be used to indicate the linguistic term values with 2TLPFS.

- The 2 TLPF ratio of data is calculated using (102)

$$\frac{\text{IMC}(\widetilde{Ep/Cj})}{\sum_{j=1}^{n} IMC(Ep/Cpj)} \quad (107)$$

where $IMC(\widetilde{Ep/Cj})$ is the amount of significance given by the p^{th} expert to the j^{th} criterion expressed by a 2TLPFS, and $\sum_{j=1}^{n} IMC(Ep/Cpj)$ is the sum of the scores of the 2 TLPF degree of significance as determined by (101), of the p^{th} expert for the n criteria. (107) is performed using (102).

- The selection criterion weights are calculated in their 2 TLPF form. Applying Equation (103), the expert evaluations for the criteria are summed together to obtain the weights $(\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)^T$

$$\tilde{w}_j = 2TLPFSWA\left(\frac{\text{IMC(Expert } 1/C_j)}{\sum_{j=1}^{n}\text{IMC(Expert } 1/C_{1j})}, \frac{\text{IMC(Expert } 2/C_j)}{\sum_{j=1}^{n}\text{IMC(Expert } 2/C_{2j})}, \dots, \frac{\text{IMC(Expert } p/C_j)}{\sum_{j=1}^{n}\text{IMC(Expert } p/C_{nj})}\right), \omega_i = \frac{1}{p} \quad (108)$$

- Using (101), the scores of the weights $(\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)^T$, found in the prior sub-step, are computed to find the crisp weights $(\overline{w}_1, \overline{w}_2, \dots, \overline{w}_n)^T$.

The crisp weights are changed because the score of 2TLPFSs might be positive or negative. If all of the results are positive, go to the next sub-step. Otherwise, the updated weights are calculated as follows:

$$\overline{w}_j' = \overline{w}_j + \sum_{j=1}^{n} |\overline{w}_j|. \quad (109)$$

- The crisp or changed crisp weights $\overline{w}_j'$ are normalized to provide the final weights $(w_1, w_2, \dots, w_n)^T$ that meet the requirement.

17- rough Fermatean fuzzy sets (RFFSs) RF-FWZIC

rough set theory is a method of data mining used for detecting hidden patterns in data and computer granularization According to various research [137]. Rough set theory is used in many modern applications. Fermatean fuzzy sets (FFSs) are more trustworthy than 'intuitionistic fuzzy sets (IFS)' and PFS, which are defined as 'the total of the cube of grade of membership and grade of non-membership is restricted by 1', according to Reference [125]. As a result, FFSs are better and more powerful than other sets (such as IFSs and PFSs) because they are capable of handling imprecision and uncertainty. Reference [138] pointed out the benefit of rough set theory over other sets, particularly in the world of data analysis, because it does not require any additional or previous understanding, such as probability in statistics or essential probability assignment in Dempster-Shafer theory, or dgree of membership or potential value in fuzzy set theory. Several advantages of rough sets were highlighted in reference [138], including effective algorithms for identifying patterns inside data; finding the minimum sets of data (data reduction); assessing the importance of data; producing sets of decision principles from data; simple understanding; and easy interpretation of the results obtained.

The experts then assessed each criterion's degree of relevance and significance using a five-point Likert scale. Table 20 displays the linguistic term' numerical counterparts.

Table 20: Numerical Scoring Scale for Linguistic Terms and FFSNs [37]

Linguistic scale	Numerical scale	M	V
Very Important (VI)	1	0.85	0.2
Important (Im)	2	0.7	0.35
Average (Av)	3	0.55	0.5
Low Important (LI)	4	0.35	0.7
Very Low Important (VLI)	5	0.2	0.85

Definition (1). For the application of FFS

Definition (1) Let F be the universe of discourse. Let $\mu, v : F \to [0,1]$ be the membership degree (MD) and NMG. Then, for any $\hbar \in F$, the terms $\mu(\hbar)$ and $v(\hbar)$ refer to the MD and non-membership degree (NMD) [139]. The FFN representation can be offered in two ways (see 110):

$$N = ((\mu(\hbar), v(\hbar)))$$
$$= (\langle\mu(\hbar), v(\hbar)\rangle)$$
$$\text{where } \mu(\hbar), v(\hbar) \in [0,1] \quad 0 \le \mu(\hbar)^3 + v(\hbar)^3 \le 1 \quad (110)$$

Definition (2) [139] the score function of FFS is define as shown in (111)

$$\text{Score}(\alpha) = \left[(\mu_\gamma)^3 - (v_\gamma)^3 \right] \tag{111}$$

The scores for each criterion are arranged from lowest to highest. The lowest FFS value in the first set represents an estimate of the lower space. The upper space estimate is then determined as the mean of the remainder FFS values employing the same criterion as in Definition (3) (see 112). This technique is utilized for all FFS values in order to meet all requirements. When a high order (final FFS) is reached, the closest approximation is selected as the FFS itself, and the lowest value is found as the arithmetic mean of the remainder values, as defined in Definition (4).

Definition (3)

$$\text{IFAM}(\gamma) = \frac{1}{n}(\gamma_1 \oplus \gamma_2 \oplus \cdots \oplus \gamma_n)$$
$$= \left(\left[1 - \Pi_{j=1}^{n} \left(1 - \mu_{\gamma_j}^3\right)^{\frac{1}{n}} \right]^{1/3}, \Pi_{j=1}^{n} v_{\gamma_j}^{\frac{1}{n}} \right) \tag{112}$$

Definition (4) Let F be the universe of discourse. Let $\mu, v : F \to [0,1]$ be the MD and NMG. Then, for any $\hbar \in F$, the terminology $\mu(\hbar)$ and $v(\hbar)$refer to the MD and NMD. This is the representation of the FFRNs

(see 113):

$$N = \left(\left[\underline{\mu}(\hbar), \underline{v}(\hbar) \right], [\bar{\mu}(\hbar), \bar{v}(\hbar)] \right) \tag{113}$$
$$= \left(\left\langle \underline{\mu}(\hbar), \underline{v}(\hbar) \right\rangle, \langle \bar{\mu}(\hbar), \bar{v}(\hbar) \rangle \right)$$

where $\underline{\mu}(\hbar), \underline{v}(\hbar), \bar{\mu}(\hbar), \bar{v}(\hbar) \in [0,1]$

$0 \le \underline{\mu}(\hbar)^3 + \underline{v}(\hbar)^3 \le 1$

$0 \le \bar{\mu}(\hbar)^3 + \bar{v}(\hbar)^3 \le 1$

The final fuzzy weight for each criterion and subcriterion is produced by aggregating the FFRS values from every expert (see Definition 5).

Definition (5) The intuitionistic fuzzy rough arithmetic mean operator of dimension n is achieved (see 114):

$$\text{FFRA}(\gamma) = \frac{1}{n}(\gamma_1 \oplus \gamma_2 \oplus \cdots \oplus \gamma_n)$$
$$= \left(\begin{array}{l} \left\{ \left[1 - \Pi_{j=1}^{n} \left(1 - \underline{\mu}_{\gamma_j}^3\right)^{\frac{1}{n}} \right]^3, \Pi_{j=1}^{n} \underline{v}_{\gamma_j}^{\frac{1}{n}} \right\}, \\ \left\{ \left[1 - \Pi_{j=1}^{n} \left(1 - \bar{\mu}_{\gamma_j}^3\right)^{\frac{1}{n}} \right]^3, \Pi_{j=1}^{n} \bar{v}_{\gamma_j}^{\frac{1}{n}}, \right\} \end{array} \right) \tag{114}$$

Definition (6) The FFRSs score function is shown (see 115) as follows:

$$\text{Score}(\alpha) = \frac{\left[2 + \underline{\mu}_\gamma^3 + \bar{\mu}_\gamma^3 - \underline{v}_\gamma^3 - \bar{v}_\gamma^3 \right]}{4} \tag{115}$$

- To calculate the weight value for every criterion, the weight values of every criteria were put together in a procedure referred to as rescaling (see 116).

$$w_j = s_j / \Sigma_{j=1}^{J} s_j \tag{116}$$

where sj is the score of each criterion’s weight value.

18- Diophantine linear fuzzy sets (LDFSs) LDFS-FWZIC

In many real-world fields, intuitionistic fuzzy sets (IFSs), q-rung orthopair fuzzy sets (q-ROFSs), and Pythagorean fuzzy sets (PFSs) are employed and have numerous applications; however, they have struggling from issue regarding membership and nonmembership grades. As a result, the idea of LDFS was established, which provides decision makers with endless freedom in choosing scores [140].This tool has been shown to be extremely successful in conveying decision makers' evaluations (DM) in MCDM; hence, it provides a straightforward technique for decision experts (DEs) to deal with imprecise and uncertain information in an extensive way [141]. Many researches have used the concept of LDFS. [142] proposed the concept of fuzzy linear Diophantine spherical groups (SLDFSs) with reference or controlling parameters. Using this LDFS, the informational ambiguity and imprecision of FWZIC may be solved. In contrast to many popular FSs, LDFS may provide the decision maker (DM) with endless freedom in modeling scores.

The significance of each attribute is assigned by the experts using the approved question and five linguistic terms of importance, as shown in Table 21.

Table 21: Importance scale for the LDFS-FWZIC method [52]

Linguistic terms	Numeric scale	LDFS	
		$A_d(\varsigma), S_d(\varsigma)$	$(,\beta)$
Very Low Important (VLI)	1	(0.1,0.8)	(0.1,0.8)
Low Important (LI)	2	(0.25,0.6)	(0.25, 0.6)
Average (Av)	3	(0.5,0.4)	(0.5, 0.4)
Important (Im)	4	(0.75,0.2)	(0.75, 0.2)
Very Important (VI)	5	(0.9,0.05)	, 0.5)

Definition 1 describes the membership and reference criteria of LDFS.

Definition 1 [140]: Assume Q is a nonempty reference set. An LDFS F on Q is an object of the form

$$F_d = \left\{\left(\varsigma, \left(A_d(\varsigma), S_d(\varsigma)\right), (\alpha, \beta)\right) : \varsigma \in Q\right\}$$

Where $A_d(\varsigma), S_d(\varsigma)$ and $\alpha, \beta\,[0,1]$ are membership, nonmembership and reference parameters, respectively. These grades satisfy the following condition:

$$0 \leq \alpha A_d(\varsigma) + \beta S_d(\varsigma) \leq 1 \forall \varsigma \in Q \text{ with } 0 \leq \alpha + \beta \leq 1$$

These reference parameters can help with system definition or classification. They extend the range of grades in LDFS and eliminate restrictions on them. The part of hesitation can be rated as follows:

$$E\pi_d = 1 - (\alpha A_d(\varsigma) + \beta S_d(\varsigma) \quad (117)$$

where E is the degree of indeterminacy-related reference parameter. Consequently, M = $(A_d, S_d), (\alpha, \beta)$ is referred to as a linear Diophantine fuzzy number (LDFN) with (LDFN)" having the properties $0 \leq \alpha A_d(\varsigma) + \beta S_d(\varsigma) \leq 1$ and $0 \leq \alpha + \beta \leq 1$.v The LDFS-EDM used to construct the weight values for every evaluation attribute. Using the LDFN operator from (117) [140], the LDFS-FNs of all five experts inside LDFS-EDM for every assessed attribute are aggregated into the following:

$$PFA(\gamma) = \frac{1}{n}(\gamma_1 \oplus \gamma_2 \oplus \cdots \oplus \gamma_n) = \left(\left[1 - \prod_{j=1}^{n} \left(1 - A_d(\varsigma)_{\gamma j}\right)^{\frac{1}{n}}\right] \prod_{j=1}^{n} S_d(\varsigma)_{\gamma j}^{\frac{1}{n}}, \left[1 - \prod_{j=1}^{n} \left(1 - \alpha_{\gamma j}\right)^{\frac{1}{n}}\right], \prod_{j=1}^{n} \beta_{\gamma j}^{\frac{1}{n}} \quad (118)$$

$$N = \left(\left(A_d(\varsigma), S_d(\varsigma)\right), \left(\alpha(\varsigma), \beta(\varsigma)\right)\right) = (< (A_d(\varsigma), S_d(\varsigma) >, < \alpha(\varsigma), \beta(\varsigma) >$$

where $A_d(\varsigma), S_d(\varsigma), \alpha(\varsigma), \beta(\varsigma) \in [0,1], 0 \leq \alpha(\varsigma) A_d(\varsigma) + \beta(\varsigma) S_d(\varsigma) \leq 1$

The final weight is obtained via defuzzification; the scoring function of LDFS is utilized in (119) to defuzzify the weight values in LDFS to their crisp values.

$$P_{Md} = P(M_d) = \frac{1}{2}[A_d - S_d] + (\alpha - \beta)] \quad (119)$$

The attribute weights summed together should equal one. The weights are rescaled using the following formula if this condition is not met:

$$w_j = s(\aleph) / \sum_{j=1}^{J} s(\aleph) \quad (120)$$

19- Z-Cloud Rough Numbers (ZCRNs)

Numerous industries have effectively applied the cloud model concept [143]. However, it has two significant shortcomings: namely (1) lacking a system to control the relationships between interpersonal data connections and (2) Individual viewpoints are not taken into account. They both influence how effectively the cloud model theories operates [144]. As a result, it is essential to make investments in the development of greater theoretical frameworks for cloud models. Rough number theory can deal with restriction number (1) in the cloud model by using both higher and lower approximations. As a result, a combination of the cloud model and rough number theories may be used to handle both individual and interpersonal uncertainty [144].

The Z-number theory [145] is an excellent method for dealing with constraint number (2) in the cloud model because it permits specialists to express their fuzzy preferences and opinion reliability in only one ordered pair (A, B) where A

indicates the foggy amount of the evaluated item and B denotes the fuzziness of A's reliability [146]. As a result, ZCRNs were utilized in this work, a model to manage uncertainty that takes into account the benefits of the cloud model,
Z-numbers, and rough number theories all at once.
The use of the Z-Cloud Rough Numbers (ZCRNs) environment tackles the problem of two kinds of uncertainty by offering a framework for managing ambiguity in data and accomplishing greater levels of data freedom. Table 22 shows the duration of each linguistic phrase using ZC.

Table 22. - Converting the linguistic terms into ZC Likert scales [39]

A				B			
Linguistic terms	Cloud Value			Linguistic terms	TFNs		
Not important	0.000	0.673	0.101	Very small (VS)	0.100	0.200	0.300
Slightly important	3.098	0.453	0.068	Small(S)	0.300	0.400	0.500
Moderately important	5.000	0.278	0.041	Medium (M)	0.600	0.700	0.800
Important	8.262	0.579	0.086	High (H)	0.700	0.800	0.900
Very important	10	0.673	0.101	Very high (VH)	0.900	1.000	1.000

Implementation of a fuzzy member function

A Z-number is an ordered pair of fuzzy numbers that appears as Z= (A, B) [145]. A Z-number is an ordered pair of fuzzy numbers that appears as Z= (A, B). In reality, the computational complicating of Z-numbers can be lowered by transforming them to ordinary fuzzy numbers [146]. The ZC model was developed with the primary goal of combining the component of the Z-second number (i.e., reliability) into the first (i.e., the fuzzy constraint) [144]. The following formulae are used to convert the Z number to ZC:

Step 1: Convert the reliability $\tilde{\mathbf{B}}$ of the element $\tilde{\mathrm{A}}$ into a real number.

$$\tilde{\alpha} = \frac{\int x\varphi_B(x)dx}{\int \varphi_B(y)dy} \qquad (121)$$

where ∫ denotes an integration in algebra.

Step 2: To obtain the weighted Z-number, first transform the judgment reliability ($\tilde{\mathbf{B}}$) value into the fuzzy restriction $\tilde{\mathrm{A}}$.

$$\tilde{Z}^{\tilde{\alpha}} = \left\{\langle x, \tilde{A}^{\sim}(x)\rangle \mid \mu_{\tilde{\tilde{A}}}(x) = \tilde{\alpha}\mu_{\tilde{A}}(x), x \in X\right\} \qquad (122)$$

For easier of use, the Z number is referred by the symbol $\tilde{Z}^{\tilde{\alpha}} = (\tilde{A}, \tilde{\alpha})$.

Step 3: Convert an unusual cloud number to a standard cloud value number.

$$\tilde{Z} = \left\{\langle x, \mu_{\tilde{Z}}(x)\rangle \mid \mu_{\tilde{Z}}(x) = \mu\left(\frac{x}{\sqrt{\tilde{\alpha}}}\right), x \in X\right\} \qquad (123)$$

As a result, the ordinary Z-number set $Z = \{\langle x, \tilde{A}_{\mu(x)}, \tilde{B}_{\varphi(x)}\rangle \mid x \in X\}$ is turned into a matching ZC set $\tilde{Z}$ with a form of the traditional cloud value, significantly minimizing the complexity of dealing with evaluation challenges using z numbers.

Following that, the steps below outline the basic ways for changing ZC numbers to ZC rough numbers [144].
Let $\tilde{Z}_i^{Ex} = \{\tilde{E}x_1, \tilde{E}x_2, \dots, \tilde{E}x_n\}, \tilde{Z}E \; = \{\tilde{E}n_1, \tilde{E}n_2, \dots, \tilde{E}n_n\}$, and $\tilde{Z}_i^{He} = \{\tilde{H}e_1, \tilde{H}e_2, \dots, \tilde{H}e_n\}$

Then, the lower approximation $\underline{\mathrm{Apr}}\,(\tilde{Z}_i)$ of $\tilde{Z}_i$ can be identified as:

$$\underline{\mathrm{Apr}}(\tilde{E}x_i) = \cup \left\{\tilde{E}x_j \in \tilde{Z}\tilde{Z}_i^{Ex} \mid \tilde{E}x_j \leqslant \tilde{E}x_i\right\} \qquad (124)$$

$$\underline{\mathrm{Apr}}(\tilde{E}n_i) = \cup \{\tilde{E}n_j \in \tilde{Z}\tilde{Z}_i^{En} \mid \tilde{E}n_j \leqslant \tilde{E}n_i\} \qquad (125)$$

$$\underline{\mathrm{Apr}}(\tilde{H}e_i) = \cup \{\tilde{H}e_j \in \tilde{Z}_i^{He} \mid \tilde{H}e_j \leqslant \tilde{H}e_i\} \qquad (126)$$

Where $(Ex_i, \tilde{E}n_i, \tilde{H}e_i)$ are elements in $(\tilde{Z}_i^{Ex}, \tilde{Z}_i^{En}, \tilde{Z}_i^{He})$ respectively; 1 i, j k The lower approximation $\underline{\mathrm{Apr}}(\tilde{E}x_i)$ of $\tilde{E}x_i$ includes all elements in $\tilde{Z}_i^{Ex}$ that have class values equal to and less than $(\tilde{E}x_i)$. And likewise for the rest.

Likewise, the upper approximation $\overline{\text{Apr}}(\tilde{Z}_i)$ of $\tilde{Z}_i$ can be identified as:

$$\overline{\text{Apr}}(\tilde{E}x_i) = \cup\{\tilde{E}x_j \in \tilde{Z}\tilde{Z}_i^{Ex} \mid \tilde{E}x_j \geqslant \tilde{E}x_i\} \quad (127)$$

$$\overline{\text{Apr}}(\tilde{E}n_i) = \cup\{\tilde{E}n_j \in \tilde{Z}_i^{En} \mid \tilde{E}n_j \geqslant \tilde{E}n_i\} \quad (128)$$

$$\overline{Apr}(\tilde{H}e_i) = \cup\{\tilde{H}e_j \in \tilde{Z}\tilde{Z}_i^{He} \mid \tilde{H}e_j \geqslant \tilde{H}e_i\} \quad (129)$$

The lower approximation $\underline{\text{Apr}}(\tilde{E}x_i)$ of $\tilde{E}x_i$ contains all objects in the set $\tilde{Z}_i^{Ex}\tilde{E}x_i$. Next, the lower limit $\underline{\text{Lim}}(\tilde{Z}_i)$ of $\tilde{Z}_i$ is calculated as:

$$\underline{\text{Lim}}(\tilde{E}x_i) = \frac{1}{\vartheta_L^{Ex}} \sum_{j=1}^{v_L^{Ex}} \tilde{E}x_j \mid \tilde{E}x_j \in \underline{\text{Apr}}(\tilde{E}x_i) \quad (130)$$

$$\underline{\text{Lim}}(\tilde{E}n_i) = \sqrt{\frac{1}{\vartheta_L^{En}} \sum_{j=1}^{v_L^{En}} (\tilde{E}n_j)^2 \mid \tilde{E}n_j \in \underline{\text{Apr}}(\tilde{E}n_i)} \quad (131)$$

$$\underline{\text{Lim}}(\tilde{E}h_i) = \sqrt{\frac{1}{\vartheta_L^{He}} \sum_{j=1}^{v_L^{He}} (\tilde{H}e_j)^2 \mid \tilde{H}e_j \in \underline{\text{Apr}}(\tilde{H}e_i)} \quad (132)$$

Where $\vartheta_L^{Ex}, \vartheta_L^{En}$, and ϑ_L^{He} show the total numbers of elements in $\underline{\text{Apr}}(\tilde{E}x_i), \underline{\text{Apr}}(\tilde{E}n_i)$, $\underline{\text{Apr}}(\tilde{H}e_i)$, respectively. For convenience, $\underline{\text{Lim}}(\tilde{E}x_i), \text{Lim}(\tilde{E}n_i)$, and $\underline{\text{Lim}}(\tilde{H}e_i)$ are expressed as $\tilde{E}x_i^L, \tilde{E}n_i^L$, and $\tilde{H}e_i^L$ in subsequent contents, respectively. Briefly the lower limit of a class ZC value is the average value of the classes contained in its lower approximal likewise, the upper limit Lim $(\tilde{Z}_i)$ of $\tilde{Z}_i$ is calculated as follows:

$$\overline{\text{Lim}}(\tilde{E}x_i) = \frac{1}{\vartheta_U^{Ex}} \sum_{j=1}^{\vartheta_U^{Ex}} \tilde{E}x_j \mid \tilde{E}x_j \in \overline{\text{Apr}}(\tilde{E}x_i) \quad (133)$$

$$\overline{\text{Lim}}(\tilde{E}n_i) = \sqrt{\frac{1}{\vartheta_U^{En}} \sum_{j=1}^{\vartheta_U^{En}} (\tilde{E}n_j)^2 \mid \tilde{E}x_j \in \overline{Apr}(\tilde{E}n_i)} \quad (134)$$

$$\overline{\text{Lim}}(\tilde{H}e_i) = \sqrt{\frac{1}{\vartheta_U^{He}} \sum_{j=1}^{\vartheta_U^{He}} (\tilde{H}e_j)^2 \mid \tilde{E}x_j \in \overline{Apr}(\tilde{H}e_i)} \quad (135)$$

Where $\vartheta_U^{Ex}, \vartheta_U^{En}$, and ϑ_U^{He} refer to the total number of elements in $\overline{\text{Apr}}(\tilde{E}x_i), \overline{\text{Apr}}(\tilde{E}n_i)$, and $\overline{Apr}(\tilde{H}e_i)$, respectively, for simplicity, $\text{Lim}(\tilde{E}x_i), \text{Lim}(\tilde{E}n_i)$, and $\text{Lim}(\tilde{H}e_i)$ are presented as $\tilde{E}x_i^U, \tilde{E}n_i^U$, and $\tilde{H}e_i^U$ in the next contents, respectively. The upper limit of a class ZC value is the average value of the classes included in its upper approximation. Once the lower limit $\underline{\text{Lim}}(\tilde{Z}_i)$ and the upper limit $(\tilde{Z}_i)$ for an arbitrary Z-cloud class $\tilde{Z}_i$ have been created, the ZCRN value ZCRN $(\tilde{Z}_i)$ of $(\tilde{Z}_i)$ can be declare as follows:

$$[\tilde{Z}_i] = [\tilde{Z}_i^L, \tilde{Z}_i^U][(\tilde{E}x_i^L, \tilde{E}n_i^L, \tilde{H}e_i^L), (\tilde{E}x_i^U, \tilde{E}n_i^U, \tilde{H}e_i^U)] \quad (136)$$

Where $[\tilde{Z}_i], \tilde{Z}_i^L$, and $\tilde{Z}_i^U$ represent the $ZCR(\tilde{Z}_i)$, the lower limit $\underline{\text{Lim}}(\tilde{Z}_i)$, and the upper limit $\text{Lim}(\tilde{Z}_i)$, respectively. The aggregation operator must be applied to obtain the final weight. This section describes the arithmetic operation of ZCRNs for processing large amounts of data utilizing the source [144].

Suppose $[\tilde{Z}_i] = [\tilde{Z}_i^L, \tilde{Z}_i^U] = [(\tilde{E}x_i^L, \tilde{E}n_i^L, \tilde{H}e_i^L), (\tilde{E}x_i^U, \tilde{E}n_i^U, \tilde{H}e_i^U)]$
(i= 1, 2, . . ., n) are nZCRNs.

The arithmetic operation of ZCRNs is defined as (137)

$$[\tilde{Z}_1] \oplus [\tilde{Z}_2] = [\tilde{Z}_1^L \oplus \tilde{Z}_2^L, \tilde{Z}_1^U \oplus \tilde{Z}_2^U] = \begin{bmatrix} \left(Ex_1^L + Ex_2^L, \sqrt{(En_1^L)^2 + (En_2^L)^2}, \sqrt{(He_1^L)^2 + (He_2^L)^2}\right) \\ \left(Ex_1^U + Ex_2^U, \sqrt{(En_1^U)^2 + (En_2^U)^2}, \sqrt{(He_1^U)^2 + (He_2^U)^2}\right) \end{bmatrix} \quad (137)$$

Defuzzify the criterion weights by using centroid method and formula is used. Keep in mind that the summation of the final weight must be 1

The following equation is used to get the essential global weight for each main criterion and associated subcriterion.

GW = LW (for main criteria) ∗LW (for its sub criteria). (138)

20- q-rung picture fuzzy sets environment.

In complicated decision issues, intuitionistic fuzzy sets (IFSs) and Pythagorean fuzzy sets (PyFSs) have limits. The total of cubic or higher powers may be more than one, resulting in uncertainty. To overcome this, [69] proposed q-rung orthopair fuzzy sets (q-ROFS). When the total of the qth powers of membership and non-membership grades cannot exceed one, q-ROFS compensates for the weaknesses of IFS and PyFS. When q is one or two, q-ROFS applies to PyFS and IFS, respectively. The q-ROFS, on the other hand, is not well suited to modeling neutral human thoughts. Picture fuzzy sets PFSs perform well in expressing human decision abstention, but they have drawbacks when positive, neutral, and negative membership grades more than 1. As a result, -RPFSs combine the best characteristics of q-ROFS and PFS, both of which are isomorphic forms for MCDM issues [147].

The selected experts can prioritize each of the characteristics using a five-point Likert scale and their associated numbers Table 23.

Table 23: The Evaluation Scales [46]

Linguistic Term	Likert Scale	q-ROPFS
Very High (VH)	5	[0.85,0.1,0.15]
Above Average (AAV)	4	[0.75,0.2,0.25]
Average (AV)	3	[0.65,0.3,0.35]
Below Average (BAV)	2	[0.25,0.2,0.75]
Very Low (VL)	1	[0.15,0.1,0.85]

Definition 1. A q-ROFS on a universe of discourse X is represented by.

$$Q = \{\langle x, \phi_Q(x), \psi_Q(x) \mid x \in X \rangle\}$$

where the pair $\phi_Q(x), \psi_Q(x)$: X→ [0, 1] denotes the membership and non-membership grades of a member x ∈ X, respectively that meet

$$\left(\phi_Q(x)\right)^q + \left(\psi_Q(x)\right)^q \leq 1, \text{ for } q \geq 1 \forall x \in X.$$

Definition 2. A PFS on a universe of discourse X is expressed by.

$$\mathcal{P} = \{\langle x, \phi_{\mathcal{P}}(x), \eta_{\mathcal{P}}(x), \psi_{\mathcal{P}}(x) \mid x \in X \rangle\}$$

Where the triplet $\phi_{\mathcal{P}}(x), \eta_{\mathcal{P}}(x), \psi_{\mathcal{P}}(x)$: X→ [0, 1] denotes the positive, neutral, and negative membership grades of an element x ∈ X, that fulfill

$$\phi_{\mathcal{P}}(x) + \eta_{\mathcal{P}}(x) + \psi_{\mathcal{P}}(x) \leq 1, \forall x \in X$$

Definition 3. A q-RPFS over the non-empty universe X is denoted by.

$$\mathbb{P} = \{\langle x, \phi_{\mathbb{P}}(x), \eta_{\mathbb{P}}(x), \psi_{\mathbb{P}}(x) \mid x \in X \rangle\}$$

where the positive, the neutral, and the negative membership degrees of an element x ∈ X, $\phi_{\mathbb{P}}(x), \eta_{\mathbb{P}}(x), \psi_{\mathbb{P}}(x)$: X→ [0, 1], respectively satisfy

$$\left(\phi_{\mathbb{P}}(x)\right)^q + \left(\eta_{\mathbb{P}}(x)\right)^q + \left(\psi_{\mathbb{P}}(x)\right)^q \leq 1, \forall x \in X.$$

To compute the weights of the criteria, firstly, the priorities given by a specialist to the criteria are scaled. This is accomplished by dividing the priority of each criterion by the sum of the total priorities. To find the total score of the criteria per specialist, formula (139) is applied and the scores are added.

Definition 4. The score function of a q-RPFS $\mathbb{P} = \{\phi_{\mathbb{P}}, \eta_{\mathbb{P}}, \eta_{\mathbb{P}}\}$ is computed by.

$$\text{Score}(\mathbb{p}) = \frac{1 + \phi_{\mathbb{P}}^{q} - \eta_{\mathbb{P}}^{q} - \psi_{\mathbb{P}}^{q}}{3} \quad (139)$$

The following scalar multiplying rule (140) is then used to scale the importance of each criterion.

$$\lambda \odot \mathbb{p} = \left\{ \left(1 - \left(1 - \phi_{\mathbb{P}}^{q}\right)^{\lambda}\right)^{\frac{1}{q}}, \eta_{\mathbb{P}}^{\lambda}, \psi_{\mathbb{P}}^{\lambda} \right\}, \lambda > 0. \quad (140)$$

the scaled scores of the expert for each criterion are summed using the weighting averaging operator (141).

$$q - RPFWA(\mathbb{P}_1, \mathbb{P}_2, \cdots, \mathbb{P}_n) = \omega_1 \mathbb{P}_1 \oplus \omega_2 \mathbb{P}_2 \oplus \cdots \oplus \omega_n \mathbb{P}_n = \left\{ \left[1 - \prod_{i=1}^{n} \left(1 - \phi_{\mathbb{P}}^{q}\right)^{\omega_i}\right]^{\frac{1}{q}}, \prod_{i=1}^{n} \eta_{\mathbb{P}}^{\omega_i}, \prod_{i=1}^{n} \psi_{\mathbb{P}}^{\omega_i} \right\}, \omega_i \in [0,1]; \sum_{i=1}^{n} \omega_i = 1 \quad (141)$$

Where ω_i is the weight of the i^{th} expert, to determine its fuzzy weight.
using (139), the score of the fuzzy weights of the criterion is obtained and then normalized to obtain the weights of the criteria.

21- probabilistic single-valued neutrosophic hesitant fuzzy set (PSVNHFS) environment

It is essential to include statistical uncertainty into actual production. The probabilistic method's effectiveness in dealing with epistemic uncertainty may be restricted. As a result, these challenges inspire researchers to combine FS and probabilistic theories to develop a new fuzzy idea.
To overcome MADM difficulties, [148] introduced the idea of probabilistic single-valued neutrosophic hesitant FS (PSVNHFS) based on hesitant FS, probabilistic dual hesitant FS, neutrosophic FS, and interval neutrosophic hesitant FS. The authors combined the SVNHFS and probability data by presenting the truth, indeterminacy, and falsity membership degree values with their related probability values. The PSVNHFS offers extra information to help in decision-making procedure [149].
The obtained data (linguistic phrases) are substituted with their numerical scale equivalents as shown in Table 24.

Table 24: Linguistic Expressions, Numerical Scale and PSVNHFNs [150].

Linguistic expressions	Numerical scale	PSVNHFNs											
		$T(x) \mid P^T(x)$				$I(x) \mid P^I(x)$				$F(x) \mid P^F(x)$			
		α_1	$P^T_{\alpha_1}$	α_2	$P^T_{\alpha_2}$	β_1	$P^I_{\beta_1}$	β_2	$P^I_{\beta_2}$	γ_1	$P^I_{\gamma_1}$	γ_2	$P^I_{\gamma_2}$
Very Important (VI)	1	0.95	0.8	0.9	0.2	0.05	0.8	0.1	0.2	0.05	0.8	0.1	0.2
Important (Im)	2	0.75	0.7	0.7	0.3	0.25	0.7	0.3	0.3	0.25	0.7	0.3	0.3
Average (Av)	3	0.55	0.5	0.5	0.5	0.45	0.5	0.5	0.5	0.45	0.5	0.5	0.5
Low Important (LI)	4	0.35	0.7	0.25	0.3	0.65	0.7	0.75	0.3	0.65	0.7	0.75	0.3
Very Low Important (VLI)	5	0.15	0.8	0.1	0.2	0.85	0.8	0.9	0.2	0.85	0.8	0.9	0.2

Application of PSVNHFS: PSVNHFS theory is performed on the produced EDM to establish PSVNHFS–EDM as follows:

$$\widetilde{\text{EDM}} = [\tilde{E}_{11} \cdots \tilde{E}_{1m} \because \tilde{E}_{l1} \cdots \tilde{E}_{km}] \quad (142)$$

In this context, all numerical values inside the EDM are substituted with their corresponding probabilistic single-valued neutrosophic hesitant fuzzy numbers (PSVNHFNs), which are listed in Table 24.
PSVNHFS's robustness may be linked back to its capability to deal with complicated and ambiguous data. Definition 1 includes a description of PSVNHFS and PSVNHFN.

Definition 1. includes a description of PSVNHFS and PSVNHFN.
Definition 1. Let X be a fixed set. A PSVNHFS on X is defined as follows:

$$NP = \{\langle x, T(x)|P^T(x), I(x)|P^I(x), F(x) \mid P^F(x)\rangle \mid x \in X\}$$

The possible elements are indicated as $T(x)|P^T(x), I(x)|P^I(x), F(x) \mid P^F(x)$ by three separate components. T(x), I(x) and F(x) are finite subsets of [0, 1] that reflect the hesitant degrees of truth, indeterminacy, and falsity of x with regard to the set X. The related probabilistic information for the three previously mentioned degree categories is represented by $P^T(x), P^I(x)$ and $P^F(x)$ which also represent subsets of [0, 1] and have the same cardinality as their related degree sets. For $\alpha_a \in T(x), \beta_b \in I(x)$, and $\gamma_c \in F(x)$, the following conditions are fulfilled:

$$P^T_{\alpha_a} \in P^T(x), P^I_{\beta_b} \in P^I(x), P^F_{\gamma_c} \in P^F(x); \sum_{i=1}^{|T(x)|} P^T_{\alpha_i} \leq 1, \sum_{i=1}^{|I(x)|} P^I_{\beta_i} \leq 1, \sum_{i=1}^{|F(x)|} P^F_{\gamma_i} \leq 1$$

And

$$0 \leq \alpha^+ + \beta^+ + \gamma^+ \leq 3$$

Where $\alpha^+ = max\{T(x)\}, \beta^+ = max\{I(x)\}$ and γ^+= {maxF(x)}; and || Denotes the cardinality of a set. An element in NP is called a PSVNHFN and is represented as:

An element in NP is called a PSVNHFN and is represented as:

$$\left\langle \left(\alpha_1 | P^T_{\alpha_1}, \ldots \alpha_{|T(x)|} | P^T_{\alpha_{|T(x)|}}\right), \left(\beta_1 | P^I_{\beta_1}, \ldots \beta_{|I(x)|} | P^I_{\beta_{|I(x)|}}\right), \left(\gamma_1 | P^F_{\gamma_1}, \ldots \gamma_{|F(x)|} | P^F_{\gamma_{|F(x)|}}\right)\right\rangle,$$

for x ∈ X. For convenience, hereafter a PSVNHFN is denoted by N = $\langle T|P^T, I|P^I, F \mid P^F\rangle$.

The probabilistic single-valued neutrosophic hesitant fuzzy weighted averaging (PSVNHFWA) operator indicated in (143) has been modified and the aggregation is performed using the probabilistic single-valued neutrosophic hesitant fuzzy averaging (PSVNHFA) operator as shown in (144).

$$\text{PSVNHFWA}(N_1, N_2, \ldots, N_r) = \left(\begin{array}{c} \bigcup_{(\alpha_j)_{j=1,\ldots,r} \in T_1 \times T_2 \times \ldots \times T_r} \left(1 - \prod_{j=1}^{r} (1-\alpha_j)^{w_j} \mid \Pi_{j=1}^{r} P^{T_j}_{\alpha_j}\right) \\ \bigcup_{(\beta_j)_{j=1,\ldots,r} \in I_1 \times I_2 \times \ldots \times I_r} \left(\prod_{j=1}^{r} (\beta_j)^{w_j} \mid \Pi_{j=1}^{r} P^{I_j}_{\beta_j}\right), \bigcup_{(\gamma_j)_{j=1,\ldots,r} \in F_1 \times F_2 \times \ldots \times F_r} \left(\prod_{j=1}^{r} (\gamma_j)^{w_j} \mid \Pi_{j=1}^{r} P^{F_j}_{\gamma_j}\right) \end{array} \right) \quad (143)$$

$$\text{PSVNHFA}(N_1, N_2, \ldots, N_r) = \left(\begin{array}{c} \bigcup_{(\alpha_j)_{j=1,\ldots,r} \in T_1 \times T_2 \times \ldots \times T_r} \left(1 - \prod_{j=1}^{r} (1-\alpha_j)^{\frac{1}{r}} \mid \Pi_{j=1}^{r} P^{T_j}_{\alpha_j}\right) \\ \bigcup_{(\beta_j)_{j=1,\ldots,r} \in I_1 \times I_2 \times \ldots \times I_r} \left(\prod_{j=1}^{r} (\beta_j)^{\frac{1}{r} | \Pi_{j=1}^{r} P^{I_j}_{\beta_j}}\right), \bigcup_{(\gamma_j)_{j=1,\ldots,r} \in F_1 \times F_2 \times \ldots \times F_r} \left(\prod_{j=1}^{r} (\gamma_j)^{\frac{1}{r}} \mid \Pi_{j=1}^{r} P^{F_j}_{\gamma_j}\right) \end{array} \right) \quad (144)$$

Let $N_j = \langle T_j | P^{T_j}, I_j | P^{I_j}, F_j \mid P^{F_j}\rangle$, for $j = 1, \ldots, r$ be PSVNHFNs. Then, the PSVNHFWA operator is defined as follows:

$$\text{PSVNHFWA}(N_1, N_2, \ldots, N_r) = w_1 N_1 \oplus w_2 N_2 \oplus \ldots \oplus w_r N_r,$$

Where $w = (w_1, w_2, \ldots, w_r)^T$ indicates the weights vector with $\sum_{j=1}^{r} w_j = 1$. As a result, the following is the results of aggregate using PSVNHFWA:

The aggregation of PSVNHFNs found in PSVNHFS-EDMs in (142)
is then $\tilde{E}_j = \text{PSVNHFA}(\tilde{E}_{1j}, \tilde{E}_{2j}, \ldots, \tilde{E}_{lj})$, for $j = 1, \ldots, m$.
The fuzzy weights of the evaluation criteria are converted into crisp weights using the PSVNHFS scoring function, as stated in (145). For any PSVNHFN N= $\langle T|P^T, I|P^I, F \mid P^F\rangle$ a scoring function is described as follows:

$$s(N) = \frac{\left(\frac{1}{|T|}\sum_{\alpha \in T} (\alpha \cdot P^T_\alpha)\right) + \left(\frac{1}{|I|}\sum_{\beta \in I} (1-\beta) \cdot P^I_\beta\right) + \left(\frac{1}{|F|}\sum_{\gamma \in F} (1-\gamma) \cdot P^F_\gamma\right)}{3} \quad (145)$$

The cumulative weights of the criterion have been set to one. If this requirement is not met, the weights are rescaled according to (146).

$$w_j = \frac{s(\bar{E}_j)}{\sum_{j=1}^{m} s(\bar{E}_j)} \tag{146}$$

Where $\bar{E}_j$ is the aggregated PSVNHFN evaluated using the PSVNHFA operator in (i), for j = 1, ..., m.

22- interval-valued spherical fuzzy sets (IvSFSs)

In [3], has utilized SFSs with the FWZIC technique for prioritization, with decision makers' opinions on the parameters of these fuzzy sets included into the model as a single value. [151] recently offered IvSFSs to allow decision makers to freely express their hesitancies in decision making with a bigger 3D domain. Furthermore, IvSFSs integrate decision makers' opinions regarding variables with an interval value rather than a single value. Furthermore, although SFS is based on a very small definition space, IvSFSs allow a larger definition area for decision makers to provide their opinions.
IvSFS-EDM is made by replacing the numerical scale in EDM with IvSFS numbers (IvSFSNs) in Table 25.

Table 25: Linguistic Expressions with Corresponding Numerical Scale and Ivsfsns [41]

Linguistic x pressions	Numerical scale	IvSFSNs					
		μ^L	μ^U	v^L	v^U	π^L	π^U
Very Important (VI)	1	0.8	0.85	0.1	0.15	0.05	0.1
Important (Im)	2	0.7	0.75	0.2	0.25	0.15	0.2
Average (Av)	3	0.5	0.55	0.45	0.5	0.2	0.25
Low Important (LI)	4	0.2	0.25	0.7	0.75	0.15	0.2
Very Low Important (VLI)	5	0.1	0.15	0.8	0.85	0.05	0.1

The terms IvSFS and IvSFSN are described below. In (147) [151], an IvSFS as of the universe of discourse U can be stated mathematically as follows:

$$\tilde{A}_S = \left\{ \left\langle u, \left(\begin{array}{c} [\mu^L_{\tilde{A}_S}(u), \mu^U_{\tilde{A}_S}(u)], \\ [v^L_{\tilde{A}_S}(u), v^U_{\tilde{A}_S}(u)], [\pi^L_{\tilde{A}_S}(u), \pi^U_{\tilde{A}_S}(u)] \end{array} \right) \right\rangle \middle| u \in U \right\} \tag{147}$$

where $0 \le \mu^L_{\tilde{A}_S}(u) \le \mu^U_{\tilde{A}_S}(u) \le 1, 0 \le v^L_{\tilde{A}_S}(u) \le v^U_{\tilde{A}_S}(u) \le 1$ and $0 \le (\mu^U_{\tilde{A}_S}(u))^2 + (v^U_{\tilde{A}_S}(u))^2 + (\pi^U_{\tilde{A}_S}(u))^2 \le 1$. For each $u \in U, \mu^U_{\tilde{A}_S}(u), v^U_{\tilde{A}_S}(u)$ and $\pi^U_{\tilde{A}_S}(u)$ are the upper degree of the positive, negative and hesitancy of u to $\tilde{A}_S$, respectively Similarly, $u \in U, \mu^L_{\tilde{A}_S}(u), v^L_{\tilde{A}_S}(u)$ and $\pi^L_{\tilde{A}_S}(u)$ are the lower degree of the positive, negative and hesitancy of u to $\tilde{A}_S$,, respectively.
For an IvSFS $\tilde{A}_S$, the pair $[\mu^L_{\tilde{A}_S}(u), \mu^U_{\tilde{A}_S}(u)], [v^L_{\tilde{A}_S}(u), v^U_{\tilde{A}_S}(u)]$and $[\pi^L_{\tilde{A}_S}(u), \pi^U_{\tilde{A}_S}(u)]$ is called IvSFSN and denoted by $\tilde{\alpha} = \langle [a, b], [c, d], [e, f] \rangle$

The weights have been determined. At first, the IvSFSNs inside the three experts' IvSFS-EDM are aggregated using IvS weighted arithmetic mean (IvSWAM), as shown in (148).

$$\text{IvSWAM}\, M_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots \dots, \tilde{\alpha}_n) = w_1 \cdot \tilde{\alpha}_1 \oplus w_2 \cdot \tilde{\alpha}_2 \oplus \cdots \oplus w_n \cdot \tilde{\alpha}_n =$$

$$\left\{\begin{array}{c}\left[\left(1-\prod_{j=1}^{n}\left(1-a_j^2\right)^{w_j}\right)^{\frac{1}{2}},\left(1-\prod_{j=1}^{n}\left(1-b_j^2\right)^{w_j}\right)^{\frac{1}{2}}\right]\\ \left[\prod_{j=1}^{n}c_j^{w_j},\prod_{j=1}^{n}d_j^{w_j}\right]\\ \left[\begin{array}{c}\left(\prod_{j=1}^{n}\left(1-a_j^2\right)^{w_j}-\prod_{j=1}^{n}\left(1-a_j^2-e_j^2\right)^{w_j}\right)^{\frac{1}{2}}\\ \left(\prod_{j=1}^{n}\left(1-b_j^2\right)^{w_j}-\prod_{j=1}^{n}\left(1-b_j^2-f_j^2\right)^{w_j}\right)^{\frac{1}{2}}\end{array}\right]\end{array}\right\} \quad (148)$$

where $w_j = 1/n$.

The fuzzy aggregating numbers are then defuzzied and transformed into crisp numbers employing the IvS score function, as shown in (149).

$$S(\tilde{\alpha}) = \frac{a^2 + b^2 - c^2 - d^2 - (e/2)^2 - (f/2)^2}{2} \quad (149)$$

Finally, if the sum of these values is less than one, the outcome's weight values are rescaled utilizing Equation (150).

$$w_j = \frac{S(\tilde{\alpha})}{\sum_{j=1}^{J} S(\tilde{\alpha})}, \quad (150)$$

23- FWZIC II intuitionistic fuzzy set (IFS)

As an extension to Zadeh's fuzzy set, the IFSs concept may consider membership and non-membership degrees with a hesitation index [124]. As a result, the IFS theory is commonly applied since it could represent inexorably imperfect or not completely reliable evaluations [152]. Furthermore, membership definitions may be used to successfully express affirmation, negation, and hesitation in IFSs. For group decision-making (GDM) to provide reliable decision results, the consistency of IFS preference relationships and the expert views gathered from these preference relations are crucial [153].
The significance degree of each criterion is defined by the experts using a five-point Likert scale, as shown in Table 26. The linguistic terms were transformed into numerical scoring scales.

Table 26: Linguistic Scoring Scale and Numerical Scoring Scale and the Corresponding IFSNs [48].

Numerical scoring scale	Linguistic scoring scale	IFSNs	
		μ	n
1	Not important (NI)	0.10	0.80
2	Slight important (SI)	0.25	0.60
3	Moderately important (MI)	0.50	0.40
4	Important (I)	0.75	0.20
5	Very important (VI)	0.90	0.05

The application of Intuitionistic fuzzy theory [154] is defined as follows:

Definition 1: Let X be the universal set:

(i) A set $\tilde{A} = \{x, m_{\tilde{A}}(x) \mid x \in X\}$ is called a fuzzy set of X, where $m_{\tilde{A}}(x): X \to [0,1]$ is a membership function; for all x ∈ X, $m_{\tilde{A}}(x)$: expresses the degree of membership of element x in A
(ii) A set $\tilde{A} = \{x, m_{\tilde{A}}(x), n_{\tilde{A}}(x) \mid x \in X\}$ is called IFS of X, where $m_{\tilde{A}}(x), n_{\tilde{A}}(x)$ are membership function and non-membership function, respectively. Thus, $0 \le m_{\tilde{A}}(x) + n_{\tilde{A}}(x) \le 1, \forall x \in X$.

(iii) In addition, $\pi_{\tilde{A}}(x) = 1 - m_{\tilde{A}}(x) - n_{\tilde{A}}(x)$ is the e hesitation degree of x.

The following equations were used in the applied arithmetic operation utilizing IFS [154]. (151) is used to perform the IFS-weighted aggregation operation (IFA):

$$IFA = (A_1, A_2, \dots, A_m) = \left(1 - \prod_{i=1}^{m} (1 - m_{A_i}), \prod_{i=1}^{m} n_{A_i}\right) \quad (151)$$

Equation (152) is used for the IFS division operation:

$$\frac{A_1}{A_2} = \begin{cases} \left\langle \frac{m_{A_1}}{m_{A_2}}, \frac{n_{A_1} - n_{A_2}}{1 - n_{A_2}} \right\rangle, \text{if } 0 \le \frac{m_{A_1}}{m_{A_2}} \le \frac{1 - n_{A_1}}{1 - n_{A_2}} \le 1 \\ \langle 1,0 \rangle, \text{otherwise.} \end{cases} \quad (152)$$

Equation (153) depicts the equation of IFS division on crisp values.

$$\frac{a}{\lambda} = \left\langle 1 - (1 - \mu_1)^{\frac{1}{\lambda}}, n_1^{\frac{1}{\lambda}} \right\rangle \quad (153)$$

In this stage, the numerical scoring scales that represent linguistic scoring scales (stage 2) are substituted with IFSN. The IFS EDM is built using (154).

$$IFS - EDM = \widehat{EDM} = \begin{matrix} E_1 \\ \vdots \\ E_f \end{matrix} \left[\begin{matrix} & C1 \dots . Cn & \\ \left(m_{\tilde{A}_{1,1}}(x), n_{\tilde{A}_{1,1}}(x)\right) & \cdots & \left(m_{\tilde{A}_{1,n}}(x), n_{\tilde{A}_{1,n}}(x)\right) \\ \vdots & \ddots & \vdots \\ \left(m_{\tilde{A}_{f,1}}(x), n_{\tilde{A}_{f,1}}(x)\right) & \cdots & \left(m_{\tilde{A}_{f,n}}(x), n_{\tilde{A}_{f,n}}(x)\right) \end{matrix} \right], \quad (154)$$

where f is the experts' number, and n is the criteria's number.

Equation (155) convert the defuzzied IFSNs into crisp numbers as follows:

$$s_j = \mu_{\tilde{t}_j} - v_{\tilde{t}_j} \quad (155)$$

The standard deviation (Std_i) is utilized to reduce preference differences across DMs based on the IFS-EDM (fuzzifed data). When $\text{Std}_i = 0$, all DMs have the exact same preference and their values change ($\text{Std}_i > 0$) based on the degree of preference variation between them. Using (156), the Std_i is determined for the membership $m_{\tilde{A}}(x)$ and non-membership $n_{\tilde{A}}(x)$ per criteria across all experts.

$$\text{Std}_i = \left\{ \text{std}\left(m_{\tilde{A}_{1,i}}(x), \dots, m_{\tilde{A}_{f,i}}(x)\right), \text{std}\left(n_{\tilde{A}_{1,i}}(x), \dots, n_{\tilde{A}_{f,i}}(x)\right) \right\} \quad (156)$$
$$\forall i = 1,2,3 \dots, n$$

The enhanced IFS-EDM is computed using (157), where each dgree of the membership $m_{\tilde{A}}(x)$ and nonmember ship $n_{\tilde{A}}(x)$ in the IFS-EDM are subtracted from their equivalent Std_i values and multiplied by the same $m_{\tilde{A}}(x)$ and $n_{\tilde{A}}(x)$.

mproved IFS_EDM

$$= \begin{matrix} E_1 \\ \vdots \\ E_f \end{matrix} \left[\begin{matrix} \left(m_{\tilde{A}_{1,1}}(x), n_{\tilde{A}_{1,1}}(x)\right) \ominus Std_1 \otimes \left(m_{\tilde{A}_{1,1}}(x), n_{\tilde{A}_{1,1}}(x)\right) \cdots & \left(m_{\tilde{A}_{1,n}}(x), n_{\tilde{A}_{1,n}}(x)\right) \ominus \text{Std}_n \otimes \left(m_{\tilde{A}_{1,n}}(x), n_{\tilde{A}_{1,n}}(x)\right) \\ \vdots \quad \ddots & \vdots \\ \left(m_{\tilde{A}_{f,1}}(x), n_{\tilde{A}_{f,1}}(x)\right) \ominus Std_1 \otimes \left(m_{\tilde{A}_{f,1}}(x), n_{\tilde{A}_{f,1}}(x)\right) \dots & \left(m_{\tilde{A}_{fn}}(x), n_{\tilde{A}_{fn}}(x)\right) \ominus \text{Std}_n \otimes \left(m_{\tilde{A}_{f,n}}(x), n_{\tilde{A}_{f,n}}(x)\right) \end{matrix} \right] \quad (157)$$

Using (151) and (152), the improved IFS-EDM is used to determine the fuzzification data ratio. IFS used with the previous equations. The process is represented symbolically by (158).

$$\widetilde{\text{Ratio}}_{ij} = \frac{(\text{Improved } \widetilde{\text{IFS\_EDM}}\,[\text{i},\text{j}])}{\sum_{\text{j}=1}^{\text{n}}(\text{Improved } \widetilde{\text{IFS\_EDM}}\,[\text{i},\text{j}])},$$
$$\text{for } i = 1,2,3,..m \text{ and } j = 1,2,3,..n, \tag{158}$$

The mean values are calculated to get the final fuzzy values for criterion weight coefficients. The IFS-EDM is used to calculate the final weight values for criteria using equations (151) and (153), whereas equation (159) represents the process symbolically.

$$\tilde{t}_j = \frac{\sum_{i=1}^{m} \widetilde{\text{Ratio}}_{ij}}{m, \text{for } i} = 1,2,3,..m \text{ and } j = 1,2,3 \tag{159}$$

To get the final weight, defuzzification is conducted using (155). As shown in (160), weight should be provided to each criterion by adding the weight values of all the criteria for rescaling purposes.

$$w_j = \frac{s_j}{\sum_{j=1}^{J} s_j} \tag{160}$$

24- circular Pythagorean fuzzy sets (C-PFSs)

In [155], proposed the idea of C-PFSs and C-PFVs to serve as a extensive extension of C-IFSs and PFSs. C-PFS is a graphical visualization of membership and nonmembership degrees that consists of a circular shape. The circle center is made up of non-negative real numbers designated as u and v, with the restriction that the sum of squares cannot be greater than 1. C-PFS is better at showing the imprecision of uncertain data because of its unique structure, that makes it possible for the modeling of information through circular points specified by a certain center and radius. As a result, the use of C-PFS allows specialists to evaluate alternatives within a larger and more adaptable space, allowing for the formulation of more detailed and complex conclusions.

The use of fuzzy expressions is a common method for collecting PFVs. The most common evaluation grade is a 5, 7, or 9. The five-grade word scale is used in this research, as shown in Table 27.

Table 27: Correspondence Between Fuzzy Terms and PFVs [156].

Grades	Fuzzy terms for C-PFS-FWZIC	PFVs
1	Very Important (VI)	(1.00,0.00)
2	Important (I)	(0.75,0.25)
3	Average (Av)	(0.50,0.50)
4	Low Important (LI)	(0.25,0.75)
5	Very Low Important (VLI)	(0.00,1.00)

Basic concepts

- C-pfss

PFSs use circular representations with a central point $(u_p(x_i), v_p(x_i))$ to denote the degrees of membership and nonmember ship of an element to an FS. This approach offers greater flexibility in defining the condition of the set $(u_p(x_i))^2 + (v_p(x_i))^2 \le 1$ than numerical representations. This notation extends not only the idea of PFS but also the concept of C-IFS. The sensitivity of the decision-making process has increased because decision-makers are now able to attain circles with certain characteristics rather than exact numerical values. PFSs and C-PFSs are defined as follows:

PFSs indicate the degrees of membership and nonmembership of an element to an FS using circular representations with a center point $(u_p(x_i), v_p(x_i))$. This method is more flexible than numerical representations in specifying the condition of the set $(u_p(x_i))^2 + (v_p(x_i))^2 \le 1$. The use of this notation extends not just the concept of PFS, but additionally the concept of C-IFS. Because decision-makers can now achieve circles with specific features rather than precise numerical values, the sensitivity of the decision-making process increased. The following are the definitions of PFSs and C-PFSs:

Definition 1. PFS A in X is expressed as:

$$A = \{\langle x, u_A(x), v_A(x)\rangle : x \in X\}$$

Where u_A, v_A: X→ are the degrees of membership and nonmembership functions, respectively, with the following condition:
$u_A^2(x) + v_A^2(x) \leq 1$

As a result, a Pythagorean fuzzy value (PFV) is expressed by the pair p=u_p,v_p. The use of fuzzy terms is a popular method for obtaining PFVs
Usually, one chooses an assessment grade of 5, 7, or 9. The five-grade term scale is used in this study and is shown in Table 27.

Definition 2 Let $r \in$. C-PFS A_r in X is expressed as follows:
$A_r = \{\langle x, u_A(x), v_A(x); r\rangle : x \in X\}$

Where u_A, v_A: X→ with the condition that $u_A^2 + v_A^2 \leq 1$. Variable ' r ' represents the radius of a circle centered at point $(u_A(x), v_A(x))$ lies in the plane. The circular representation describes the degrees of membership and nonmembership of element × within set X.
Definition 3. Let u_p, v_p be functions with codomain [0, 1], subject to the condition $u_p^2 + v_p^2 \leq 1$ and $r_p \in$ [0, 1]. A C-PFV is represented by the triple $p = \langle u_p, v_p; r_p\rangle$. A set of C-PFVs can be regarded as a C-PFS.
Proposition 1 For finite set X, let $\{\langle u_{i,1}, v_{i,1}\rangle, \langle u_{i,2}, v_{i,2}\rangle, \cdots, \langle u_{i,k_i}, v_{i,k_i}\rangle\}$ be a set of assigned PFVs for xi ∈ X, then following:

$$A_r = \{\langle x_i, u(x_i), v(x_i); r_i\rangle : x_i \in X\} \text{ is a C-PFS, where } \langle u(x_i), v(x_i)\rangle = \left\langle \sqrt{\frac{\sum_{j=1}^{k_i} u_{i,j}^2}{k_i}}, \sqrt{\frac{\sum_{j=1}^{k_i} v_{i,j}^2}{k_i}} \right\rangle$$

And
$$r_i = min\left\{ \max_{1\leqslant i\leqslant k_i} \sqrt{\left(u(x_i) - u_{i,j}\right)^2 + \left(v(x_i) - v_{i,j}\right)^2}, 1\right\}$$
with $0 \leq r_i \leq 1$ for each i.

Definition 4. Let $\{p_i =< u_i, v_i >: i = 1, \cdots, n\}$ be a collection of PFVs. Here is an expression for the algebraic arithmetic aggregation operator:

$$PA(p_1, \cdots, p_n) = \left\langle \sqrt{1 - \prod_{i=1}^{n} (1 - u_i^2)^{\frac{1}{n}}}, \prod_{i=1}^{n} v_i^{\frac{1}{n}} \right\rangle$$

Definition 5. The score function of a C-PFV in A_r can be expressed as follows:
Score $= u_A^2(x) - v_A^2(x) - r^2(x)$

Definition 6. The variation between C-PFSs $A_r = \{\langle x, u_A(x), v_A(x); r\rangle : x \in X\}$ and $B_t = \{\langle x, u_B(x), v_B(x); t\rangle : x \in X\}$, which is the Hamming distance, is expressed as follows:

$$\xi(A_r, B_t) = \frac{1}{2} \cdot \left(|u_A^2(x) - u_B^2(x)| + |v_A^2(x) - v_B^2(x)| + |r^2(x) - t^2(x)|\right).$$

- The sum of the weights allocated to all factors in a layer have to be equal to one. Hence, as a result, (161) is used to calibrate the score values in order to figure out the final weight of the evaluation elements.

$$w_j = s_j \frac{\sum_{j=1}^{n} s_j}{.} \quad (161)$$

3.2 Aggregation Operators

Aggregation operators are useful in numerous areas, including decision-making [157]. Several aggregation operators have been created in the literature by academics to aggregate numerical data in various scenarios [158]. The goal of the aggregation phase is to combine a group of criteria in such a way that the final aggregate output takes into consideration all of the single criterion. The final classification selection naturally results from this collection of overall degrees; hence, useful classifications are not eliminated because they fail to match a few criteria [159]. Furthermore, we found that the number of approaches of aggregation operators for addressing MCDM issues, such as Geometric Bonferroni Mean (GBM), Bonferroni Mean (BM), ordered weighted averaging (OWA), and other hybrid aggregation, will expand in the future. Table 28 summarizes different types of aggregation operators and the fuzzy equations that used in the literature within each FWZIC version to find the weighting result.

Table 28. **Aggregation Operators using Fuzzy Equations.**

Ref.	Type of aggregation	Equations
[51]	As the same type of development	$\mathbf{PHFWA}(\mathbf{h_1}(\mathbf{p}), \mathbf{h_2}(\mathbf{p}), \cdots, \mathbf{h_n}(\mathbf{p})) = \bigoplus_{i=1} \omega_i \mathbf{h}_i(\mathbf{p}) =$ $\cup_{\gamma_{1_1} \in \mathbf{h}_1, \gamma_{2_1} \in \mathbf{h}_2, \cdots, \mathbf{h}_{\mathbf{h}_1} \in \mathbf{h}_n} \left\{ \left[1 - \prod_{\mathbf{i}_{i=1}}^{\mathbf{n}} (1 - \gamma_{i_1})^{\omega_i} \right] \left(\prod_{i=1}^{\mathbf{n}} \mathbf{P}_{i_1} / \prod_{i=1}^{n} \left(\sum_{l=1}^{\lvert h_i(p) \rvert} \mathbf{p}_{i_1} \right) \right) \right\}$
[3]	arithmetic mean	w= $(w_1, w_2 \dots, w_n); w_i \in [01]; \sum_{i=1}^{n} w_i = 1$ $\mathbf{SWAM}_w(\tilde{A}_{S1}, \dots, \tilde{A}_{Sn}) = w_1 \tilde{A}_{S1} + w_1 \tilde{A}_{S1} + \cdots + w_n \tilde{A}_{Sn}$ $= \left\{ \left[1 - \prod_{i=1}^{n} (1 - \mu_{\tilde{A}_{Si}}^2)^{w_i} \right]^{\frac{1}{2}}, \prod_{i=1}^{n} v_{\tilde{A}_{Si}}^{w_i} \left[\prod_{i=1}^{n} (1 - \mu_{\tilde{A}_{Si}}^2)^{w_i} - \prod_{i=1}^{n} (1 - \mu_{\tilde{A}_{Si}}^2 - \pi_{\tilde{A}_{Si}}^2)^{w_i} \right]^{\frac{1}{2}} \right\}.$
[43]	arithmetic mean	$\mathrm{q-ROFA}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \left\langle \left(1 - \prod_{k=1}^{n} (1 - \mu_k^q) \right)^{\frac{1}{q}}, \prod_{k=1}^{n} v_k \right\rangle$
[25]	arithmetic mean	$\mathbf{PFAG}(\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_n) = \left(\sqrt{1 - \prod_{j=1}^{n} \left(1 - (\mu_j)^2 \right)} = \prod_{j=1}^{n} (v_j) \right)$
[21]	As the same type of development	$\mathrm{PPHFWA}(\aleph_1, \aleph_2, \cdots, \aleph_r) = \mathbb{K}_1 \aleph_1 \oplus \mathbb{K}_2 \aleph_2 \oplus \cdots \cdots \oplus \mathbb{K}_r \aleph_r$ $\mathrm{PPHFWA}(\aleph_1, \aleph_2, \cdots, \aleph_r) = \bigcup_{\hbar_j \in \tau_{\aleph_j}, \tilde{p}_{\aleph_j} \in \tilde{p}_{\aleph_j}} \sqrt{1 - \Pi_{j=1}^{r} \left(1 - \left(\hbar_{\aleph_j} \right)^2 \right)^{\mathbb{K}_j}} / \Pi_{j=1}^{r} \tilde{p}_{\aleph_j}, \bigcup_{\varrho_{\aleph_j} \in \delta_{\aleph_j}, b\aleph_j \in b\aleph_j} \Pi_{j=1}^{r} (\varrho_{\aleph_j})^{\mathbb{K}_j} / \Pi_{j=1}^{r} b_{\aleph_j}$ where $\mathbb{K} = (\mathbb{K}_1, \mathbb{K}_2, \cdots, \mathbb{K}_r)^T$ denotes the weights of $\aleph_j \in [0,1]$ with $\sum_{\hat{j}=1}^{r} \mathbb{K}_{\hat{j}} = 1.$
[45]	arithmetic mean	$\mathrm{T-SAM}(\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n) = \left\{ \left[1 - \prod_{i=1}^{n} (1 - \mu_{\tilde{p}_i}^2) \right]^{1/T}, \right.$ $\left. \prod_{i=1}^{n} v_{\tilde{p}_i}, \left[\prod_{i=1}^{n} (1 - \mu_{\tilde{p}_i}^2) - \prod_{i=1}^{n} (1 - \mu_{\tilde{p}_i}^2 - s_{\tilde{p}_i}^2) \right]^{1/T} \right\}.$
[95]	arithmetic mean	$\tilde{\boldsymbol{A}} \oplus \tilde{\boldsymbol{B}} = \left(\boldsymbol{a}_1^T + \boldsymbol{b}_i^T ; \boldsymbol{min} \left(\boldsymbol{H}_1(\tilde{\boldsymbol{A}}^T), \boldsymbol{H}_1(\tilde{\boldsymbol{B}}^T) \right), \boldsymbol{min} \left(\boldsymbol{H}_2(\tilde{\boldsymbol{A}}^T), \boldsymbol{H}_2(\tilde{\boldsymbol{B}}^T) \right) : \boldsymbol{T} \in \{ \boldsymbol{U}, \boldsymbol{L} \}, \boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4} \right).$
[100]	arithmetic mean	$\mathbf{CPFA}(\mathbf{e_1 e_2 \dots, e_n}) = \left(\begin{array}{c} \left\langle \left[\sqrt{1 - \Pi_{i=1}^{n} \left(1 - (\mu_{e_i}^{L})^2 \right)}, \sqrt{1 - \Pi_{i=1}^{n} \left(1 - (\mu_{e_i}^{U})^2 \right)} \right] \right. \\ \left[\Pi_{i=1}^{n} (v_e^{L}), \Pi_{i=1}^{n} (v_e^{U}) \right] \\ \left(\sqrt{1 - \Pi_{i=1}^{n} \left(1 - (\mu_{e_i})^2 \right)}, \prod_{i=1}^{n} (v_{e_e}) \right) \end{array} \right).$
[107]	Geometric	$\mathrm{SNWG}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_j^{\omega_j} = \left(\prod_{j=1}^{n} \rho_j^{\omega_j}, 1 - \prod_{j=1}^{n} (1 - \sigma_j)^{\omega_j}, 1 - \prod_{j=1}^{n} (1 - \tau_j)^{\omega_j} \right)$
[110]	arithmetic mean	$\mathbf{FPHFAM}\ (\aleph_1, \aleph_2, \dots, \aleph_r) = 1lr\aleph_1 \oplus 1lr\aleph_2 \oplus \cdots \oplus 1lr\aleph_r.$

		$= \left(\bigcup_{\hbar_j \in \tau_{\aleph_j}, \tilde{p}_{\aleph_j} \in \tilde{p}_{\aleph_j}} \sqrt[3]{1 - \frac{\prod_{j=1}^{r} \left(1-\left(\hbar_{\aleph_j}\right)^3\right)^{\frac{3}{r}}}{\prod_{j=1}^{r} \tilde{p}_{\aleph_j}}}, \frac{\bigcup_{\varrho_{\aleph_j}\in\partial_{\aleph_j}, b_{\aleph_j}\in b_{\aleph_j}} \prod_{j=1}^{r} \left(\varrho_{\aleph_j}\right)^{\frac{1}{r}}}{\prod_{j=1}^{r} b_{\aleph_j}} \right).$ $= \left(\bigcup_{\hbar_j \in \tau_{\aleph_j}, \tilde{p}_{\aleph_j} \in \tilde{p}_{\aleph_j}} \sqrt[3]{1 - \prod_{j=1}^{r} \left(1-\left(\hbar_{\aleph_j}\right)^3\right)^{w_j} / \prod_{j=1}^{r} \tilde{p}_{\aleph_j}}, \bigcup_{\varrho_{\aleph_j}\in\partial_{\aleph_j}, b_{\aleph_j}\in b_{\aleph_j}} \prod_{j=1}^{r} \left(\varrho_{\aleph_j}\right)^{w_j} / \prod_{j=1}^{r} b_{\aleph_j} \right)$
[53]	arithmetic mean	$\text{IVPFRWA}(g(b_1), g(b_1), \ldots, g(b_n)) = \left\{ \oplus_{i=1}^{n} w_i \underline{g}(b_i), \oplus_{i=1}^{n} w_i \bar{g}(b_i) \right\}$ $= \left\{ \begin{array}{l} \left\{ \left[\sqrt{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\mu}^{l}_{\tilde{A}_{si}}\right)^2\right)^{\frac{1}{n}}\right)}, \sqrt{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\mu}^{u}_{\tilde{A}_{si}}\right)^2\right)^{\frac{1}{n}}\right)} \right], \left[\prod_{i=1}^{n}\left(\underline{v}^{l}_{\tilde{A}_{si}}\right)^{\frac{1}{n}}, \prod_{i=1}^{n}\left(\underline{v}^{u}\tilde{A}_{si}\right)^{\frac{1}{n}} \right] \right\} \\ \left\{ \left[\sqrt{\left(1-\prod_{i=1}^{n}\left(1-\left(\bar{\mu}^{l}_{\tilde{A}_{si}}\right)^2\right)^{1/n}\right)}, \sqrt{\left(1-\prod_{i=1}^{n}\left(1-\left(\bar{\mu}^{u}_{\tilde{A}_{si}}\right)^2\right)^{1/n}\right)} \right], \left[\prod_{i=1}^{n}\left(\bar{v}^{l}_{\tilde{A}_{si}}\right)^{1/n}, \prod_{i=1}^{n}\left(\bar{v}^{u}\tilde{A}_{si}\right)^{1/n} \right] \right\} \end{array} \right.$
[42]	arithmetic mean	$NCNWAAA(s1, s2, \cdots, sn) = \left(\left[1 - \prod_{j=1}^{n} \left(1-T_j^{L}\right)^{w_j}, 1 - \prod_{j=1}^{n} \left(1-T_j^{U}\right)^{w_j} \right] \right.$ $\left. \left[\prod_{j=1}^{n} \left(I_j^{L}\right)^{w_j}, \prod_{j=1}^{n} \left(I_j^{U}\right)^{w_j} \right] \left[\prod_{i=1}^{n} \left(F_j^{L}\right)^{w_j}, \prod_{i=1}^{n} \left(F_j^{U}\right)^{w_j} \right], 1 - \prod_{j=1}^{n} \left(1-\lambda_{Tj}\right)^{w_j}, \prod_{j=1}^{n} \left(\lambda_{Ij}\right)^{w_j}, \prod_{j=1}^{n} \left(\lambda_{Fj}\right)^{w_j} \right)$ where $w = (w_1, w_2 \ldots, w_n)$; $w_i \in [0,1]$, $\sum_{i=1}^{n} w_i = 1$, and $w_i = 1/n$
[40]	arithmetic mean	$= \left(\bigcup_{\mu_{Q_i}\in\mathcal{M}_i, v_{Q_i}\in\mathcal{N}_i} \left\{ \left\{ \sqrt[q]{1 - \prod_{i=1}^{r} \left(1-\left(\mu_{Q_i}\right)^q\right)^{\frac{1}{r}}} \mid \prod_{i=1}^{r} p_{\mu_{Q_i}} \right\} \left\{ \prod_{i=1}^{r} \left(v_{Q_i}\right)^{\frac{1}{r}} \mid \prod_{i=1}^{r} p_{v_{Q_i}} \right\} \right\} \right)$
[130]	arithmetic mean	$\text{SWAM}_w\left(\tilde{A}_{S1}, \ldots, \tilde{A}_{Sn}\right) = w_1 \tilde{A}_{S1} + w_1 \tilde{A}_{S1} + \cdots + w_n \tilde{A}_{Sn}$ $= \left\{ \left[1 - \prod_{i=1}^{n} \left(1-\mu^2_{\tilde{A}_{Si}}\right)^{1/n} \right]^{1/2} \right.$ $\left. , \prod_{i=1}^{n} v_{\tilde{A}_{Si}}^{w_i}, \left[\prod_{i=1}^{n} \left(1-\mu^2_{\tilde{A}_{Si}}\right)^{1/n} - \prod_{i=1}^{n} \left(1-\mu^2_{\tilde{A}_{Si}} - \pi^2_{\tilde{A}_{Si}}\right)^{1/n} \right]^{1/2} \right\}$
[132]	arithmetic mean	$\text{DHFA}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right) = \oplus_{j=1}^{n} \left(\tilde{d}_j\right) = \bigcup_{\tilde{\gamma}_j\in\tilde{h}_j, \tilde{\eta}_j\in\tilde{g}_j} \left\{ \left\{ 1 - \prod_{j=1}^{n} (1-\tilde{\gamma}) \right\}, \left\{ \prod_{j=I}^{n} (\tilde{\eta}) \right\} \right\}$
[38]	Geometric+AVE RGE	$2\,\text{TLPFSWA}\left(\tilde{P}_1, \tilde{P}_2, \ldots, \tilde{P}_n\right) = \left\{ \Delta\left(K \sqrt[2]{1 - \prod_{i=1}^{n} \left(1 - \left(\frac{\Delta^{-1}(l_{u_i}, \mu_i)}{K}\right)^2\right)^{\omega_i}} \right), \Delta\left(K \prod_{i=1}^{n} \left(\frac{\Delta^{-1}(l_{v_i}, v_i)}{K}\right)^{\omega_i} \right) \right\}.$ The 2 TLPF weighting geometric operator:

		$2\,\text{TLPFSWG}(\tilde{P}_1,\tilde{P}_2,\ldots,\tilde{P}_n)=\left\{\Delta\left(\text{K}\prod_{i=1}^{n}\left(\frac{\Delta^{-1}(l_{u_i},\mu_i)}{\text{K}}\right)^{\omega_i}\right),\Delta\left(\text{K}\sqrt[2]{1-\prod_{i=1}^{n}\left(1-\left(\frac{\Delta^{-1}(l_{v_i},v_i)}{\text{K}}\right)^{2}\right)^{\omega_i}}\right)\right\}$
[37]	arithmetic mean	$\text{FFRA}(\gamma)=\frac{1}{n}(\gamma_1\oplus\gamma_2\oplus\cdots\oplus\gamma_n)=\left(\begin{array}{c}\left\{\left[1-\prod_{j=1}^{n}\left(1-\underline{\mu}_{\gamma_j}^{3}\right)^{\frac{1}{n}}\right]^{3},\prod_{j=1}^{n}\underline{v}_{\gamma_j}^{\frac{1}{n}}\right\},\\ \left\{\left[1-\prod_{j=1}^{n}\left(1-\bar{\mu}_{\gamma_j}^{3}\right)^{\frac{1}{n}}\right]^{3},\prod_{j=1}^{n}\bar{v}_{\gamma_j}^{\frac{1}{n}},\right\}\end{array}\right)$
[52]	As the same type of development	$DPFA(\gamma)=\frac{1}{n}(\gamma_1\oplus\gamma_2\oplus\cdots\oplus\gamma_n)=\left(\left[1-\prod_{j=1}^{n}\left(1-A_d(\varsigma)_{\gamma j}\right)^{\frac{1}{n}}\right]\prod_{j=1}^{n}S_d(\varsigma)_{\gamma j}^{\frac{1}{n}},\left[1-\prod_{j=1}^{n}\left(1-\alpha_{\gamma j}\right)^{\frac{1}{n}}\right],\prod_{j=1}^{n}\beta_{\gamma j}^{\frac{1}{n}}\right.$ $N=\left(\left(A_d(\varsigma),S_d(\varsigma)\right),\left(\alpha(\varsigma),\beta(\varsigma)\right)\right)=(<(A_d(\varsigma),S_d(\varsigma)>,<\alpha(\varsigma),\beta(\varsigma)>$ where $A_d(\varsigma),S_d(\varsigma),\alpha(\varsigma),\beta(\varsigma)\in[0,1],0\le\alpha(\varsigma)A_d(\varsigma)+\beta(\varsigma)S_d(\varsigma)\le 1$
[39]	arithmetic mean	Suppose $[\tilde{Z}_i]=[\tilde{Z}_i^L,\tilde{Z}_i^U]=[(\tilde{E}x_i^L,\tilde{E}n_i^L,\tilde{H}e_i^L),(\tilde{E}x_i^U,\tilde{E}n_i^U,\tilde{H}e_i^U)]$ (i= 1, 2, . . ., n) are n ZCRNs. The arithmetic operation of ZCRNs is defined as (17) $[\tilde{Z}_1]\oplus[\tilde{Z}_2]=[\tilde{Z}_1^L\oplus\tilde{Z}_2^L,\tilde{Z}_1^U\oplus\tilde{Z}_2^U]=\left[\begin{array}{c}\left(Ex_1^L+Ex_2^L,\sqrt{(En_1^L)^2+(En_2^L)^2},\sqrt{(He_1^L)^2+(He_2^L)^2}\right)\\ \left(Ex_1^U+Ex_2^U,\sqrt{(En_1^U)^2+(En_2^U)^2},\sqrt{(He_1^U)^2+(He_2^U)^2}\right)\end{array}\right]$
[46]	Average operator	$q-RPFWA(\mathbb{P}_1,\mathbb{P}_2,\cdots,\mathbb{P}_n)=\omega_1\mathbb{P}_1\oplus\omega_2\mathbb{P}_2\oplus\cdots\oplus\omega_n\mathbb{P}_n=$ $\left\{\left[1-\prod_{i=1}^{n}\left(1-\phi_{\text{p}}^{q}\right)^{\omega_i}\right]^{\frac{1}{q}},\prod_{i=1}^{n}\eta_p^{\omega_i},\prod_{i=1}^{n}\psi_{\mathbb{P}}^{\omega_i}\right\},\omega_i\in[0,1];\sum_{i=1}^{n}\omega_i=1$
[150]	averaging (PSVNHFA) operator	$\text{PSVNHFA}(N_1,N_2,\ldots,N_r)=\left(\begin{array}{c}\bigcup_{(\alpha_j)_{j=1,\ldots,r}\in T_1\times T_2\times\ldots\times T_r}\left(1-\prod_{j=1}^{r}(1-\alpha_j)^{\frac{1}{r}}\mid\Pi_{j=1}^{r}P_{\alpha_j}^{T_j}\right)\\ \bigcup_{(\beta_j)_{j=1,\ldots,r}\in I_1\times I_2\times\ldots\times I_r}\left(\prod_{j=1}^{r}(\beta_j)^{\frac{1}{r}\mid\Pi_{j=1}^{r}P_{\beta_j}^{I_j}}\right),\bigcup_{(\gamma_j)_{j=1,\ldots,r}\in F_1\times F_2\times\ldots\times F_r}\left(\prod_{j=1}^{r}(\gamma_j)^{\frac{1}{r}}\mid\Pi_{j=1}^{r}P_{\gamma_j}^{F_j}\right)\end{array}\right)$
[41]	arithmetic mean	$\mathbf{IvSWAM}\,M_w(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots\ldots,\tilde{\alpha}_n)=w_1\cdot\tilde{\alpha}_1\oplus w_2\cdot\tilde{\alpha}_2\oplus\cdots\oplus w_n\cdot\tilde{\alpha}_n=$ $\left\{\begin{array}{c}\left[\left(1-\prod_{j=1}^{n}\left(1-a_j^2\right)^{w_j}\right)^{\frac{1}{2}},\left(1-\prod_{j=1}^{n}\left(1-b_j^2\right)^{w_j}\right)^{\frac{1}{2}}\right]\\ \left[\prod_{j=1}^{n}c_j^{w_j},\prod_{j=1}^{n}d_j^{w_j}\right]\\ \left[\begin{array}{c}\left(\prod_{j=1}^{n}\left(1-a_j^2\right)^{w_j}-\prod_{j=1}^{n}\left(1-a_j^2-e_j^2\right)^{w_j}\right)^{\frac{1}{2}}\\ \left(\prod_{j=1}^{n}\left(1-b_j^2\right)^{w_j}-\prod_{j=1}^{n}\left(1-b_j^2-f_j^2\right)^{w_j}\right)^{\frac{1}{2}}\end{array}\right]\end{array}\right\}$

[48]	arithmetic mean	$IFA = (A_1, A_2, \dots, A_m) = \left(1 - \prod_{i=1}^{m} (1 - m_{A_i}), \prod_{i=1}^{m} n_{A_i}\right)$
[156]	The algebraic arithmetic	$PA(p_1, \cdots, p_n) = \left\langle \sqrt{1 - \prod_{i=1}^{n} (1 - u_i^2)^{\frac{1}{n}}}, \prod_{i=1}^{n} v_i^{\frac{1}{n}} \right\rangle$

Figure 4 shows the variety of aggregation operators available to tackle MCDM issues from 2021 to 2023, such as Geometric Mean (GM), Bonferroni Mean (BM), Arithmetic Mean (AM), and others. The results presented in Figure 4 was generated through simulations of the data conducted using Microsoft Office Excel.

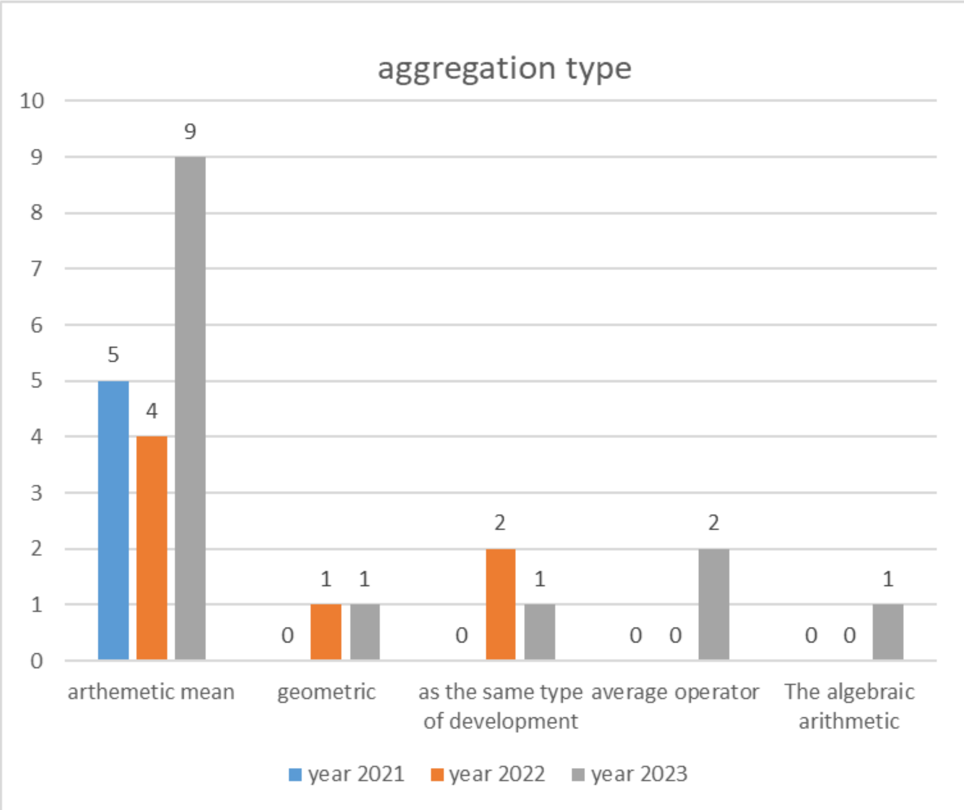

FIGURE 4. - The Aggregation Operators for Fuzzy Types

3.3 The Integration Method

To tackle numerous complex MCDM problems in the studies. FWZIC outperformed other MCDM weighting methods, due to its ability to determine the weights of the criteria with zero inconsistency [110]. several scholars in the academic literature have recently concentrated on integrating them with MCDM approaches [36] and other techniques to prioritize the list of the criteria and to identifying the alternatives and evaluation criteria and to address the ambiguity, uncertainty and vagueness issues [51]. these integration of a new formulation of the FWZIC and other methods can provide a dynamic distribution mechanism for priorities, successfully overcoming the inconsistency problem and the distance measurement [25]. Table 29 provides an overview of FWZIC methods which have been integrated with another ranking method to achieve the goals associated with these development studies.

Table 29. - Integrated Methods with FWZIC

	Ref.	Fuzzy type	The Integrated method	year
1.	[51]	P-H-FWZIC. probabilistic hesitant fuzzy set-	Fawzic + MULTIMOORA	2022

		fuzzy weighted zero-inconsistency	(multiplicative multi-objective Optimisation by ratio analysis)	
2.	[3]	spherical FWZIC (S-FWZIC).	Fawzic + GRA-TOPSIS (grey relational analysis–technique for order of preference by similarity to ideal solution	2022
3.	[43] [160] [161]	q-rung orthopair fuzzy-weighted zero-inconsistency (q-ROFWZIC)	Fawzic + FDOSM	2021,2022,2023
4.	[25]	Pythagorean fuzzy-weighted zero-inconsistency PFWZIC.	Fawzic + FDOSM.	2021
5.	[21]	Pythagorean probabilistic hesitant fuzzy sets and fuzzy weighted zero inconsistency (PPH–FWZIC)	Fawzic + MARCOS (measurement of alternatives and ranking according to the compromise solution)	2022
6.	[45]	T-spherical FWZIC	Fawzic + FDOSM	2021
7.	[95]	interval type 2 trapezoidal-fuzzy weighted with zero inconsistency (IT2TR-FWZIC)	Fawzic +VIKOR (VIekriterijumsko KOmpromisno Rangiranje)	2021
8.	[100]	Cubic Pythagorean CP-FWZIC	Fawzic + FDOSM	2021
9.	[107]	neutrosophic FWZIC (NS-FWZIC	Fawzic + FDOSM	2022
10.	[110]	Fermatean probabilistic hesitant fuzzy weighted zero inconsistency FPH–FWZIC	Fawzic + FDOSM+multi attributive ideal-real comparative analysis (MAIRCA)	2023
11.	[53]	interval-valued Pythagorean fuzzy rough set IVPFRS–FWZIC	(Fawzic + EDAS) evaluation based on distance from average solution	2023
12.	[42]	neutrosophic cubic sets NCS–FWZIC	(Fawzic + MABAC) multi-attributive border	2023

			approximation area comparison	
13.	[40]	q-rung orthopair probabilistic hesitant fuzzy set q-ROPHFS–FWZIC	Fawzic + FDOSM+ MULTIMOORA	2023
14.	[130]	Spherical Fuzzy Rough-Weighted Zero-Inconsistency (SFR-WZIC),	Fawzic + FDOSM	2023
15.	[132]	dual hesitant fuzzy weighted zero inconsistency (DH-FWZIC)	Fawzic + FDOSM	2022
16.	[38]	2-tuple linguistic Pythagorean fuzzy-weighted zero-inconsistency (2 TLP-FWZIC)	(Fawzic + MABAC) Modified multi-attributive border approximation area comparison	2023
17.	[37]	rough Fermatean fuzzy sets RF-FWZIC,	Fawzic + FDOSM	2023
18.	[52]	Diophantine linear fuzzy sets LDFS-FWZIC	Fawzic + MULTIMOORA	2023
19.	[39]	Z-Cloud Rough Numbers (ZCRNs) environment	Fawzic + FDOSM	2023
20.	[46]	q-rung picture	Fawzic +simple additive weighting (SAW)	2023
21.	[150]	probabilistic single-valued neutrosophic hesitant fuzzy set (PSVNHFS)	Fawzic + (DLBD) dynamic localisation-based decision	2023
22.	[41]	interval-valued spherical fuzzy sets (IvSFSs)	Fawzic+ COPRAS (complex proportional assessment)	2023
23.	[48]	Fwzic II intuitionistic fuzzy set (IFS)	Fawzic + FDOSM	2022
24.	[156]	circular Pythagorean fuzzy sets (C-PFSs)	Fawzic + CPOS (conditional probabilities by	2023

opinion scores)

Figure 5 show a comprehensive view of all presented methodology which integrated with FWZIC Method to solve different challenging MCDM issues in various research according to the years of publication from 2020 to 2023.

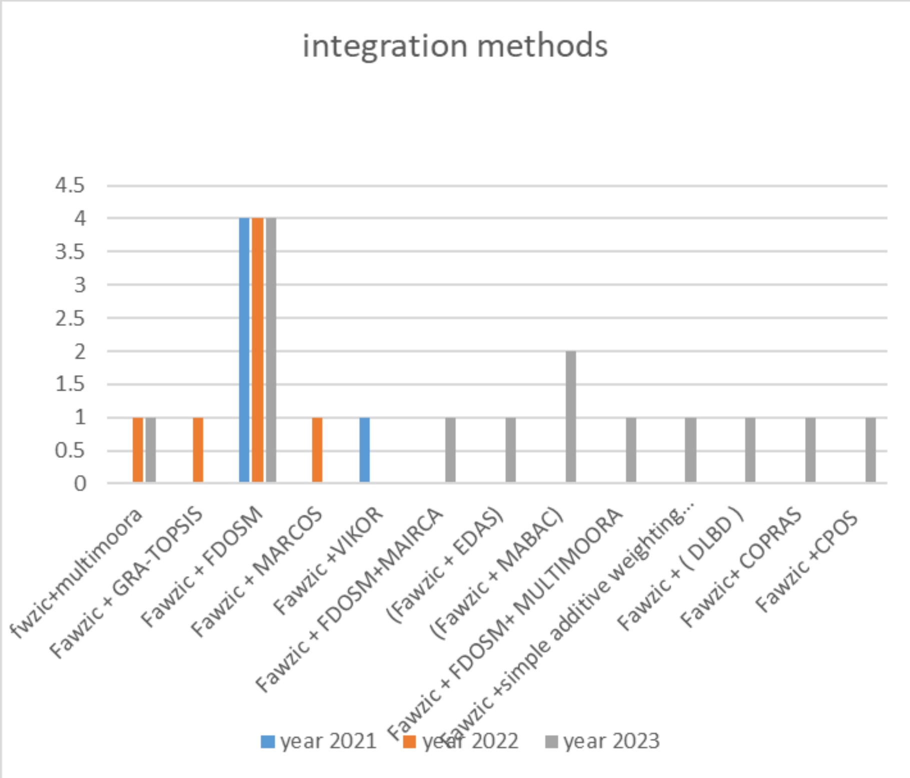

FIGURE 5. - Several Included Articles in Different Categories by Year of Publication

3.4 Case Study

Case studies demonstrate how MCDM approaches may help decision-makers make better informed, transparent, and defensible decisions in a variety of real-world circumstances. Multiple methods have been used to solve complicated decision problems across a wide range of areas. technical related and the others are related to medical cases. Figure 6 illustrates the number of various case studies used in these studies.

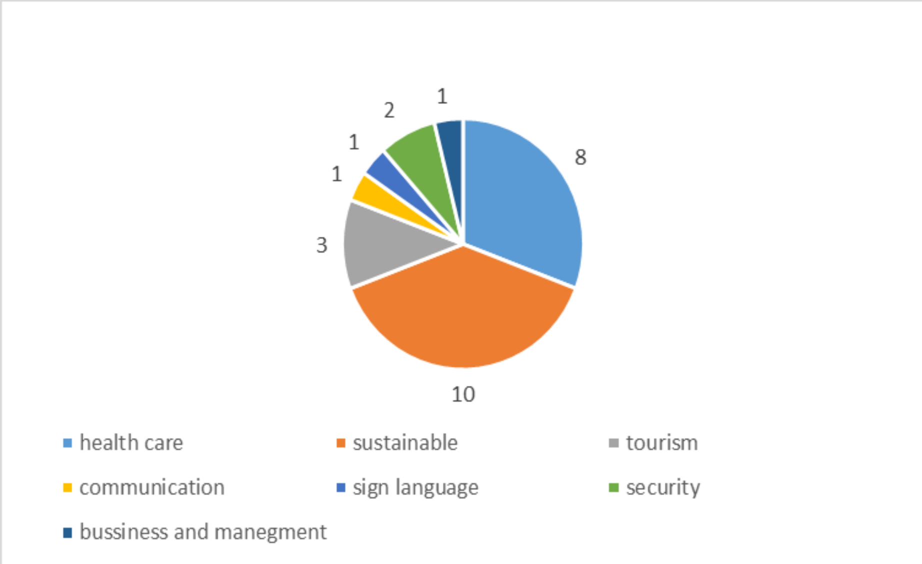

FIGURE 6. - Various Case Studies Used in These Papers

4. Recommendations

In this section has present a summary of recommendations were mentioned in the final set articles. Several sub-sections reported below:

4.1. Advancements and Integration in FWZIC and FDOSM

Several researchers have suggested that FWZIC be extended to various fuzzy environments, such as the interval-valued Fermatean fuzzy rough set and soft hesitant fuzzy rough set [51] [3] [21] [95] [53] [52] [41]. Other studies suggest extending both FWZIC and FDOSM under another type of fuzzy set, such as Gaussian and complex interval-valued Pythagorean fuzzy set [25] [45] [100] [107] [160] [132] [39]. In the same way, literature [48] suggests adopting other Intuitionistic FS (IFS) families with FWZIC II and FDOSM II, such as spherical fuzzy sets, T-spherical fuzzy sets, or Fermatean fuzzy sets. Likewise, Additional studies suggest implementing an alternative fuzzy set, such as complex neutrosophic, neutrosophic cubic hesitant, or neutrosophic soft set with FWZIC [42]. Also, extending FDOSM under the probabilistic single-valued neutrosophic hesitant (PSVNH) environment [150], all of these recommendations aim to compare and assess whether these types can effectively handle the vagueness problem and improve the final judgment with greater certainty and accuracy. For using other linguistic scales (Likert scales) (e.g., 7, 10, or 11), a recommendation is suggested by these studies to evaluate the proposed method's suitability, construct the Expert Decision Matrix (EDM), and create positive and negative opinion matrices [51] [3] [21] [100] [107] [160] [53] [110] [132] [37] [156]. Furthermore, despite their level of experience, all experts received treatment equally. Hence, Literature[51] [21] [107] [160] [110] [53] [42] [40] [132] [38] [52] suggest giving the experts a certain amount of effect based on their knowledge that can be used in determining the criteria weights and providing more reasonable results. Following that, expert weights may be considered in the proposed studies [100] [37] for extending this research and implementing the methodologies to other types and circumstances of MCDM problems utilizing various case studies and alternatives. In addition, two research focused on building interval-valued Pythagoreanfuzzy rough set fuzzy weighted with zero inconsistency (IVPFRS-FWZIC) [53] and Diophantine linear fuzzy sets fuzzy weighted with zero inconsistency (LDFS-FWZIC) [52] to solve problems of unreliable, imprecise, and incomplete data. Similarly, these studies combine evaluation based on distance from average solution (EDAS) with interval-valued Pythagorean fuzzy rough set (IVPFRS) [53] to address the ambiguity issue and Multiplicative multiple objective optimisation by ratio analysis (MULTIMOORA) with Diophantine linear fuzzy sets(LDFS) [52]. On the other hand, additional MCDM ranking techniques have the potential to be integrated with the interval type 2 trapezoidal-fuzzy weighted with zero inconsistency (IT2TR-FWZIC) method and used to examine the recently revealed benchmarking outcomes [95]. Various studies recommend utilizing more than one aggregation and defuzzification technique to provide the final weighting attributes like in FWZIC[3] [21] [53] [42] [132] [41] or with both FWZIC and the ranking alternatives in FDOSM [110] [40] [130] [38] [150] [48] or only with MULTIMOORA [40] or with MULTIMOORA and FWZIC [52]. Other studies recommend using other aggregation operators with FWZIC [51] [160] or for the ranking alternatives in FDOSM [107]. Similarly, some studies suggest using the application of different defuzzification methods to weigh criteria for FWZIC [95], with both FWZIC and complex proportional assessment (CPOS) [156] or only for FDOSM for the ranking alternatives [132] or for both FWZIC and FDOSM[107] [160] .

4.2. Future Research and Development: Expanding Horizons for FWZIC and FDOSM

Literature [43] [25] [45] recommend further research and development of FWZIC and FDOSM as follows: (1) Providing and processing a large-scale dataset of COVID-19 vaccine recipients, taking into account all probabilities that are often increased for each alternative and distribution criterion. (2) Implementing the proposed MCDM methods on two levels: first, each vaccination recipient membership will be prioritized, and then, each alternative inside each membership will be prioritized, followed by effective accumulation. Another article [51] suggests using the proposed methodology to evaluate and benchmark any future approaches in the transportation industry, choice of portfolio based on firm financial performance, observation process modeling in the context of cognition processes, and shape memory alloy wire actuators. Furthermore, the suggested framework may be utilized to benchmark innovative systems in other categories of healthcare Industry 4.0 systems, and research [3]recommends integrating additional MCDM ranking methods with S-FWZIC to investigate new benchmarking outcomes. To solve the uncertainty issue, literature [21] also suggests applying the suggested method for comparing potential future fuel supply system modeling approaches (FSSMAs) for electric vehicle (EV) in the transportation industry and extending MARCOS to fuzzy environments. Moreover, article [95] provides numerous recommendations First, the proposed decision-making framework can be utilized with any future category of smart e-tourism data management apps to assess and benchmark the new applications. Secondly, for improved variation in the data of smart e-tourism data management apps, the twelve important criteria might be assessed using a five-point Likert scale. Also, the proposed method recommended by [160] can be used to benchmark any future possible energy systems in the transportation industry. As with paper the [110], an alternative methodology can be specified and used, which ranks alternatives based on median similarity (RAMS) and selects the best one. RAMS is an extension of the most recently developed technique that used perimeter similarity (RAPS). On this basis, it can be used as a further tool that combines the RAMS method with the multiple criteria ranking by alternative trace (MCRAT) methodology using a majority index and the concept of the VIKOR method. The trace to median index (RATMI) is used to rank the alternatives using this tool. For the selection problem, an illustration of the usage of RAMS and RATMI will be applicable by evaluating the agriculture food 4.0 supply chain in different environments. Similarly, Research [40] suggests developing a comprehensive assessment based on the connection of Construction and demolition waste (CDW) management strategies and the driver with barrier attributes of reuse redistribution. Besides this, a study [132] indicates that fuzzy failure mode impact analysis can be used to weight pavement criteria. Furthermore, [156] recommends future research that provides more comprehensive and practical proposals for ranking and grading SSL systems. Through another divergence method, frameworks an alternative approach might be used to derive the conditional probabilities and threshold rules for TWD. Moreover, the C-PFS-CPOS method can solve problems with missing or absent data and immeasurable factors such as binomial factors (yes/no responses), polynomial factors (such as color gradations), textual factors (such as brand names), and categorical factors (interval values or ranges). Finally, research (45) suggests that FWZIC II and FDOSM II be used to benchmark the numerous security and privacy features for intelligent medical systems based on federated learning and blockchain technology.

5. CONCLUSIONS AND FUTURE WORK

Researchers across diverse disciplines have consistently employed Multi-Criteria Decision-Making (MCDM) methods to enhance their respective fields, utilizing both conventional and innovative approaches. The selection of a weighting mechanism for evaluation criteria is crucial in addressing MCDM problems. Recognizing the importance of staying abreast of methodological advancements, this study undertook a comprehensive review of various innovative methods integrated with FWZIC. The analysis involved scrutinizing papers retrieved from prominent databases, namely IEEE Xplore, ScienceDirect, Scopus, and PubMed, spanning from August 23, 2023, to October 30, 2023. A total of 26 articles were meticulously chosen based on predefined inclusion and exclusion criteria for this systematic review. Utilizing bibliometric and content analysis, the study explored emerging trends associated with FWZIC, including study components such as sources, authors, countries, affiliations, areas of application, case studies, fuzzy implementations, hybrid studies (involving other weighting methods), and application tools for these methods. The findings of this literature systematic review (LSR) offer a comprehensive overview of each new development related to the weighting method and its applications. As a results for our research: 1- Extracting the development types that are employed in the FWZIC approach based on the Fuzzy Set, 2- Extracting aggregation operator types, 3- Integration Method with FWZIC (hybrid with other methods), and 4- Case studies types that show how MCDM approaches may help decision-makers in a variety of decisions. In conclusion, this research contributes valuable insights and expertise, making it a beneficial resource for academics and practitioners working in the domain of multi-criteria decision-making. For future directions, Extend FWZIC to include different fuzzy types, such as interval-valued intuitionistic fuzzy rough sets and soft hesitant fuzzy rough sets, to compare their effectiveness in addressing uncertainty and ambiguity problems. Also, apply these developed fuzzy sets to other case studies, by implementing different aggregation operators, and compare them to the operators employed in these studies.

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CONFLICTS OF INTEREST

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REFERENCES

[1] M. Hadid, Q. M. Hussein, Z. Al-Qaysi, M. Ahmed, M. M. J. I. J. F. C. S. Salih, and Mathematics, "An Overview of Content-Based Image Retrieval Methods And Techniques," vol. 4, no. 3, pp. 66-78, 2023.

[2] A. D. Ahmed, M. M. Salih, and Y. R. J. I. A. Muhsen, "Opinion Weight Criteria Method (OWCM): A New Method for Weighting Criteria with Zero Inconsistency," 2024.

[3] S. Qahtan *et al.*, "Novel multi security and privacy benchmarking framework for blockchain-based IoT healthcare industry 4.0 systems," *IEEE Transactions on Industrial Informatics*, vol. 18, no. 9, pp. 6415-6423, 2022.

[4] M. Tao and X. Wang, "An Integrated MCDM Model for Sustainable Course Planning: An Empirical Case Study in Accounting Education," *Sustainability*, vol. 15, no. 6, p. 5024, 2023.

[5] N. F. Silva, M. dos Santos, C. F. S. Gomes, and L. P. de Andrade, "An integrated CRITIC and Grey Relational Analysis approach for investment portfolio selection," *Decision Analytics Journal*, p. 100285, 2023.

[6] B. Ayan, S. Abacıoğlu, and M. P. Basilio, "A Comprehensive Review of the Novel Weighting Methods for Multi-Criteria Decision-Making," *Information*, vol. 14, no. 5, p. 285, 2023.

[7] R. Bhardwaj and S. Garg, "An MCDM Approach to Analytically Identify the Air Pollutants' Impact on Health," *Atmosphere*, vol. 14, no. 6, p. 909, 2023.

[8] C.-N. Wang, F.-C. Yang, T. M. N. Vo, V. T. T. Nguyen, and M. Singh, "Enhancing efficiency and cost-effectiveness: A groundbreaking bi-algorithm MCDM approach," *Applied Sciences*, vol. 13, no. 16, p. 9105, 2023.

[9] H. Taherdoost and M. Madanchian, "Multi-criteria decision making (mcdm) methods and concepts. Encyclopedia 3 (1): 77–87," ed, 2023.

[10] A. Balali, A. Yunusa-Kaltungo, and R. Edwards, "A systematic review of passive energy consumption optimisation strategy selection for buildings through multiple criteria decision-making techniques," *Renewable and Sustainable Energy Reviews*, vol. 171, p. 113013, 2023.

[11] M. S. Al-Samarraay *et al.*, "A new extension of FDOSM based on Pythagorean fuzzy environment for evaluating and benchmarking sign language recognition systems," *Neural Computing and Applications*, pp. 1-19, 2022.

[12] M. H. Jasim *et al.*, "Emotion detection among Muslims and non-Muslims while listening to Quran recitation using EEG," vol. 9, p. 14, 2019.

[13] S. J. Ghoushchi *et al.*, "Assessing Sustainable Passenger Transportation Systems to Address Climate Change Based on MCDM Methods in an Uncertain Environment," *Sustainability*, vol. 15, no. 4, p. 3558, 2023.

[14] M. L. Shuwandy *et al.*, "Sensor-Based Authentication in Smartphone; a Systematic Review," 2024.

[15] J.-F. Tsai, S.-P. Shen, and M.-H. Lin, "Applying a Hybrid MCDM Model to Evaluate Green Supply Chain Management Practices," *Sustainability*, vol. 15, no. 3, p. 2148, 2023.

[16] R. A. Aljanabi, Z. Al-Qaysi, M. Ahmed, M. M. J. I. J. F. C. S. Salih, and Mathematics, "Hybrid Model for Motor Imagery Biometric Identification," vol. 5, no. 1, pp. 1-12, 2024.

[17] Z. Al-Qaysi, M. M. Salih, M. L. Shuwandy, M. Ahmed, Y. S. J. A. D. S. Altarazi, and Analysis, "Multi-Tiered CNN Model for Motor Imagery Analysis: Enhancing UAV Control in Smart City Infrastructure for Industry 5.0," vol. 2023, pp. 88-101, 2023.

[18] S. U. Selvan *et al.*, "Toward multi-species building envelopes: A critical literature review of multi-criteria decision-making for design support," *Building and Environment*, p. 110006, 2023.

[19] A. Indelicato, J. C. Martín, and R. Scuderi, "A comparison of attitudes towards immigrants from the perspective of the political party vote," *Heliyon*, vol. 9, no. 3, 2023.

[20] A. Rasmussen, H. Sabic, S. Saha, and I. E. Nielsen, "Supplier selection for aerospace & defense industry through MCDM methods," *Cleaner Engineering and Technology*, vol. 12, p. 100590, 2023.

[21] S. Qahtan, H. A. Alsattar, A. Zaidan, M. Deveci, D. Pamucar, and W. Ding, "A novel fuel supply system modelling approach for electric vehicles under Pythagorean probabilistic hesitant fuzzy sets," *Information Sciences,* vol. 622, pp. 1014-1032, 2023.
[22] M. M. Salih, B. Zaidan, and A. Zaidan, "Fuzzy decision by opinion score method," *Applied Soft Computing,* vol. 96, p. 106595, 2020.
[23] R. M. Maher, M. M. Salih, H. A. Hussein, and M. A. Ahmed, "A New Development of FDOSM Based on a 2-Tuple Fuzzy Environment: Evaluation and Benchmark of Network Protocols as a Case Study," *Computers,* vol. 11, no. 7, p. 109, 2022.
[24] X. Chew, K. W. Khaw, A. Alnoor, M. Ferasso, H. Al Halbusi, and Y. R. Muhsen, "Circular economy of medical waste: novel intelligent medical waste management framework based on extension linear Diophantine fuzzy FDOSM and neural network approach," *Environmental Science and Pollution Research,* pp. 1-27, 2023.
[25] O. Albahri *et al.*, "Novel dynamic fuzzy decision-making framework for COVID-19 vaccine dose recipients," *Journal of advanced research,* vol. 37, pp. 147-168, 2022.
[26] M. Alaa *et al.*, "Assessment and ranking framework for the English skills of pre-service teachers based on fuzzy Delphi and TOPSIS methods," *IEEE access,* vol. 7, pp. 126201-126223, 2019.
[27] K. H. Abdulkareem *et al.*, "A novel multi-perspective benchmarking framework for selecting image dehazing intelligent algorithms based on BWM and group VIKOR techniques," *International Journal of Information Technology & Decision Making,* vol. 19, no. 03, pp. 909-957, 2020.
[28] M. M. Salih, B. Zaidan, A. Zaidan, and M. A. Ahmed, "Survey on fuzzy TOPSIS state-of-the-art between 2007 and 2017," *Computers & Operations Research,* vol. 104, pp. 207-227, 2019.
[29] M. Alsalem *et al.*, "Multi-criteria decision-making for coronavirus disease 2019 applications: a theoretical analysis review," *Artificial Intelligence Review,* vol. 55, no. 6, pp. 4979-5062, 2022.
[30] A. N. Jasim, L. C. Fourati, and O. Albahri, "Evaluation of Unmanned Aerial Vehicles for Precision Agriculture Based on Integrated Fuzzy Decision-Making Approach," *IEEE Access,* 2023.
[31] A. Alamoodi *et al.*, "Systematic review of MCDM approach applied to the medical case studies of COVID-19: trends, bibliographic analysis, challenges, motivations, recommendations, and future directions," *Complex & intelligent systems,* pp. 1-27, 2023.
[32] R. Mohammed *et al.*, "Determining importance of many-objective optimisation competitive algorithms evaluation criteria based on a novel fuzzy-weighted zero-inconsistency method," *International Journal of Information Technology & Decision Making,* vol. 21, no. 01, pp. 195-241, 2022.
[33] N. Basil, M. Alqaysi, M. Deveci, A. Albahri, O. Albahri, and A. Alamoodi, "Evaluation of autonomous underwater vehicle motion trajectory optimization algorithms," *Knowledge-Based Systems,* p. 110722, 2023.
[34] M. Alqaysi, A. Albahri, and R. A. Hamid, "Hybrid diagnosis models for autism patients based on medical and sociodemographic features using machine learning and multicriteria decision-making (MCDM) techniques: An evaluation and benchmarking framework," *Computational and Mathematical Methods in Medicine,* vol. 2022, 2022.
[35] S. S. Joudar, A. Albahri, and R. A. Hamid, "Triage and priority-based healthcare diagnosis using artificial intelligence for autism spectrum disorder and gene contribution: a systematic review," *Computers in Biology and Medicine,* vol. 146, p. 105553, 2022.
[36] S. Qahtan, K. Yatim, H. Zulzalil, M. H. Osman, A. Zaidan, and H. Alsattar, "Review of healthcare industry 4.0 application-based blockchain in terms of security and privacy development attributes: Comprehensive taxonomy, open issues and challenges and recommended solution," *Journal of Network and Computer Applications,* vol. 209, p. 103529, 2023.
[37] O. Albahri *et al.*, "Rough Fermatean fuzzy decision-based approach for modelling IDS classifiers in the federated learning of IoMT applications," *Neural Computing and Applications,* pp. 1-19, 2023.
[38] O. Albahri *et al.*, "Multi-perspective evaluation of integrated active cooling systems using fuzzy decision making model," *Energy Policy,* vol. 182, p. 113775, 2023.
[39] Y. R. Muhsen, N. A. Husin, M. B. Zolkepli, N. Manshor, and A. A. J. Al-Hchaimi, "Evaluation of the Routing Algorithms for NoC-Based MPSoC: A Fuzzy Multi-Criteria Decision-Making Approach," *IEEE Access,* 2023.
[40] H. Ghailani *et al.*, "Developing sustainable management strategies in construction and demolition wastes using a q-rung orthopair probabilistic hesitant fuzzy set-based decision modelling approach," *Applied Soft Computing,* vol. 145, p. 110606, 2023.
[41] A. Zaidan, H. Alsattar, S. Qahtan, M. Deveci, D. Pamucar, and B. B. Gupta, "Secure Decision Approach for Internet of Healthcare Things Smart Systems-Based Blockchain," *IEEE Internet of Things Journal,* 2023.
[42] S. Qahtan, A. A. Zaidan, H. A. Ibrahim, M. Deveci, W. Ding, and D. Pamucar, "A decision modeling approach for smart training environment with motor Imagery-based brain computer interface under neutrosophic cubic fuzzy set," *Expert Systems with Applications,* vol. 224, p. 119991, 2023.
[43] A. S. Albahri *et al.*, "Integration of fuzzy-weighted zero-inconsistency and fuzzy decision by opinion score methods under a q-rung orthopair environment: a distribution case study of COVID-19 vaccine doses," *Computer Standards & Interfaces,* vol. 80, p. 103572, 2022.

[44] M. Al-Samarraay *et al.*, "An integrated fuzzy multi-measurement decision-making model for selecting optimization techniques of semiconductor materials," *Expert Systems with Applications,* vol. 237, p. 121439, 2024.

[45] M. Alsalem *et al.*, "Based on T-spherical fuzzy environment: a combination of FWZIC and FDOSM for prioritising COVID-19 vaccine dose recipients," *Journal of infection and public health,* vol. 14, no. 10, pp. 1513-1559, 2021.

[46] O. Albahri *et al.*, "Evaluation of organizational culture in companies for fostering a digital innovation using q-rung picture fuzzy based decision-making model," *Advanced Engineering Informatics,* vol. 58, p. 102191, 2023.

[47] A. ALSEREIDI, "Federal Multi Criteria Decision Making Framework in Distribution of Anti-SARS-CoV-2 Monoclonal Antibody to Eligible High-Risk Patients as Case Study," The British University in Dubai (BUiD), 2022.

[48] U. Mahmoud *et al.*, "DAS benchmarking methodology based on FWZIC II and FDOSM II to support industrial community characteristics in the design and implementation of advanced driver assistance systems in vehicles," *Journal of Ambient Intelligence and Humanized Computing,* vol. 14, no. 9, pp. 12747-12774, 2023.

[49] S. S. Joudar, A. Albahri, and R. A. Hamid, "Intelligent triage method for early diagnosis autism spectrum disorder (ASD) based on integrated fuzzy multi-criteria decision-making methods," *Informatics in Medicine Unlocked,* vol. 36, p. 101131, 2023.

[50] Z. Al-qaysi, A. Albahri, M. Ahmed, M. M. J. N. C. Salih, and Applications, "Dynamic decision-making framework for benchmarking brain–computer interface applications: a fuzzy-weighted zero-inconsistency method for consistent weights and VIKOR for stable rank," pp. 1-24, 2024.

[51] S. Qahtan, H. Alsattar, A. Zaidan, D. Pamucar, and M. Deveci, "Integrated sustainable transportation modelling approaches for electronic passenger vehicle in the context of industry 5.0," *Journal of Innovation & Knowledge,* vol. 7, no. 4, p. 100277, 2022.

[52] Z. Mohammed *et al.*, "Bitcoin network-based anonymity and privacy model for metaverse implementation in Industry 5.0 using linear Diophantine fuzzy sets," *Annals of Operations Research,* pp. 1-41, 2023.

[53] H. A. Ibrahim, A. A. Zaidan, S. Qahtan, and B. B. Zaidan, "Sustainability assessment of palm oil industry 4.0 technologies in a circular economy applications based on interval-valued Pythagorean fuzzy rough set-FWZIC and EDAS methods," *Applied Soft Computing,* vol. 136, p. 110073, 2023.

[54] A. H. Alamoodi *et al.*, "Sentiment analysis and its applications in fighting COVID-19 and infectious diseases: A systematic review," *Expert systems with applications,* vol. 167, p. 114155, 2021.

[55] Z. Al-Qaysi *et al.*, "Systematic review of training environments with motor imagery brain–computer interface: coherent taxonomy, open issues and recommendation pathway solution," *Health and Technology,* vol. 11, no. 4, pp. 783-801, 2021.

[56] L. A. Zadeh, "Fuzzy sets," *Information and control,* vol. 8, no. 3, pp. 338-353, 1965.

[57] D. Simić, I. Kovačević, V. Svirčević, and S. Simić, "50 years of fuzzy set theory and models for supplier assessment and selection: A literature review," *Journal of Applied Logic,* vol. 24, pp. 85-96, 2017.

[58] I. A. Najm, M. Ismail, J. Lloret, K. Z. Ghafoor, B. Zaidan, and A. A.-r. T. Rahem, "Improvement of SCTP congestion control in the LTE-A network," *Journal of Network and Computer Applications,* vol. 58, pp. 119-129, 2015.

[59] F. Al-Sharqi, A. Al-Quran, M. U. J. I. J. f. C. S. Romdhini, and Mathematics, "Decision-making techniques based on similarity measures of possibility interval fuzzy soft environment," vol. 4, no. 4, pp. 18-29, 2023.

[60] L. Abdullah, "Fuzzy multi criteria decision making and its applications: a brief review of category," *Procedia-Social and Behavioral Sciences,* vol. 97, pp. 131-136, 2013.

[61] V. Torra, "Hesitant fuzzy sets," *International journal of intelligent systems,* vol. 25, no. 6, pp. 529-539, 2010.

[62] R. M. Rodríguez, L. Martínez, V. Torra, Z. Xu, and F. Herrera, "Hesitant fuzzy sets: state of the art and future directions," *International journal of intelligent systems,* vol. 29, no. 6, pp. 495-524, 2014.

[63] B. Zhu, "Decision method for research and application based on preference relation," *Southeast University, Nanjing,* 2014.

[64] Z. Xu and W. Zhou, "Consensus building with a group of decision makers under the hesitant probabilistic fuzzy environment," *Fuzzy Optimization and Decision Making,* vol. 16, pp. 481-503, 2017.

[65] F. Jiang and Q. Ma, "Multi-attribute group decision making under probabilistic hesitant fuzzy environment with application to evaluate the transformation efficiency," *Applied Intelligence,* vol. 48, pp. 953-965, 2018.

[66] F. Kutlu Gündoğdu and C. Kahraman, "Spherical fuzzy sets and spherical fuzzy TOPSIS method," *Journal of intelligent & fuzzy systems,* vol. 36, no. 1, pp. 337-352, 2019.

[67] M. Mathew, R. K. Chakrabortty, and M. J. Ryan, "A novel approach integrating AHP and TOPSIS under spherical fuzzy sets for advanced manufacturing system selection," *Engineering Applications of Artificial Intelligence,* vol. 96, p. 103988, 2020.

[68] F. Kutlu Gündoğdu and C. Kahraman, "Optimal site selection of electric vehicle charging station by using spherical fuzzy TOPSIS method," in *Decision making with spherical fuzzy sets: theory and applications*: Springer, 2020, pp. 201-216.
[69] R. R. Yager, "Generalized orthopair fuzzy sets," *IEEE Transactions on Fuzzy Systems,* vol. 25, no. 5, pp. 1222-1230, 2016.
[70] A. Hussain, M. Ali, and T. Mahmood, "Hesitant q-rung orthopair fuzzy aggregation operators with their applications in multi-criteria decision making," *Iranian Journal of Fuzzy Systems,* vol. 17, no. 3, pp. 117-134, 2020.
[71] Z. Pawlak, "Rough sets," *International journal of computer & information sciences,* vol. 11, pp. 341-356, 1982.
[72] S. Ashraf, N. Rehman, A. Hussain, H. AlSalman, and A. H. Gumaei, "q-rung orthopair fuzzy rough einstein aggregation information-based EDAS method: applications in robotic agrifarming," *Computational Intelligence and Neuroscience,* vol. 2021, 2021.
[73] H. Wang, Y. Zhang, and J. Yao, "An extended VIKOR method based on q-rung orthopair shadowed set and its application to multi-attribute decision making," *Symmetry,* vol. 12, no. 9, p. 1508, 2020.
[74] P. Liu and P. Wang, "Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making," *International Journal of Intelligent Systems,* vol. 33, no. 2, pp. 259-280, 2018.
[75] R. R. Yager, "Pythagorean membership grades in multicriteria decision making," *IEEE Transactions on fuzzy systems,* vol. 22, no. 4, pp. 958-965, 2013.
[76] M. Akram and G. Ali, "Hybrid models for decision-making based on rough Pythagorean fuzzy bipolar soft information," *Granular Computing,* vol. 5, pp. 1-15, 2020.
[77] M. Akram and S. Naz, "Energy of Pythagorean fuzzy graphs with applications," *Mathematics,* vol. 6, no. 8, p. 136, 2018.
[78] L. Wang and N. Li, "Pythagorean fuzzy interaction power Bonferroni mean aggregation operators in multiple attribute decision making," *International Journal of Intelligent Systems,* vol. 35, no. 1, pp. 150-183, 2020.
[79] W. W. Mohd, L. Abdullah, B. Yusoff, C. Taib, and J. Merigo, "An integrated MCDM model based on Pythagorean fuzzy sets for green supplier development program," *Malaysian Journal of Mathematical Sciences,* vol. 13, pp. 23-37, 2019.
[80] X. Peng and Y. Yang, "Pythagorean fuzzy Choquet integral based MABAC method for multiple attribute group decision making," *International Journal of Intelligent Systems,* vol. 31, no. 10, pp. 989-1020, 2016.
[81] G. Wei and M. Lu, "Pythagorean fuzzy power aggregation operators in multiple attribute decision making," *International Journal of Intelligent Systems,* vol. 33, no. 1, pp. 169-186, 2018.
[82] X. Peng and Y. Yang, "Some results for Pythagorean fuzzy sets," *International Journal of Intelligent Systems,* vol. 30, no. 11, pp. 1133-1160, 2015.
[83] G. Qian, H. Wang, and X. Feng, "Generalized hesitant fuzzy sets and their application in decision support system," *Knowledge-based systems,* vol. 37, pp. 357-365, 2013.
[84] M. S. A. Khan, S. Abdullah, A. Ali, N. Siddiqui, and F. Amin, "Pythagorean hesitant fuzzy sets and their application to group decision making with incomplete weight information," *Journal of Intelligent & Fuzzy Systems,* vol. 33, no. 6, pp. 3971-3985, 2017.
[85] B. Batool, M. Ahmad, S. Abdullah, S. Ashraf, and R. Chinram, "Entropy based Pythagorean probabilistic hesitant fuzzy decision making technique and its application for fog-haze factor assessment problem," *Entropy,* vol. 22, no. 3, p. 318, 2020.
[86] B. Batool, S. Abdullah, S. Ashraf, and M. Ahmad, "Pythagorean probabilistic hesitant fuzzy aggregation operators and their application in decision-making," *Kybernetes,* vol. 51, no. 4, pp. 1626-1652, 2022.
[87] H. Garg, M. Munir, K. Ullah, T. Mahmood, and N. Jan, "Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators," *Symmetry,* vol. 10, no. 12, p. 670, 2018.
[88] P. Liu, B. Zhu, and P. Wang, "A multi-attribute decision-making approach based on spherical fuzzy sets for Yunnan Baiyao's R&D project selection problem," *International Journal of Fuzzy Systems,* vol. 21, pp. 2168-2191, 2019.
[89] T. Mahmood, J. Ahmmad, Z. Ali, D. Pamucar, and D. Marinkovic, "Interval valued T-spherical fuzzy soft average aggregation operators and their applications in multiple-criteria decision making," *Symmetry,* vol. 13, no. 5, p. 829, 2021.
[90] S. Ashraf and S. Abdullah, "Emergency decision support modeling for COVID-19 based on spherical fuzzy information," *International Journal of Intelligent Systems,* vol. 35, no. 11, pp. 1601-1645, 2020.
[91] T. Mahmood, K. Ullah, Q. Khan, and N. Jan, "An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets," *Neural Computing and Applications,* vol. 31, pp. 7041-7053, 2019.

[92] D. Wu and W. W. Tan, "Genetic learning and performance evaluation of interval type-2 fuzzy logic controllers," *Engineering Applications of Artificial Intelligence,* vol. 19, no. 8, pp. 829-841, 2006.
[93] J. M. Mendel, "Type-2 fuzzy sets and systems: an overview," *IEEE computational intelligence magazine,* vol. 2, no. 1, pp. 20-29, 2007.
[94] H. Hu, Y. Wang, and Y. Cai, "Advantages of the enhanced opposite direction searching algorithm for computing the centroid of an interval type-2 fuzzy set," *Asian Journal of Control,* vol. 14, no. 5, pp. 1422-1430, 2012.
[95] E. Krishnan *et al.*, "Interval type 2 trapezoidal-fuzzy weighted with zero inconsistency combined with VIKOR for evaluating smart e-tourism applications," *International Journal of Intelligent Systems,* vol. 36, no. 9, pp. 4723-4774, 2021.
[96] M. K. Ghorabaee, "Developing an MCDM method for robot selection with interval type-2 fuzzy sets," *Robotics and Computer-Integrated Manufacturing,* vol. 37, pp. 221-232, 2016.
[97] M. K. Ghorabaee, M. Amiri, J. S. Sadaghiani, and E. K. Zavadskas, "Multi-criteria project selection using an extended VIKOR method with interval type-2 fuzzy sets," *International Journal of Information Technology & Decision Making,* vol. 14, no. 05, pp. 993-1016, 2015.
[98] P. Talukdar and P. Dutta, "Distance measures for cubic Pythagorean fuzzy sets and its applications to multicriteria decision making," *Granular Computing,* vol. 6, pp. 267-284, 2021.
[99] M. S. A. Khan, F. Khan, J. Lemley, S. Abdullah, and F. Hussain, "Extended topsis method based on Pythagorean cubic fuzzy multi-criteria decision making with incomplete weight information," *Journal of Intelligent & Fuzzy Systems,* vol. 38, no. 2, pp. 2285-2296, 2020.
[100] A. Alamoodi *et al.*, "New extension of fuzzy-weighted zero-inconsistency and fuzzy decision by opinion score method based on cubic pythagorean fuzzy environment: a benchmarking case study of sign language recognition systems," *International Journal of Fuzzy Systems,* vol. 24, no. 4, pp. 1909-1926, 2022.
[101] L. A. Zadeh, G. J. Klir, and B. Yuan, *Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers*. World scientific, 1996.
[102] F. Smarandache, "Neutrosophic set-a generalization of the intuitionistic fuzzy set," in *2006 IEEE international conference on granular computing*, 2006, pp. 38-42: IEEE.
[103] M. N. Jafar, M. Saeed, M. Saqlain, and M.-S. Yang, "Trigonometric similarity measures for neutrosophic hypersoft sets with application to renewable energy source selection," *Ieee Access,* vol. 9, pp. 129178-129187, 2021.
[104] M. N. Jafar, M. Zia, A. Saeed, M. Yaqoob, and S. Habib, "Aggregation operators of bipolar neutrosophic soft sets and it's applications in auto car selection," *International Journal of Neutrosophic Science,* vol. 9, no. 1, pp. 37-7-46, 2021.
[105] A. A. Supciller and F. Toprak, "Selection of wind turbines with multi-criteria decision making techniques involving neutrosophic numbers: A case from Turkey," *Energy,* vol. 207, p. 118237, 2020.
[106] D. Pamucar, M. Deveci, D. Schitea, L. Erişkin, M. Iordache, and I. Iordache, "Developing a novel fuzzy neutrosophic numbers based decision making analysis for prioritizing the energy storage technologies," *International Journal of Hydrogen Energy,* vol. 45, no. 43, pp. 23027-23047, 2020.
[107] A. Alamoodi *et al.*, "Based on neutrosophic fuzzy environment: a new development of FWZIC and FDOSM for benchmarking smart e-tourism applications," *Complex & Intelligent Systems,* vol. 8, no. 4, pp. 3479-3503, 2022.
[108] R. Şahin and G. D. Küçük, "Group decision making with simplified neutrosophic ordered weighted distance operator," *Mathematical Methods in the Applied Sciences,* vol. 41, no. 12, pp. 4795-4809, 2018.
[109] J.-j. Peng, J.-q. Wang, X.-h. Wu, H.-y. Zhang, and X.-h. Chen, "The fuzzy cross-entropy for intuitionistic hesitant fuzzy sets and their application in multi-criteria decision-making," *International Journal of Systems Science,* vol. 46, no. 13, pp. 2335-2350, 2015.
[110] S. Qahtan *et al.*, "Evaluation of agriculture-food 4.0 supply chain approaches using Fermatean probabilistic hesitant-fuzzy sets based decision making model," *Applied Soft Computing,* vol. 138, p. 110170, 2023.
[111] E. Bartl and J. Konecny, "Rough fuzzy concept analysis," *Fundamenta Informaticae,* vol. 156, no. 2, pp. 141-168, 2017.
[112] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning-III," *Information sciences,* vol. 9, no. 1, pp. 43-80, 1975.
[113] K. T. Atanassov and S. Stoeva, "Intuitionistic fuzzy sets," *Fuzzy sets and Systems,* vol. 20, no. 1, pp. 87-96, 1986.
[114] R. R. Yager and A. M. Abbasov, "Pythagorean membership grades, complex numbers, and decision making," *International Journal of Intelligent Systems,* vol. 28, no. 5, pp. 436-452, 2013.
[115] X. Peng and Y. Yang, "Fundamental properties of interval-valued Pythagorean fuzzy aggregation operators," *International journal of intelligent systems,* vol. 31, no. 5, pp. 444-487, 2016.
[116] D. Dubois and H. Prade, "Rough fuzzy sets and fuzzy rough sets," *International Journal of General System,* vol. 17, no. 2-3, pp. 191-209, 1990.

[117] B. Sun, Z. Gong, and D. Chen, "Fuzzy rough set theory for the interval-valued fuzzy information systems," *Information Sciences,* vol. 178, no. 13, pp. 2794-2815, 2008.
[118] H.-L. Yang and J.-J. Zhou, "Interval-valued pythagorean fuzzy rough approximation operators and its application," *Journal of Intelligent & Fuzzy Systems,* vol. 39, no. 3, pp. 3067-3084, 2020.
[119] M. Ali, I. Deli, and F. Smarandache, "The theory of neutrosophic cubic sets and their applications in pattern recognition," *Journal of intelligent & fuzzy systems,* vol. 30, no. 4, pp. 1957-1963, 2016.
[120] J. J. S. C. Ye, "Operations and aggregation method of neutrosophic cubic numbers for multiple attribute decision-making," vol. 22, no. 22, pp. 7435-7444, 2018.
[121] J. Ye, "Operations and aggregation method of neutrosophic cubic numbers for multiple attribute decision-making," *Soft Computing,* vol. 22, no. 22, pp. 7435-7444, 2018.
[122] A. Alamleh *et al.*, "Federated learning for IoMT applications: A standardization and benchmarking framework of intrusion detection systems," *IEEE Journal of Biomedical and Health Informatics,* vol. 27, no. 2, pp. 878-887, 2022.
[123] M. M. Shahri, A. E. Jahromi, and M. Houshmand, "Failure Mode and Effect Analysis using an integrated approach of clustering and MCDM under pythagorean fuzzy environment," *Journal of Loss Prevention in the Process Industries,* vol. 72, p. 104591, 2021.
[124] K. T. Atanassov and K. T. Atanassov, *Intuitionistic fuzzy sets*. Springer, 1999.
[125] T. Senapati and R. R. Yager, "Fermatean fuzzy sets," *Journal of Ambient Intelligence and Humanized Computing,* vol. 11, pp. 663-674, 2020.
[126] A. Alnoor *et al.*, "Toward a sustainable transportation industry: oil company benchmarking based on the extension of linear diophantine fuzzy rough sets and multicriteria decision-making methods," *IEEE Transactions on Fuzzy Systems,* vol. 31, no. 2, pp. 449-459, 2022.
[127] I. Gokasar, D. Pamucar, M. Deveci, and W. Ding, "A novel rough numbers based extended MACBETH method implementation in the prioritization of the connected autonomous vehicles," 2022.
[128] M. Deveci, P. R. Brito-Parada, D. Pamucar, and E. A. Varouchakis, "Rough sets based Ordinal Priority Approach to evaluate sustainable development goals (SDGs) for sustainable mining," *Resources Policy,* vol. 79, p. 103049, 2022.
[129] C.-N. Huang, S. Ashraf, N. Rehman, S. Abdullah, and A. Hussain, "A novel spherical fuzzy rough aggregation operators hybrid with TOPSIS method and their application in decision making," *Mathematical Problems in Engineering,* vol. 2022, pp. 1-20, 2022.
[130] R. Mohammed *et al.*, "A decision modeling approach for smart e-tourism data management applications based on spherical fuzzy rough environment," *Applied Soft Computing,* vol. 143, p. 110297, 2023.
[131] H. Zhang, L. Shu, S. Liao, and C. Xiawu, "Dual hesitant fuzzy rough set and its application," *Soft Computing,* vol. 21, pp. 3287-3305, 2017.
[132] S. Ismael *et al.*, "Toward sustainable transportation: A pavement strategy selection based on the extension of dual-hesitant fuzzy multicriteria decision-making methods," *IEEE Transactions on Fuzzy Systems,* vol. 31, no. 2, pp. 380-393, 2022.
[133] L. D. D. R. Calache, L. G. Zanon, R. F. M. Arantes, L. Osiro, and L. C. R. Carpinetti, "Risk prioritization based on the combination of FMEA and dual hesitant fuzzy sets method," *Production,* vol. 31, 2021.
[134] W. S. Du, "Subtraction and division operations on intuitionistic fuzzy sets derived from the Hamming distance," *Information Sciences,* vol. 571, pp. 206-224, 2021.
[135] Y. Zhang, Y. Zhang, C. Gong, H. Dinçer, and S. Yüksel, "An integrated hesitant 2-tuple Pythagorean fuzzy analysis of QFD-based innovation cost and duration for renewable energy projects," *Energy,* vol. 248, p. 123561, 2022.
[136] M. Akram, R. Bibi, and M. Deveci, "An outranking approach with 2-tuple linguistic Fermatean fuzzy sets for multi-attribute group decision-making," *Engineering Applications of Artificial Intelligence,* vol. 121, p. 105992, 2023.
[137] J. Dai and J. Chen, "Feature selection via normative fuzzy information weight with application into tumor classification," *Applied Soft Computing,* vol. 92, p. 106299, 2020.
[138] R. K. Nowicki and R. K. Nowicki, "Rough set theory fundamentals," *Rough Set–Based Classification Systems,* pp. 7-16, 2019.
[139] T. Senapati and R. R. Yager, "Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods," *Engineering Applications of Artificial Intelligence,* vol. 85, pp. 112-121, 2019.
[140] M. Riaz and M. R. Hashmi, "Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems," *Journal of Intelligent & Fuzzy Systems,* vol. 37, no. 4, pp. 5417-5439, 2019.
[141] A. Iampan, G. S. García, M. Riaz, H. M. Athar Farid, and R. Chinram, "Linear Diophantine fuzzy Einstein aggregation operators for multi-criteria decision-making problems," *Journal of Mathematics,* vol. 2021, pp. 1-31, 2021.

[142] M. R. Hashmi, S. T. Tehrim, M. Riaz, D. Pamucar, and G. Cirovic, "Spherical linear diophantine fuzzy soft rough sets with multi-criteria decision making," *Axioms,* vol. 10, no. 3, p. 185, 2021.
[143] S. Lou, Y. Feng, Z. Li, H. Zheng, Y. Gao, and J. Tan, "An edge-based distributed decision-making method for product design scheme evaluation," *IEEE Transactions on Industrial Informatics,* vol. 17, no. 2, pp. 1375-1385, 2020.
[144] G. Huang, L. Xiao, W. Pedrycz, D. Pamucar, G. Zhang, and L. Martínez, "Design alternative assessment and selection: A novel Z-cloud rough number-based BWM-MABAC model," *Information Sciences,* vol. 603, pp. 149-189, 2022.
[145] L. A. Zadeh, "A note on Z-numbers," *Information sciences,* vol. 181, no. 14, pp. 2923-2932, 2011.
[146] A. K. Yazdi, A. R. Komijan, P. F. Wanke, and S. Sardar, "Oil project selection in Iran: A hybrid MADM approach in an uncertain environment," *Applied Soft Computing,* vol. 88, p. 106066, 2020.
[147] M. Akram, "Multi-criteria decision-making methods based on q-rung picture fuzzy information," *Journal of Intelligent & Fuzzy Systems,* vol. 40, no. 5, pp. 10017-10042, 2021.
[148] S. Shao, X. Zhang, Y. Li, and C. Bo, "Probabilistic single-valued (interval) neutrosophic hesitant fuzzy set and its application in multi-attribute decision making," *Symmetry,* vol. 10, no. 9, p. 419, 2018.
[149] R. Şahin and F. Altun, "Decision making with MABAC method under probabilistic single-valued neutrosophic hesitant fuzzy environment," *Journal of Ambient Intelligence and Humanized Computing,* vol. 11, pp. 4195-4212, 2020.
[150] H. A. Alsattar *et al.*, "Developing deep transfer and machine learning models of chest X-ray for diagnosing COVID-19 cases using probabilistic single-valued neutrosophic hesitant fuzzy," *Expert Systems with Applications,* vol. 236, p. 121300, 2024.
[151] F. K. Gündoğdu and C. Kahraman, "A novel fuzzy TOPSIS method using emerging interval-valued spherical fuzzy sets," *Engineering Applications of Artificial Intelligence,* vol. 85, pp. 307-323, 2019.
[152] A. Pankowska and M. Wygralak, "General IF-sets with triangular norms and their applications to group decision making," *Information Sciences,* vol. 176, no. 18, pp. 2713-2754, 2006.
[153] H. Behret, "Group decision making with intuitionistic fuzzy preference relations," *Knowledge-Based Systems,* vol. 70, pp. 33-43, 2014.
[154] Q. Lei and Z. Xu, "Derivative and differential operations of intuitionistic fuzzy numbers," *International Journal of Intelligent Systems,* vol. 30, no. 4, pp. 468-498, 2015.
[155] M. C. Bozyiğit, M. Olgun, and M. Ünver, "Circular Pythagorean fuzzy sets and applications to multi-criteria decision making," *arXiv preprint arXiv:2210.15483,* 2022.
[156] H. A. Alsattar *et al.*, "Three-way decision-based conditional probabilities by opinion scores and Bayesian rules in circular-Pythagorean fuzzy sets for developing sustainable smart living framework," *Information Sciences,* vol. 649, p. 119681, 2023.
[157] S. Khan, S. Abdullah, S. Ashraf, R. Chinram, and S. Baupradist, "Decision Support Technique Based on Neutrosophic Yager Aggregation Operators: Application in Solar Power Plant Locations—Case Study of Bahawalpur, Pakistan," *Mathematical Problems in Engineering,* vol. 2020, pp. 1-21, 2020.
[158] R. R. Yager, "On ordered weighted averaging aggregation operators in multicriteria decisionmaking," *IEEE Transactions on systems, Man, and Cybernetics,* vol. 18, no. 1, pp. 183-190, 1988.
[159] A. Mardani, M. Nilashi, E. K. Zavadskas, S. R. Awang, H. Zare, and N. M. Jamal, "Decision making methods based on fuzzy aggregation operators: Three decades review from 1986 to 2017," *International Journal of Information Technology & Decision Making,* vol. 17, no. 02, pp. 391-466, 2018.
[160] S. Qahtan, H. A. Alsattar, A. Zaidan, M. Deveci, D. Pamucar, and D. Delen, "Performance assessment of sustainable transportation in the shipping industry using a q-rung orthopair fuzzy rough sets-based decision making methodology," *Expert Systems with Applications,* vol. 223, p. 119958, 2023.
[161] A. Alamoodi, O. Albahri, A. Zaidan, H. Alsattar, B. Zaidan, and A. Albahri, "Hospital selection framework for remote MCD patients based on fuzzy q-rung orthopair environment," *Neural Computing and Applications,* vol. 35, no. 8, pp. 6185-6196, 2023.