Iraqi Journal for Computer Science and Mathematics

Journal Homepage: http://journal.esj.edu.iq/index.php/IJCM
e-ISSN: 2788-7421 p-ISSN: 2958-0544

Constant-Stress Partially Accelerated Life Testing for Weibull Inverted Exponential Distribution with Censored Data

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DOI: https://doi.org/10.52866/ijcsm.2024.05.02.009
Received September 2023; Accepted January 2024; Available online March 2024

ABSTRACT: The novelty of this article is estimating the parameters of Weibull inverted exponential (WIE) distribution with a constant stress partially accelerated life test (PALT) under adaptive type-II progressively censored samples. Moreover, the maximum likelihood estimators (MLEs), their asymptotic variances confidence intervals, and Bayes estimators (BEs) of the model parameters and the acceleration factor are obtained. Furthermore, the approximate bootstrap and credible confidence intervals of the estimators are acquired. The accuracy of the MLEs and BEs for the model parameters and the acceleration factor is investigated through the simulation studies.

Keywords: Partially accelerated life tests; Constant stress; Adaptive type-II progressively censoring; Weibull inverted exponential; Bayesian approach; bootstrap confidence interval, simulation study.

1. INTRODUCTION

The developments of technologies and global competition have emphasized the need for more accurate estimation of reliability of a product, system or component in a shorter time. In standard life data analysis, failure data is examined to determine the product, component, or system's life characteristics. Collecting lifetimes on extremely dependable items with very long lifetimes is often exceedingly difficult, but not impossible, because very few, if any, failures can occur during a restricted testing time under normal conditions. Given this difficulty, reliability practitioners have attempted to introduce methods to make failures quickly by subjecting the products to severer environmental conditions without introducing additional failure modes other than those observed under normal operating conditions. ALT is one of the most modern approaches that used to observe enough failure data, in a short period. In such testing, products are tested at higher than normal levels of stress (e.g., temperature, voltage, humidity, vibration or pressure) to induce quickly failures.

PALT is one type of the ALT schemes. The stress loading in PALTs can be applied in different types, commonly used types are constant-stress, step-stress and progressive-stress.

In step-stress PALT, the stress on every unit is increasing step by step at prespecified times or simultaneous the occurrence of a constant number of failures, for a brief review on step-stress model, see Miller and Nelson [1], Bai et al. [2], Soliman et al. [3] and Balakrishnan et al. [4].

In progressive-stress model, the stress on each test unit is continuously increasing in time. If an ALT contains linearly increasing stress, for more details see Yin and Sheng [5], Abdel-Hamid and AL-Hussaini [6].

In a constant stress PALT experiment, the total test units are first divided into two groups, the items of one of the groups are allocated to a normal condition, and the items of the other group are allocated to a stress condition, each unit is run at a constant level of stress until the unit fails. The constant stress PALTs have advantages in certain aspects including conducting experiments, modeling distributions, and estimating parameters, the statistical models of most types of distributions based on constant stress PALTs are more effectively developed compared with those of step-stress PALTs. The calculation of parameter estimation of distributions in constant stress PALTs is generally easy to perform as well, for a brief review on constant stress PALTs, Nelson [7] pointed out that the constant-stress testing has several advantages. Bai and Chung [8] provided the ML method to estimate the scale parameter and acceleration factor

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for the exponential distribution under two types of PALT which are step and constant stresses in case of type-I censoring Abdel-Hamid [9] studied the constant stress PALTs for Burr type-XII distribution with progressive type-II censoring. Estimating the Burr XII parameters in constant stress PALTs under multiple censored data was considered by Cheng and Wang [10]. Zarrin et al. [11] analyzed the maximum likelihood method for estimating the acceleration factor and the parameters of Rayleigh distribution for constant stress PALT. Estimating the generalized exponential distribution parameters and the acceleration factor under constant stress PALTs with type-II censoring reported by Ismail [12]. Kamal et al. [13] used maximum likelihood approach for estimating the acceleration factor and parameters for inverted Weibull distribution based on type-I censored data. The constant stress PALT under progressive censoring for the generalized exponential distribution was introduced by Jaheen et al. [14]. Abushal and soliman [15] applied the maximum likelihood and Bayesian approaches to estimate the parameters of the Pareto distribution under progressive censoring data for constant stress PALTs. Estimation and optimum constant stress PALTs for Gompertz distribution with type-I censoring discussed by Li and Zheng [16]. Ahmed et al. [17] discussed how to obtain maximum likelihood and Bayesian estimation of parameters in exponentiated Weibull distribution under partially acceleration life tests. For units with a Gompertz distribution and units that fail under two independent causes of failure, Alghamdi [18] used a Type-I generalized hybrid censoring technique. Alam et al. [19] proposed constant stress PALTs and estimated costs of maintenance service policy for the generalized inverted exponential distribution. Mahmoud MAW et al. [20] discussed Estimating the modified Weibull parameters in presence of constant stress PALTs. Inference on Nadarajah-Haghighi distribution with constant stress PALT under progressive type-II censoring was reported by Dey S. et al. [21]. Ahmadini A, et al. [22] proposed estimation of constant stress PALT for Fréchet distribution with type-I censoring. Analysis of the modified Kies exponential distribution with constant stress PALTs under Type-II censoring was covered by Nassar M. and Alam FMA [23].

In reliability experiments, the another way to save time and reduce cost is censored data, see Balakrishnan and Kundu [24]. There are several conventional censoring schemes, such as, type-I censoring scheme, type-II censoring scheme, hybrid censoring scheme and progressive censoring scheme. The inability to remove units at locations other than the experiment's terminal point is one of the disadvantages of the traditional type-I, type-II, or hybrid censoring schemes. Because of that, a more general censoring scheme called progressive censoring has been introduced. The progressive censoring scheme (Balakrishnan & Aggarwala, [25]) has the flexibility of allowing removal of units at points other than the terminal point of the experiment. Another advantage of progressive censoring is that the degeneration information of the test units is obtained from those removed units. Although the scheme is more flexible in terms of the removal of units, it still has disadvantages. The drawback of the type- II progressive censoring, similar to the conventional type- II censoring, is that it can take a lot of time to get to the m^{th} failure time and the running time T of the experiment is still unknown. In order to assure the number of failures, Ng et al. [26] proposed another censoring scheme called the adaptive type-II progressively hybrid censoring scheme.

This type of censoring scheme can be described as follows: the observed sample size is always m and the R_i may change accordingly during the experiment. If $T > X_{m\,:\,m\,:\,n}$, we will get m observed failures before stopping the experiment and all R_i remain unchanged. If $X_{j\,:\,m\,:\,n} < T < X_{j+1\,:\,m\,:\,n}$ for $0 < j < m$, where $X_{0\,:\,m\,:\,n} = 0$, we will continue to run the experiment until the m^{th} failure occurs and we adjust R_i to R_i^*, where $R_i^* = 0$ for $i = j+1, j+2, ..., m-1$ and $R_m^* = n - m - \sum_{k=1}^{j} R_k$. Thus, this setting can be viewed as a design, in which one would ideally like to have m observed failure times for efficiency of inference, and at the same time have the total time on test to be not too far away from the ideal test duration T. The value of T plays an important role in the determination of the values of It is used to compromise between a shorter experimental time and a higher chance to observe extreme failures. We will have a usual progressive type-II censoring scheme with pre-fixed R_i ' s when $T \to \infty$. And, we will have a conventional type-II censoring scheme with $R_1 = ... = R_{m-1} = 0$ and $R_m = n - m$ when $T \to 0$.

In addition to this introductory section this article includes some more sections too. In section 2 a description of the lifetime model and assumption are presented. The MLEs of model parameters of our model and the acceleration factor are derived for constant stress PALT using adaptive progressive type-II censored data in section 3. In section 4 the BEs of model parameters using Markov chain Monte Carlo simulation method are obtained. In Section 5 intervals estimation of the model parameter and acceleration factor are described such as asymptotic, bootstrap and credible confidence intervals. Numerical studies to illustrate the theoretical results are given in section 6. The conclusion is made of the study in section 7.

2. Description of the model

2.1. The WIE distribution: As a lifetime model

Chandrakant et al. [27] introduced a three-parameter WIE distribution, which is considered an extension of the inverted exponential distribution. The WIE distribution is flexible in nature and can take several shapes, such as J-reversed, symmetric, and positively skewed as well. Additionally, the shape of the WIE distribution is either be decreasing or unimodal. The shape of the hazard function can be decreasing, increasing and an inverted bathtub (depending upon the values of the parameters). According to the previous features, the WIE distribution can be used to fit different data in several vital fields, such as engineering, industry, biomedical studies, and medicine, to contribute to solving many obstacles. In our study, the failure times are assumed to be from the WIE (λ, γ, η) distribution. For some statistical properties as well as the properties of order statistics of the WIE distribution.

For conducting a constant stress PALT experiment, the total test items are divided into two groups. The items of one group are allocated to the normal condition and the items of the other group are allocated to the stress condition. The lifetime of the test unit under the normal and stress conditions is assumed to follow the WIE distribution. The probability density function (PDF) of the WIE distribution and the cumulative distribution function (CDF) of the lifetime of a unit for the normal condition

$$f_{T_1}(t)=\frac{\lambda\gamma\eta}{t^2}\frac{(e^{-\frac{\eta}{t}})^{\gamma}}{(1-e^{-\frac{\eta}{t}})^{\gamma+1}}e^{-\lambda\left(\frac{e^{-\frac{\eta}{t}}}{1-e^{-\frac{\eta}{t}}}\right)^{\gamma}}, \tag{1}$$

and

$$F_{T_1}(t)=1-e^{-\lambda\left(\frac{e^{-\frac{\eta}{t}}}{1-e^{-\frac{\eta}{t}}}\right)^{\gamma}}, \tag{2}$$

the survival and hazard rate functions of the WIE (λ, γ, η) distribution are

$$S_{T_1}(t)=e^{-\lambda\left(\frac{e^{-\frac{\eta}{t}}}{1-e^{-\frac{\eta}{t}}}\right)^{\gamma}}, \tag{3}$$

and

$$h_{T_1}(t)=\frac{\lambda\gamma\eta}{t^2}\frac{(e^{-\frac{\eta}{t}})^{\gamma}}{(1-e^{-\frac{\eta}{t}})^{\gamma+1}}. \tag{4}$$

The hazard rate of an item tested at accelerated condition is given by $h_{T_2}(t)=\alpha h_{T_1}(t)$, where is an acceleration factor satisfying $\alpha>1$. Therefore the hazard rate function, survival function, CDF and PDF under accelerated condition are given, respectively, by

$$h_{T_2}(t)=\frac{\alpha\lambda\gamma\eta}{t^2}\times\frac{(e^{-\frac{\eta}{t}})^{\gamma}}{(1-e^{-\frac{\eta}{t}})^{\gamma+1}}, \tag{5}$$

$$S_{T_2}(t)=\exp\left(-\int h_2(z)dz\right)=e^{-\alpha\lambda\left(\frac{e^{-\frac{\eta}{t}}}{1-e^{-\frac{\eta}{t}}}\right)^{\gamma}}, \tag{6}$$

$$F_{T_2}(t)=1-e^{-\alpha\lambda\left(\frac{e^{-\frac{\eta}{t}}}{1-e^{-\frac{\eta}{t}}}\right)^{\gamma}}, \tag{7}$$

and

$$f_{T_2}(t)=\frac{\alpha\lambda\gamma\eta}{t^2}\times\frac{(e^{-\frac{\eta}{t}})^{\gamma}}{(1-e^{-\frac{\eta}{t}})^{\gamma+1}}\times e^{-\alpha\lambda\left(\frac{e^{-\frac{\eta}{t}}}{1-e^{-\frac{\eta}{t}}}\right)^{\gamma}}, \tag{8}$$

where T_1 and T_2 represent lifetimes of units under the normal condition and the stress condition, respectively, λ, γ is the shape parameter, η is the scale parameter, α is the acceleration factor, and $\lambda>0$, $\gamma>0$, $\eta>0$, $\alpha>1$.

2.2. Constant-stress PALT

According to constant stress PALTs under adaptive progressive type-II censoring, group 1 consists of n_1 items randomly chosen among n test items is subjected to use condition and group 2 consists of $n_2 = n - n_1$ remaining items are subjected to an accelerated condition. The observed sample size is always m_j and the R_{ji}, $i = 1,\ldots,m_j$, $j = 1,2$ may change accordingly during the experiment. If $T > X_{m_j : m_j : n_j}$, we will get m_j observed failures before stopping the experiment and all R_{ij} remain unchanged. If $X_{jk_j : m_j : n_j} < T < X_{j(k_j+1): m_j : n_j}$ for $0 < k_j < m_j$, where $X_{j0 : m_j : n_j} = 0$, we will continue to run the experiment until the m_j^{th} failure occurs and we adjust R_{ji} to R_{ji}^*, where $R_{ji}^* = 0$ for $i = k_j + 1, k_j + 2, \ldots, m_j - 1$ and $R_{mj}^* = n_j - m_j - \sum_{l=1}^{k_j} R_l$. The joint PDF for $X_{j1 : m_j, n_j}^{R_{j1}} < X_{j2 : m_j, n_j}^{R_{j2}} < \ldots < X_{jm_j : m_j, n_j}^{R_{jm_j}}$ is given by

$$L(\alpha,\lambda,\gamma,\eta \mid \underline{x}) \propto \prod_{j=1}^{2}\prod_{i=1}^{m_j} f(x_{ji : m_j : n_j}) \prod_{i=1}^{k_j}[1-F(x_{ji : m_j : n_j})]^{R_{ji}} \prod_{i=k+1}^{m_j-1}[1-F(x_{ji : m_j : n_j})]^{R_{ji}^*}\left[1-F(x_{m_j : m_j : n_j})\right]^{R_{ji}^*}$$
$$\propto \prod_{j=1}^{2}\prod_{i=1}^{m_j} f(x_{ji : m_j : n_j}) \prod_{i=1}^{k_j}[1-F(x_{ji : m_j : n_j})]^{R_{ji}}\left[1-F(x_{m_j : m_j : n_j})\right]^{n_j - m_j - \sum_{i=1}^{k_j} R_{ji}}, \tag{9}$$

for $-\infty < x_{j1 : m_j : n_j} < x_{j2 : m_j : n_j} < \ldots < x_{jk_j : m_j : n_j} < T < x_{j(k_j+1)m_j : n_j} < x_{j(k_j+2) : m_j : n_j} < \ldots < x_{jm_j : m_j : n_j} < \infty.$

3. Maximum likelihood estimation

In this section, the MLEs of the model parameters are constructed. Let $x_{jm_j : m_j : n_j} = x_{jm_j}$ be the observed values of the lifetime obtained from adaptive progressive type-II censoring under constant stress PALT.
The likelihood function for group 1 is given by

$$L\left(\lambda,\ \gamma,\ \eta \mid \underline{x_1}\right) \propto \prod_{i=1}^{m_1} \frac{\lambda\gamma\eta}{x_{1i}^2} e^{\frac{\eta}{x_{1i}}} (e^{\frac{\eta}{x_{1i}}} - 1)^{-\gamma-1} \times \exp(-\lambda(e^{\frac{\eta}{x_{1i}}} - 1)^{-\gamma})$$
$$\times \prod_{i=1}^{k_1}\left[\exp(-\lambda(e^{\frac{\eta}{x_{1i}}} - 1)^{-\gamma})\right]^{R_{1i}} \times \left[\exp(-\lambda(e^{\frac{\eta}{x_{m_1}}} - 1)^{-\gamma})\right]^{A_{m_1}}, \tag{10}$$

where $X_1 = (x_{11}, x_{12}, x_{13,\ldots,} x_{1m_1})$, $A_{m_1} = n_1 - m_1 - \sum_{i=1}^{k_1} R_{1i}$,
The likelihood function for group 2 is given by

$$L\left(\alpha,\lambda,\ \gamma,\ \eta \mid \underline{x_2}\right) \propto \prod_{i=1}^{m_2} \frac{\alpha\lambda\gamma\eta}{x_{2i}^2} e^{\frac{\eta}{x_{2i}}} (e^{\frac{\eta}{x_{2i}}} - 1)^{-\gamma-1} \times \exp(-\alpha\lambda(e^{\frac{\eta}{x_{2i}}} - 1)^{-\gamma})$$
$$\times \prod_{i=1}^{k_2}\left[\exp(-\alpha\lambda(e^{\frac{\eta}{x_{2i}}} - 1)^{-\gamma})\right]^{R_{2i}} \times \left[\exp(-\alpha\lambda(e^{\frac{\eta}{x_{m_2}}} - 1)^{-\gamma})\right]^{A_{m_2}}, \tag{11}$$

where $X_2 = (x_{21}, x_{22}, x_{23,\ldots,} x_{2m_2})$, $A_{m_2} = n_2 - m_2 - \sum_{i=1}^{k_2} R_{2i}$,
based on Eqs. (10) and (11) The likelihood function for group 1 and group 2 can be written in one equation as follow:

$$L\left(\alpha,\lambda,\ \gamma,\ \eta\,|\,\underline{x}\right)\propto\prod_{j=1}^{2}\prod_{i=1}^{m_j}\frac{\alpha^{j-1}\lambda\gamma\eta}{x_{ji}^{2}}e^{\frac{\eta}{x_{ji}}}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma-1}\times\exp(-\alpha^{j-1}\lambda(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma})$$
$$\times\prod_{i=1}^{k_j}\left[\exp(-\alpha^{j-1}\lambda(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma})\right]^{R_{ji}}\times\left[\exp(-\alpha^{j-1}\lambda(e^{\frac{\eta}{x_{m_j}}}-1)^{-\gamma})\right]^{A_{m_j}}, \tag{12}$$

where $X=(x_{j1},x_{j2},x_{j3},...,x_{jm_2})$, $A_{m_j}=n_j-m_j-\sum_{i=1}^{k_j}R_{ji}$.

It is easier to maximize the natural logarithm of the likelihood function $\log L(\alpha,\lambda,\ \gamma,\ \eta\,|\,\underline{x})$ than the likelihood function. Therefore, the log-likelihood function is given by

$$\ell(\alpha,\lambda,\ \gamma,\ \eta\,|\,\underline{x})=\log L\left(\alpha,\lambda,\ \gamma,\ \eta\,|\,\underline{x}\right)\propto\sum_{j=1}^{2}\left(m_j\log(\lambda\gamma\eta)+(j-1)m_j\log(\alpha)-(\gamma+1)\sum_{i=1}^{m_j}\log(e^{\frac{\eta}{x_{ji}}}-1)\right.$$
$$\left.+\sum_{i=1}^{m_j}\frac{\eta}{x_{ji}}-\alpha^{j-1}\lambda\sum_{i=1}^{m_j}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}-\sum_{i=1}^{k_j}R_{ji}\alpha^{j-1}\lambda(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}-A_{m_j}\alpha^{j-1}\lambda(e^{\frac{\eta}{x_{m_j}}}-1)^{-\gamma}\right). \tag{13}$$

The first order partial derivatives of log-likelihood function with respect to $\eta,\ \lambda,\ \gamma$, and α are given by:

$$\frac{\partial\ell}{\partial\lambda}=\sum_{j=1}^{2}\left(\frac{m_j}{\lambda}-\alpha^{j-1}\sum_{i=1}^{m_j}W^{-\gamma}\left(x_{ji},\eta\right)-\sum_{i=1}^{k_j}R_{ji}\alpha^{j-1}W^{-\gamma}\left(x_{ji},\eta\right)-A_{m_j}\alpha^{j-1}W^{-\gamma}\left(x_{m_j},\eta\right)\right), \tag{14}$$

$$\frac{\partial\ell}{\partial\gamma}=\sum_{j=1}^{2}\left(\frac{m_j}{\gamma}-\sum_{i=1}^{m_j}\log W\left(x_{ji},\eta\right)+\alpha^{j-1}\lambda\sum_{i=1}^{m_j}W^{-\gamma}\left(x_{ji},\eta\right)\log W\left(x_{ji},\eta\right)\right.$$
$$\left.+\sum_{i=1}^{k_j}R_{ji}\alpha^{j-1}\lambda W^{-\gamma}\left(x_{ji},\eta\right)\log W\left(x_{ji},\eta\right)+A_{m_j}\alpha^{j-1}\lambda W^{-\gamma}\left(x_{m_j},\eta\right)\log W\left(x_{m_j},\eta\right)\right), \tag{15}$$

$$\frac{\partial\ell}{\partial\eta}=\sum_{j=1}^{2}\left(\frac{m_j}{\eta}+\sum_{i=1}^{m_j}\frac{1}{x_{ji}}-(\gamma+1)\sum_{i=1}^{m_j}\frac{\frac{1}{x_{ji}}e^{\frac{\eta}{x_{ji}}}}{(e^{\frac{\eta}{x_{ji}}}-1)}+\alpha^{j-1}\lambda\gamma\sum_{i=1}^{m_j}W_1\left(x_{ji},\gamma,\eta\right)\right.$$
$$\left.+\sum_{i=1}^{k_j}R_{ji}\alpha^{j-1}\lambda\gamma\sum_{i=1}^{m_j}W_1\left(x_{ji},\gamma,\eta\right)+A_{m_j}\alpha^{j-1}\lambda\gamma W_1\left(x_{m_j},\gamma,\eta\right)\right), \tag{16}$$

$$\frac{\partial\ell}{\partial\alpha}=\frac{m_2}{\alpha}-Z\left(x_{2i},\lambda,\ \gamma,\ \eta\right), \tag{17}$$

where

$$W\left(y,\eta\right)=(e^{\frac{\eta}{y}}-1),\ W_1\left(y,\gamma,\eta\right)=\frac{1}{y}e^{\frac{\eta}{y}}(e^{\frac{\eta}{y}}-1)^{-\gamma-1},$$
$$Z\left(x_{2i},\lambda,\ \gamma,\ \eta\right)=\lambda\sum_{i=1}^{m2}(e^{\frac{\eta}{x_{2i}}}-1)^{-\gamma}+\sum_{i=1}^{k_2}R_{2i}\lambda(e^{\frac{\eta}{x_{2i}}}-1)^{-\gamma}+A_{m_2}\lambda(e^{\frac{\eta}{x_{m_2}}}-1)^{-\gamma}. \tag{18}$$

We can show from the likelihood equations that, for given $\lambda,\ \gamma$ and η, the MLE of α, from Eq. (17), is

$$\hat{\alpha}(\lambda,\ \gamma,\ \eta)=\frac{-m_2}{Z\left(x_{2i},\lambda,\ \gamma,\ \eta\right)}. \tag{19}$$

By replacing α by $\hat{\alpha}(\lambda,\ \gamma,\ \eta)$ in (14), (15) and (16), we obtain the profile likelihood equations for $\lambda,\ \gamma$ and η. Once $\hat{\lambda},\ \hat{\gamma}$ and $\hat{\eta}$ are obtained as the solution of the system of equations $\frac{\partial\ell}{\partial\lambda}=\frac{\partial\ell}{\partial\gamma}=\frac{\partial\ell}{\partial\eta}=0$.

4. Bayes estimation

This section deals with obtaining the Bayesian estimation for the unknown parameters when the data are obtained from WIE model under different loss functions based on constant stress PALT under adaptive progressive type-II censored data.

4.1. Prior and posterior distribution

In practical works the parameters cannot be treated as a constant during the life testing time. Therefore, considering the lifetime model's parameters to be random variables would be accurate. The type of prior information we have access to frequently determines the prior distribution that is chosen. When we don't know much or anything about the parameter, It is best to employ a non-informative prior (NIP). In many practical situations, the information about the parameters are available in an independent manner. In this section, we take an informative prior distribution for the parameter λ as the gamma with the scale parameter a and shape parameter b, and the parameters $\gamma,\ \eta$ and α have NIP, thus

$$\pi_1(\lambda)\propto\lambda^{a-1}e^{-\lambda b},\ \lambda>0,a>0,b>0.$$

$$\pi_2(\gamma)\propto\frac{1}{\gamma},\quad \gamma>0.$$

$$\pi_3(\eta)\propto\frac{1}{\eta},\quad \eta>0.$$

$$\pi_4(\alpha)\propto\frac{1}{\alpha},\quad \alpha>1.$$

In case of NIP, we take $a\to 0$ and $b\ \to 0$.

Therefore, the joint prior of the parameters $\lambda,\ \gamma,\ \eta$ and α can be expressed by

$$g(\lambda,\ \gamma,\ \eta,\alpha)\propto\lambda^{a-1}\gamma^{-1}\eta^{-1}\alpha^{-1}e^{-\lambda b},\ \lambda>0,\ \gamma>0,\ \eta>0,\ \alpha>1. \tag{20}$$

The joint posterior density function of the parameters $\lambda,\ \gamma,\ \eta,\alpha$ can be expressed by using $L(\alpha,\lambda,\ \gamma,\ \eta\,|\,\underline{x})$ and $g(\lambda,\ \gamma,\ \eta,\ \alpha)$ from Eqs. (12) and (20), we get after simplification, the posterior distribution as

$$\pi_\theta(\alpha,\lambda,\ \gamma,\ \eta\,|\,\underline{x})\propto\lambda^{m_1+m_2+a-1}\gamma^{m_1+m_2-1}\eta^{m_1+m_2-1}\alpha^{m_2-1}\times\prod_{j=1}^{2}\prod_{i=1}^{m_j}\tfrac{1}{x_{ji}^2}e^{\frac{\eta}{x_{ji}}}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma-1}$$
$$\times\exp\left[-\lambda\left(b+\sum_{j=1}^{2}\left(\sum_{i=1}^{m_j}\alpha^{j-1}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}+\sum_{i=1}^{k_j}\alpha^{j-1}R_{ji}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}\right)+\alpha A_{m_j}(e^{\frac{\eta}{x_{m_j}}}-1)^{-\gamma}\right)\right]. \tag{21}$$

We noted that the Bayes estimators' form is implicit and cannot be analytically resolved. By establishing a Markov chain Monte Carlo (MCMC) with a limiting distribution that is equal to the target. In such a situation, BEs and highest posterior density (HPD) credible intervals of the parameters will be computed using the MCMC methods using Gibbs sampler and Metropolis-Hastings (see Hastings [28]) algorithm.

Thus for implementing the Gibbs algorithm, the full conditional posterior densities of $\lambda,\ \gamma,\ \eta$ and α are given by

$$\pi_\alpha(\alpha\,|\,\lambda,\ \gamma,\ \eta,\underline{x})\propto\alpha^{m_2-1}\times\exp\left[-\alpha T\left(x_{2i},\lambda,\ \gamma,\ \eta\right)\right], \tag{22}$$

where,

$$T\left(x_{2i},\lambda,\ \gamma,\ \eta\right)=\lambda\times\left(\sum_{i=1}^{m_2}(e^{\frac{\eta}{x_{2i}}}-1)^{-\gamma}+\sum_{i=1}^{k_2}R_{2i}(e^{\frac{\eta}{x_{2i}}}-1)^{-\gamma}+A_{m_2}(e^{\frac{\eta}{x_{m_2}}}-1)^{-\gamma}\right), \tag{23}$$

$$\pi_\lambda(\lambda\,|\,\alpha,\ \gamma,\ \eta,\underline{x})\propto\lambda^{m_1+m_2+a-1}$$
$$\times\exp\left[-\lambda\left(b+\sum_{j=1}^{2}\left(\sum_{i=1}^{m_j}\alpha^{j-1}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}+\sum_{i=1}^{k_j}\alpha^{j-1}R_{ji}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}\right)+\alpha A_{m_j}(e^{\frac{\eta}{x_{m_j}}}-1)^{-\gamma}\right)\right], \tag{24}$$

$$\pi_\gamma(\gamma\,|\,\alpha,\lambda,\ \eta,\underline{x}) \propto \gamma^{m_1+m_2-1}\prod_{j=1}^{2}\prod_{i=1}^{m_j}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma-1}$$
$$\times\exp\left[-\lambda\left(b+\sum_{j=1}^{2}\left(\sum_{i=1}^{m_j}\alpha^{j-1}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}+\sum_{i=1}^{k_j}\alpha^{j-1}R_{ji}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}\right)+\alpha A_{m_j}(e^{\frac{\eta}{x_{m_j}}}-1)^{-\gamma}\right)\right], \tag{25}$$

and

$$\pi_\eta(\gamma\,|\,\alpha,\lambda,\ \gamma,\underline{x}) \propto \eta^{m_1+m_2-1}\times\prod_{j=1}^{2}\prod_{i=1}^{m_j}\tfrac{1}{x_{ji}^2}e^{\frac{\eta}{x_{ji}}}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma-1}$$
$$\times\exp\left[-\lambda\left(b+\sum_{j=1}^{2}\left(\sum_{i=1}^{m_j}\alpha^{j-1}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}+\sum_{i=1}^{k_j}\alpha^{j-1}R_{ji}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}\right)+\alpha A_{m_j}(e^{\frac{\eta}{x_{m_j}}}-1)^{-\gamma}\right)\right]. \tag{26}$$

Therefore, samples of α and λ generated by using a gamma distribution. However, the posterior distribution of γ and η cannot be reduced analytically to a well-known distribution, and therefore it is not possible to sample directly by standard methods. Therefore, we use the Metropolis-Hasting algorithm with the normal proposal distribution to generate a random sample from the posterior densities of $\lambda,\ \gamma,\ \eta$ and α . We use the following algorithm to compute the BE of $\lambda,\ \gamma,\ \eta$ and α

.Algorithm (1)

Step 1: Start with an ($\lambda^{(0)},\gamma^{(0)},\eta^{(0)},\alpha^{(0)}$).

Step 2: Set $j=1$.

Step 3: Generate $\alpha^{(j)}$ from Gamma ($m_2,T\left(x_{2i},\lambda^{(j-1)},\ \gamma^{(j-1)},\ \eta^{(j-1)}\right)$), where $T\left(x_{2i},\lambda^{(\bullet)},\ \gamma^{(\bullet)},\ \eta^{(\bullet)}\right)$ is as given by Eq. (23).

Step 4: Generate $\lambda^{(j)}$ from

Gamma ($m_{1+}m_2,(b+\sum_{j=1}^{2}(\sum_{i=1}^{m_j}\alpha^{j-1}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma}+\sum_{i=1}^{k_j}\alpha^{j-1}R_{ji}(e^{\frac{\eta}{x_{ji}}}-1)^{-\gamma})+\alpha A_{m_j}(e^{\frac{\eta}{x_{m_j}}}-1)^{-\gamma})$).

Step 5: Using the proposal distribution and the Metropolis-Hasting algorithm, generate $\gamma^{(j)}$ from $\pi_\gamma(\gamma^{(j-1)}\,|\,\alpha^{(j)},\lambda^{(j)},\eta^{(j-1)},\underline{x})$, as follows:

- Generate $\gamma^{(*)}$ from the proposal distribution $q(\gamma)$.
- Calculate the acceptance probability
$$\rho(\gamma,\gamma^{(*)})=\min\left[1,\frac{\pi_\gamma(\gamma^{(*)}\,|\,\alpha^{(j)},\lambda^{(j)},\eta^{(j-1)},\underline{x})q(\gamma^{(j-1)})}{\pi_\gamma(\gamma^{(j-1)}\,|\,\alpha^{(j)},\lambda^{(j)},\eta^{(j-1)},\underline{x})q(\gamma^{(*)})}\right].$$
- Generate from u from Uniform $(0,1)$. $u\le\rho(\gamma,\gamma^{(*)})$ accept the proposal and set $\gamma^{(*)}=\gamma^{(j)}$. Otherwise, reject the proposal and set $\gamma^{(*)}=\gamma^{(j-1)}$.

Step 6: Repeate the previous step, using Metropolis-Hasting algorithm, generate $\eta^{(j)}$ from $\pi_\eta(\eta^{(j-1)}\,|\,\alpha^{(j)},\lambda^{(j)},\gamma^{(j)}\underline{x})$, with the $N(\eta^{(j-1)},\widehat{s}e\ (\widehat{\eta}))$, proposal distribution.

Step 7: Set $j=j+1$.

Step 8: Repeat Steps 2 through 6 , N times, and obtain the posterior samples $\lambda^{(j)}$, $\gamma^{(j)}$, $\eta^{(j)}$, and $\alpha^{(j)}$, $j=1,2,3,...,N$.

The initial iteration value in this study is MLEs rather than arbitrary estimation. These samples are used to compute the BEs, and to construct the HPD credible intervals for λ,γ,η and α .

4.2. Bayes estimation based on balanced loss function

In Bayesian approach, to select a single value that represents the best estimate of an unknown parameter, one must specify a loss function. This paper proposes the use of balanced loss function, which creates a balance between classical and Bayesian approaches, and provides an estimate that is a linear combination of ML and BEs. Ahmadi et al. [27] suggested the use of so-called balanced loss function, to be in the form

$$L_{\rho,\omega,\delta_0}(\theta,\delta)=\omega q(\theta)\rho(\delta_0,\delta)+(1-\omega)q(\theta)\rho(\theta,\delta). \tag{27}$$

where $\omega\in[0,1)$, $q(\theta)$ is a suitable positive weight function and $\rho(\theta,\delta)$ is an arbitrary loss function when estimating θ by δ. The parameter δ_0 is a chosen prior estimator of θ, obtained for example from the criterion of ML, least squares or moment among others. A general development with regard to BEs under $L_{\rho,\omega,\delta_0}(\theta,\delta)$ is given, namely by relating such estimators to Bayes solutions to the unbalanced case, i.e., $L_{\rho,\omega,\delta_0}(\theta,\delta)$; with $\omega=0$. $L_{\rho,\omega,\delta_0}(\theta,\delta)$ can be specialized to various choices of loss function, such as for squares error (SE) and linear exponential (LINEX) loss functions.

By choosing $\rho(\theta,\delta)=(\delta-\theta)^2$ and $q(\theta)=1$, the balanced loss function (27) reduced to the balanced SE loss (BSEL) function, in the form

$$\delta_{\omega,\delta_0}(\theta,\delta)=\omega(\delta-\delta_0)^2+(1-\omega)(\delta-\theta)^2, \tag{28}$$

and the corresponding BE of the function θ is given by

$$\hat{\theta}_{BS}=\omega\delta_0+(1-\omega)E(\theta\,|\,\underline{x}). \tag{29}$$

By choosing $q(\theta)=1$ and $\rho(\theta,\delta)=e^{c(\delta-\theta)}-c(\delta-\theta)-1$ in Eq. (27) reduced to the balanced LINEX loss (BLINEXL) function in the form:

$$\hat{\theta}_{BL}=-\frac{1}{c}\log\left[\omega e^{-c\delta_0}+(1-\omega)E\left(e^{-c\theta}\,|\,\underline{x}\right)\right], \tag{30}$$

where $c\neq 0$ is the shape parameter of BLINEXL function.

Using Eqs. (27) - (30) the approximate BEs under the BSEL and BLINEXL functions for $\theta=(\lambda,\gamma,\eta,\alpha)$ are given, respectively by:

$$\hat{\theta}_S=\omega\,\hat{\theta}_{ML}+(1-\omega)\frac{\sum_{i=M+1}^{N}\theta^{(i)}}{N}, \tag{31}$$

and

$$\hat{\theta}_L=-\frac{1}{c}\log\left[\omega e^{-a\hat{\theta}_{ML}}+(1-\omega)\frac{\sum_{i=M+1}^{N}e^{-a\theta^{(i)}}}{N}\right]. \tag{32}$$

5. Interval estimation

In this section, the approximate, Bootstrap and credible intervals of the parameters λ, γ, η and α are derived.

5.1. Asymptotic confidence interval

In this subsection, the approximate confidence intervals of the parameters are obtained based on the asymptotic distributions of the MLEs of the elements of the vector of unknown parameters. It is known that the asymptotic distribution of the MLEs of λ, γ, η and α is given, the exact solution for the confidence intervals is not possible since the distributions of MLEs are not explicitly defined here. However, one can obtain the asymptotic confidence intervals using large sample property of the MLE.

Under this property, the asymptotic sampling distribution of $\begin{pmatrix}\hat{\lambda}-\lambda\\ \hat{\gamma}-\gamma\\ \hat{\eta}-\eta\end{pmatrix}$ is $N_3(0,\Omega^{-1})$, where, Ω is the observed Fisher information matrix and which is defined as follows:

$$\Omega=\begin{pmatrix} -\frac{\partial^2\ell}{\partial\lambda^2} & -\frac{\partial^2\ell}{\partial\lambda\partial\gamma} & -\frac{\partial^2\ell}{\partial\lambda\partial\eta} & -\frac{\partial^2\ell}{\partial\lambda\partial\alpha} \\ -\frac{\partial^2\ell}{\partial\gamma\partial\lambda} & -\frac{\partial^2\ell}{\partial\gamma^2} & -\frac{\partial^2\ell}{\partial\gamma\partial\eta} & -\frac{\partial^2\ell}{\partial\gamma\partial\alpha} \\ -\frac{\partial^2\ell}{\partial\eta\partial\lambda} & -\frac{\partial^2\ell}{\partial\eta\partial\gamma} & -\frac{\partial^2\ell}{\partial\eta^2} & \frac{\partial^2\ell}{\partial\eta\partial\alpha} \\ -\frac{\partial^2\ell}{\partial\alpha\partial\lambda} & -\frac{\partial^2\ell}{\partial\alpha\partial\gamma} & -\frac{\partial^2\ell}{\partial\alpha\partial\eta} & -\frac{\partial^2\ell}{\partial\alpha^2} \end{pmatrix}_{(\hat{\lambda},\,\hat{\gamma},\,\hat{\eta},\hat{\alpha})}, \tag{33}$$

whose elements are given in Appendix.

The diagonal elements of Ω^{-1} provides the asymptotic variances for the parameters $\lambda,\ \gamma,\ \eta$ and α respectively. Then two-sided 100(1- ϑ)% normal approximation confidence interval of $\lambda,\ \gamma,\ \eta$ and α can be obtained as

$$\left(\hat{\lambda}\pm z_{\vartheta/2}\,\widehat{se}(\hat{\lambda})\right),\left(\hat{\gamma}\pm z_{\vartheta/2}\,\widehat{se}(\hat{\gamma})\right),\left(\hat{\eta}\pm z_{\vartheta/2}\,\widehat{se}(\hat{\eta})\right) \text{ and } \left(\hat{\alpha}\pm z_{\vartheta/2}\,\widehat{se}(\hat{\alpha})\right), \tag{34}$$

Where $se(.)$ is the square root of the diagonal element of Δ^{-1} corresponding to each parameter, and $z_{\vartheta/2}$ is the quantile $100(1-\vartheta/2)\%$ of the standard normal distribution.

5.2. Bootstrap confidence intervals

In this section, confidence intervals based on the parametric bootstrap method for the unknown parameters $\lambda,\ \gamma,\ \eta$ and α using percentile interval are derived, for more details see Efron and Tibshirani [29]. The following algorithm is construct to obtain a bootstrap sample.

1. From the original two sample $\{x_1, x_2,..., x_n\}$ compute MLEs $\hat{\lambda},\ \hat{\gamma},\ \hat{\eta},$ and $\hat{\alpha}$.
2. Using $\hat{\lambda},\ \hat{\gamma},\ \hat{\eta},$ and $\hat{\alpha}$ to generate a bootstrap sample $\{x_1^*, x_2^*,..., x_n^*\}$ and compute the bootstrap estimate of $\hat{\lambda}^*,\ \hat{\gamma}^*,\ \hat{\eta}^*$ and $\hat{\alpha}^*$.
3. Repeat steps (1)-(2), B times and arrange each estimate in ascending order to obtain the bootstrap samples $\{\hat{\lambda}_1^*,\ \hat{\lambda}_2^*,\ ...,\ \hat{\lambda}_B^*\}$, $\{\hat{\gamma}_1^*,\ \hat{\gamma}_2^*,...,\ \hat{\gamma}_B^*\}$, $\{\hat{\eta}_1^*,\ \hat{\eta}_2^*,\ ...,\ \hat{\eta}_B^*\}$ and $\{\hat{\alpha}_1^*,\ \hat{\alpha}_2^*,\ ...,\ \hat{\alpha}_B^*\}$.

   The approximate $100(1-\frac{\vartheta}{2})\%$ confidence interval for θ_i is given by

   $(\hat{\theta}^*_{i\left(N\frac{\vartheta}{2}\right)},\ \hat{\theta}^*_{i\left(1-N\frac{\vartheta}{2}\right)}), i=1,2,3,4,$

   where $\hat{\theta}_1^* = \hat{\lambda}^*,\ \hat{\theta}_2^* = \hat{\gamma}^*,\ \hat{\theta}_3^* = \hat{\eta}^*,\ \hat{\theta}_4^* = \hat{\alpha}^*$.

5.3. Credible confidence intervals

A $100\,(1-\vartheta)\%$ Bayesian credible or posterior interval for a random quantity θ is the interval that has the posterior probability $(1-\vartheta)$ that θ lies in the interval.

The following algorithm is performed to obtain credible confidence intervals of $\lambda,\ \gamma\ ,\eta$ and α.

Algorithm (2)

1. Repeat steps (1) to (6) in algorithm (1)
2. Then using the algorithm proposed by Chen and Shao [30], the Bayesian credible interval for the parameter is obtained by using the generated MCMC samples. By arranging the posterior sample $\theta^{(j)},\ j=1,2,3,..., N$ as $\theta_{(1)} < \theta_{(2)} < ... < \theta_{\theta_{(N)}}$, the $100(1-\vartheta)\%$ HPD credible intervals for $\theta=(\lambda,\gamma,\eta,\alpha)$ is given by

$$(\theta_{(J)},\ \theta_{(J+[N(1-\vartheta)])}), \tag{35}$$

where J is chosen such that

$$\theta_{(J+[N(1-\vartheta)])} - \theta_{(J)} = \min_{1\le i\le \vartheta N}\left(\theta_{(i+[N(1-\vartheta)])} - \theta_{(i)}\right);\ j=1,2,..., N.$$

6. Numerical exploration

To show how the suggested method may be used, we perform out a simulation study and examine an example in this section.

6.1. Simulation study

To investigate how the ML and BEs respond in terms of their mean square errors (MSEs) and coverage probabilities depending on various sample size choices a simulation study is carried out, let $n_1 = 20, 25, 30, 40$ and $n_2 = 20, 25, 40, 50$; different effective sample sizes, $m_1 = 10, 15, 18, 20$ and $m_2 = 10, 18, 20, 25$; and 18 different censoring schemes; details of the schemes are given in Table 1. Different progressive censoring schemes(CSs) are considered with notation that ($5, 0^3$) means ($5, 0, 0, 0$).

In all cases we have used $\lambda = 0.08,\ \gamma = 0.3,\ \eta = 0.2$, $\alpha = 1.5$, and $T = 0.85$. For a given n_j , m_j , T , $\lambda,\ \gamma$, η , α and schemes $R_{ji},\ i = 1, 2, ..., m_j,\ j = 1, 2$, using the algorithm proposed by Ng et al. [26] . we have generated a sample for a given CS.

1. For $j = 1, 2$, generate m_j independent and identically distributed random numbers ($U_{j1}, U_{j2}, ..., U_{jm_j}$) from uniform distribution $U[0,1]$.
2. Determine the values of the censored scheme R_{ji} , $i = 1, 2, ..., m_j$, such that $\sum_{i=1}^{m_j} R_{ji} = n_j - m_j,\ j = 1, 2.$
3. Set $E_{ji} = U_{ji}^{1/\left[i + \sum_{h=m_j-i+1}^{m_j} R_{jh}\right]},\ i = 1, 2, ..., m_j,\ j = 1, 2.$
4. Generate the progressive type-II censored sample ($U^*_{j1}, U^*_{j2}, ..., U^*_{jm_j}$), where $U^*_{ji} = 1 - \prod_{h=m_j-i+1}^{m_j} E_{jh}$,

   $i = 1, 2, ..., m_j,\ j = 1, 2.$
5. The order observations $x_{j1:m_j:n_j}, x_{j2:m_j:n_j}, ..., x_{jm_j:m_j:n_j}$, are calculated as follows

   x_{ji} [illegible] log [illegible] log [illegible] U^*_{ji} [illegible] 1/ [illegible] 1 [illegible], i = $1, 2, \ldots, m_j$, j = $1, 2$
6. Determine the value of k_j , where $X_{j_{k_j}:m_j:n_j} < T < X_{j(k_j+1):m_j:n_j}$, and discard the sample $X_{j(k_j+2):m_j:n_j}$,..., $X_{j_{m_j}:m_j:n_j}$.
7. Generate the first $m_j - k_j - 1$ order statistics from a truncated distribution $f(x_j)/[1 - F(x_{k_j+1})]$, with sample size ($n_j - \sum_{i=1}^{k_j} R_{ji} - k_j - 1$) as $X_{j(k_j+2):m_j:n_j}, ..., X_{j_{m_j}:m_j:n_j}$.

We calculate the acceleration factor and the MLEs of the unknown parameters $\lambda,\ \gamma$ and η and the acceleration factor α using the generated data. The Newton-Raphson method is applied for solving the nonlinear system to obtain the MLEs of the parameters and compute the approximate intervals. We also compute the BEs of the unknown parameters based on the MCMC sampling procedure. For BE, we are used informative prior for the parameter λ , we have used the hyper parameters value as $a = 0.1$ and $b = 1$. Based on 1000 replications, we calculate the average estimates (AE) and the average MSE of the estimations. Results are reported in tables (2-5). In all cases BSEL and BLINEXL functions, with $\omega = 0.2, 0.8$, have been used for computing the BEs.

The following points are quite clear from the tables (2-5). As sample size ($n_j,\ j = 1, 2$) increases, the MSEs of estimates of all the unknown parameters decrease. For fixed $n_j,\ j = 1, 2$, the MSEs of estimators of model parameters decrease as $m_j,\ j = 1, 2$ increases. From tables 6 and 7 we discover that the Bayesian credible intervals and approximate confidence intervals' coverage probability are relatively near to the nominal level. Also, in most cases, the Bayesian credible intervals are marginally shorter length than that of the approximate confidence intervals. Hence, we recommend to use Bayesian credible intervals over approximate confidence intervals. When $\omega = 0.8$ all results of BEs under both BSEL and BLINEXL functions for the parameters are quite similar to corresponding MLEs.

6.2. Numerical example

The simulated data observed based on $n_1 = 35$, $n_2 = 45$, $m_1 = 25$, $m_2 = 35$, $\lambda = 0.1$, $\gamma = 0.25$, $\eta = 0.35$, $\alpha = 1.5$, $R_1 = (0^{10}, 1^5, 0^{10})$ and $R_2 = (0^{15}, 1^5, 0^{15})$. The LEs of model parameters obtained by using N Maximize option of Mathematica 10 are

$\hat{\lambda} = 0.0912$, $\hat{\gamma} = 0.2423$, $\hat{\eta} = 0.3085$, $\hat{\alpha} = 1.5057$.

The 95% asymptotic confidence intervals for the parameters are:

$0.0641 \le \lambda \le 0.1113$, $0.1604 \le \gamma \le 0.2792$, $0.2710 \le \eta \le 0.3874$ and $1.1156 \le \alpha \le 1.9983$,

The bootstrap estimate of model parameters are obtained as

$\hat{\lambda} = 0.0841$, $\hat{\gamma} = 0.1984$, $\hat{\eta} = 0.3421$, $\hat{\alpha} = 1.5447$.

95% bootstrap confidence intervals for the parameters λ, γ, η and the acceleration factor α are

$0.0754 \le \lambda \le 0.1089$, $0.1784 \le \gamma \le 0.2867$, $0.2548 \le \eta \le 0.3751$ and $1.1234 \le \alpha \le 2.0097$,

we compute the BEs of λ, γ, η and the acceleration factor α. Since we do not have any prior information, we assume $a = b = 0$. Figure 1 shows the trace plots of 10000 MCMC samples for posterior distribution of λ, γ, η and α. It show that the MCMC procedure converges very well. Therefore.

Hence, under BSEL function ($\omega = 0$), we compute the approximate Bayes estimates of λ, γ, η and α using MCMC method and they are

$\hat{\lambda} = 0.0794$, $\hat{\gamma} = 0.2378$, $\hat{\eta} = 0.3512$, $\hat{\alpha}_{BS} = 1.4236$,

and the associated 95% symmetric credible intervals are given by

$0.0550 \le \lambda \le 0.1851$, $0.2214 \le \gamma \le 0.3810$, $0.1845 \le \eta \le 0.6421$ and $1.0210 \le \alpha \le 1.8940$, ,

Under BLINEXL function ($\omega = 0$), with $c_1 = 1$ and $c_2 = 5$, we compute the approximate BEs of λ, γ, η and α, and they are

$\hat{\lambda} = 0.0921$, $\hat{\gamma} = 0.2401$, $\hat{\eta} = 0.3608$, $\hat{\alpha}_{BL} = 1.4006$,

$\hat{\lambda} = 0.1105$, $\hat{\gamma} = 0.2604$, $\hat{\eta} = 0.3508$, $\hat{\alpha}_{BL} = 1.3169$,

We also compute the approximate BEs of λ, γ, η and α under both BSEL and BLINEXL function with $\omega = 0.2, 0, 8$ and they are in Table 8.

7. Concluding remarks

In this paper, we have considered the constant stress PALT when the observed data come from WIE distribution under adaptive progressive type-II censoring. We derived ML and Bayes estimators of the parameters and the acceleration parameter using NIP and gamma informative priors under both BSEL and BLINEXL functions. These estimates cannot be obtained in closed form, but can be computed numerically. Asymptotic confidence intervals based on observed Fisher information and HPD credible intervals of the parameter are developed. We made use of MH algorithm for BE. A simulation study was carried out to compare and contrast how well the suggested approaches performed for various sample sizes and CSs. Based on the simulation study, we find that in terms of MSEs, the Bayes estimates perform better than MLEs. Also, In terms of average length and coverage probability, the HPD credible intervals based on the Metropolis-Hastings algorithm perform better than asymptotic confidence intervals. Furthermore, As sample size increases, the length of the confidence interval likewise shortens, and for all sets of parameters taken into consideration, the coverage probability is about equal to the nominal value.

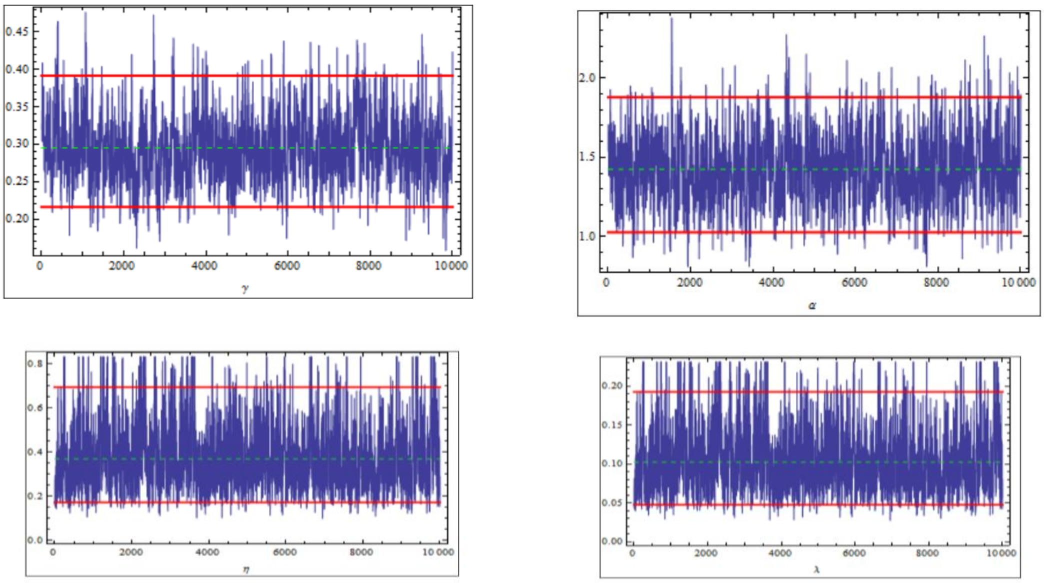

Figure 1. MCMC iterations and the posterior samples' kernel histograms for each parameter.

Table 1. Several CSs for the simulation study.

CS	n_1	n_2	m_1	m_2	R_1	R_2	*CS*	n_1	n_2	m_1	m_2	R_1	R_2
[1]	30	40	15	20	$(15,0^{14})$	$(20,0^{19})$	[10]	20	20	10	10	$(10,0^{9})$	$(10,0^{9})$
[2]					(1^{15})	(1^{20})	[11]					(1^{10})	(1^{10})
[3]					$(0^{14},15)$	$(0^{19},20)$	[12]					$(0^{9},10)$	$(0^{9},10)$
[4]	30	40	20	25	$(10,0^{19})$	$(15,0^{24})$	[13]	20	20	18	18	$(12,0^{17})$	$(12,0^{17})$
[5]					$(5,0^{18},5)$	$(10,0^{23},5)$	[14]					$(6,0^{16},6)$	$(6,0^{16},6)$
[6]					$(0^{19},10)$	$(0^{24},15)$	[15]					$(0^{17},12)$	$(0^{17},12)$
[7]	40	50	20	25	$(20,0^{19})$	$(25,0^{24})$	[16]	25	25	18	18	$(7,0^{17})$	$(7,0^{17})$
[8]					$(10,0^{18},10)$	$(13,0^{23},12)$	[17]					$(4,0^{16},3)$	$(4,0^{16},3)$
[9]					$(0^{19},20)$	$(0^{24},25)$	[18]					$(0^{17},7)$	$(0^{17},7)$

Table 2. AE and MSEs for ML and BEs under BSEL and BLINEXL functions of α.

CS	*MLE*	*BE*									
		BSEL			BLINEX						
		ω=0	ω=0.2	ω=0.8	ω=0		ω=0.2		ω=0.8		
					c=1	c=5	c=1	c=5	c=1	c=5	
[1]	1.3970	1.4527	1.4415	1.4081	1.3251	1.3311	1.3193	1.4417	1.4024	1.3805	
	(0.0494)	(0.0282)	(0.0394)	(0.0436)	(0.0440)	(0.0401)	(0.0460)	(0.0414)	(0.0530)	(0.0499)	
[2]	1.4115	1.3421	1.3560	1.3976	1.322	1.2521	1.3389	1.2735	1.3925	1.3633	
	(0.0501)	(0.0309)	(0.0359)	(0.0440)	(0.0297)	(0.0415)	(0.0313)	(0.0380)	(0.0433)	(0.0382)	
[3]	1.4883	1.4315	1.4428	1.4769	1.4106	1.2387	1.4253	1.2593	1.4718	1.4441	
	(0.0557)	(0.0487)	(0.0333)	(0.0444)	(0.0329)	(0.0468)	(0.0343)	(0.0430)	(0.044)	(0.0405)	
[4]	1.5122	1.5396	1.5341	1.5177	1.5181	1.4424	1.5168	1.4540	1.5133	1.4953	
	(0.0414)	(0.0405)	(0.0413)	(0.0415)	(0.0331)	(0.0322)	(0.0347)	(0.0301)	(0.0306)	(0.0307)	
[5]	1.5052	1.4049	1.4250	1.4851	1.2874	1.2252	1.4096	1.2496	1.4802	1.4517	
	(0.0427)	(0.0374)	(0.0365)	(0.0396)	(0.0398)	(0.0335)	(0.0377)	(0.0771)	(0.0371)	(0.0365)	
[6]	1.5162	1.2918	1.4167	1.4913	1.2752	1.2165	1.4016	1.2426	1.4861	1.4545	
	(0.0428)	(0.0386)	(0.0365)	(0.0390)	(0.0392)	(0.0354)	(0.0379)	(0.0379)	(0.0384)	(0.0399)	
[7]	1.5071	1.5543	1.5449	1.5166	1.5329	1.4572	1.5276	1.4655	1.5122	1.4952	
	(0.0510)	(0.0473)	(0.0476)	(0.0497)	(0.0435)	(0.0377)	(0.0447)	(0.0392)	(0.0492)	(0.0467)	
[8]	1.4993	1.4425	1.4539	1.4880	1.4260	1.2672	1.4400	1.3865	1.4840	1.4627	
	(0.0382)	(0.0261)	(0.0276)	(0.0348)	(0.0274)	(0.0366)	(0.0281)	(0.0336)	(0.0344)	(0.0320)	
[9]	1.4992	1.4278	1.4421	1.4850	1.4127	1.3586	1.4291	1.3786	1.4810	1.4593	
	(0.0346)	(0.0231)	(0.0240)	(0.0308)	(0.0248)	(0.0349)	(0.0249)	(0.0314)	(0.0305)	(0.0284)	
[10]	1.5242	1.5891	1.5762	1.5372	1.5472	1.4149	1.5423	1.4299	1.5285	1.4920	

	(0.1024)	(0.0848)	(0.0872)	(0.0978)	(0.0724)	(0.0589)	(0.0777)	(0.0617)	(0.0957)	(0.0830)
[11]	1.5115	1.4637	1.4733	1.5019	1.4322	1.3303	1.4468	1.3525	1.4942	1.4508
	(0.0846)	(0.0428)	(0.0494)	(0.0744)	(0.0430)	(0.0582)	(0.0482)	(0.0550)	(0.0727)	(0.0599)
[12]	1.4896	1.4414	1.4510	1.4800	1.4121	1.3163	1.4264	1.3377	1.4728	1.4324
	(0.0821)	(0.0439)	(0.0498)	(0.0727)	(0.0453)	(0.0625)	(0.0497)	(0.0590)	(0.0715)	(0.0614)
[13]	1.5200	1.5269	1.5255	1.5214	1.5021	1.4163	1.5056	1.4325	1.5163	1.4933
	(0.0637)	(0.0558)	(0.0573)	(0.0620)	(0.0520)	(0.0490)	(0.05416)	(0.0490)	(0.0611)	(0.0563)
[14]	1.5147	1.4746	1.4827	1.5067	1.4515	1.3718	1.4637	1.3925	1.5017	1.4750
	(0.0577)	(0.0462)	(0.0481)	(0.0549)	(0.0453)	(0.0512)	(0.0468)	(0.0484)	(0.0542)	(0.0494)
[15]	1.5180	1.4387	1.4546	1.5021	1.4168	1.3409	1.4361	1.3653	1.4968	1.4661
	(0.0553)	(0.0483)	(0.0485)	(0.0527)	(0.0490)	(0.0596)	(0.0483)	(0.0541)	(0.0519)	(0.0474)
[16]	1.5276	1.5494	1.5450	1.5320	1.5247	1.4393	1.5252	1.4534	1.5270	1.5054
	(0.0456)	(0.0350)	(0.0337)	(0.0379)	(0.0349)	(0.0324)	(0.0401)	(0.0403)	(0.0430)	(0.0423)
[17]	1.5044	1.4410	1.4537	1.4917	1.4200	1.3469	1.4360	1.3685	1.4866	1.4581
	(0.0497)	(0.0473)	(0.0367)	(0.0455)	(0.0363)	(0.0379)	(0.0370)	(0.0379)	(0.0449)	(0.0411)
[18]	1.4866	1.3694	1.3929	1.4632	1.3510	1.2865	1.5095	1.3127	1.4577	1.4246
	(0.0595)	(0.0524)	(0.0443)	(0.0463)	(0.0509)	(0.04942)	(0.0465)	(0.0403)	(0.0469)	(0.0408)

Table 3. AE and MSEs for ML and BEs under BSEL and BLINEXL functions of λ.

CS	*MLE*	*BE*									
		BSEL			BLINEX						
		ω=0	ω=0.2	ω=0.8	ω=0		ω=0.2		ω=0.8		
					c=1	c=5	c=1	c=5	c=1	c=5	
[1]	0.0622	0.0764	0.0736	0.0850	0.0854	0.0823	0.0891	0.0688	0.0861	0.0742	
	(0.3656)	(0.2477)	(0.2575)	(0.3782)	(0.1785)	(0.1595)	(0.1946)	(0.1495)	(0.3338)	(0.1494)	
[2]	0.0860	0.0809	0.0828	0.0884	0.0754	0.0897	0.0841	0.0813	0.0848	0.0832	
	(0.3792)	(0.2475)	(0.2689)	(0.3479)	(0.2402)	(0.2305)	(0.2599)	(0.2070)	(0.3049)	(0.1405)	
[3]	0.0867	0.0856	0.0858	0.0865	0.0740	0.0610	0.0745	0.0614	0.0860	0.0734	
	0.5256	(0.3523)	(0.3809)	(0.4849)	(0.2186)	(0.2333)	(0.2429)	(0.2098)	(0.4064)	(0.1453)	
[4]	0.0861	0.0869	0.0734	0.0628	0.0607	0.0826	0.0570	0.0730	0.0607	0.0745	
	(0.2314)	(0.1684)	(0.1873)	(0.2158)	(0.1577)	(0.1463)	(0.1668)	(0.1380)	(0.1944)	(0.1521)	
[5]	0.0865	0.0743	0.0747	0.0861	0.0632	0.0809	0.0738	0.0613	0.0878	0.0743	
	(0.3683)	(0.2301)	(0.2465)	(0.3294)	(0.1878)	(0.1563)	(0.1927)	(0.2096)	(0.2881)	(0.2090)	
[6]	0.0850	0.0738	0.0741	0.0951	0.0729	0.0762	0.0733	0.0711	0.0749	0.0731	
	(0.3299)	(0.2260)	(0.2744)	(0.3122)	(0.2206)	(0.2153)	(0.2552)	(0.2080)	(0.2299)	(0.2436)	
[7]	0.0899	0.0827	0.0816	0.0812	0.0859	0.0682	0.0847	0.0614	0.0895	0.0740	
	(0.2984)	(0.2558)	(0.27417)	(0.2772)	(0.1670)	(0.1400)	(0.1766)	(0.1338)	(0.1786)	(0.1471)	
[8]	0.0812	0.0828	0.0847	0.08160	0.0717	0.0616	0.0753	0.0620	0.0875	0.0686	
	(0.3027)	(0.2554)	(0.2710)	(0.3014)	(0.1714)	(0.1872)	(0.1859)	(0.1684)	(0.1763)	(0.1267)	
[9]	0.0879	0.0767	0.0789	0.0857	0.0623	0.0923	0.0408	0.0658	0.0529	0.0734	
	(0.3010)	(0.2611)	(0.2847)	(0.2986)	(0.2187)	(0.2096)	(0.2315)	(0.1879)	(0.2515)	(0.1302)	
[10]	0.0842	0.0904	0.0901	0.0882	0.0864	0.0612	0.0867	0.0616	0.0878	0.0636	
	(0.1103)	(0.0824)	(0.1100)	(0.1102)	(0.1119)	(0.1181)	(0.2096)	(0.2279)	(0.2533)	(0.1844)	
[11]	0.0983	0.0976	0.0978	0.0982	0.0644	0.0588	0.0802	0.0566	0.0712	0.0626	
	(0.4332)	(0.2354)	(0.3041)	(0.3427)	(0.3618)	(0.3257)	(0.4169)	(0.2946)	(0.4184)	(0.1902)	
[12]	0.0981	0.0976	0.0977	0.0980	0.0743	0.0504	0.0748	0.0641	0.0769	0.0625	
	(0.2954)	(0.2505)	(1.1089)	(1.2438)	(0.3727)	(0.3420)	(0.4277)	(0.3104)	(0.4197)	(0.2027)	
[13]	0.0765	0.0970	0.0769	0.0966	0.0853	0.0620	0.0855	0.0655	0.0624	0.0641	
	(0.3143)	(0.2357)	(0.3008)	(0.3100)	(0.1468)	(0.1143)	(0.2099)	(0.1693)	(0.3071)	(0.1490)	
[14]	0.0918	0.0860	0.0871	0.0860	0.0720	0.0528	0.0755	0.0664	0.0873	0.0657	
	(0.3526)	(0.2389)	(0.3044)	(0.3396)	(0.1974)	(0.2066)	(0.2034)	(0.1947)	(0.3024)	(0.1395)	
[15]	0.0815	0.0775	0.0805	0.0887	0.06491	0.0578	0.0949	0.0615	0.0852	0.0620	
	(0.3793)	(0.2858)	(0.3000)	(0.3561)	(0.2085)	(0.2040)	(0.2281)	(0.2264)	(0.3103)	(0.1408)	
[16]	0.0864	0.0926	0.0970	0.0801	0.0805	0.0822	0.0887	0.0606	0.0743	0.0743	
	(0.3007)	(0.2778)	(0.2920)	(0.3001)	(0.2433)	(0.1681)	(0.2664)	(0.1556)	(0.3197)	(0.1472)	
[17]	0.0811	0.0800	0.0822	0.0889	0.0720	0.1069	0.0928	0.0731	0.0549	0.1278	
	(0.3886)	(0.2803)	(0.2973)	(0.3623)	(0.2026)	(0.2002)	(0.2168)	(0.2161)	(0.3585)	(0.1476)	
[18]	0.0824	0.0522	0.0547	0.0823	0.0917	0.0658	0.0669	0.0744	0.0933	0.0857	
	(0.4230)	(0.2814)	(0.2989)	(0.3838)	(0.2059)	(0.2023)	(0.2298)	(0.2607)	(0.3309)	(0.1550)	

Table 4. AE and MSEs for ML and BEs under BSEL and BLINEXL functions of γ.

CS	*MLE*	*BE*								
		BSEL			BLINEX					
		ω=0	ω=0.2	ω=0.8	ω=0		ω=0.2		ω=0.8	
					c=1	c=5	c=1	c=5	c=1	c=5
[1]	0.3395	0.3097	0.3264	0.3765	0.2638	0.2565	0.2879	0.2837	0.3654	0.2862
	(0.1269)	(0.0931)	(0.0980)	(0.1183)	(0.0851)	(0.0503)	(0.08910)	(0.0867)	(0.1135)	(0.0805)
[2]	0.3129	0.3234	0.3105	0.3373	0.2807	0.2936	0.2654	0.3066	0.3248	0.3695
	(0.1307)	(0.1436)	(0.1361)	(0.1284)	(0.1112)	(0.0525)	(0.1109)	(0.0567)	(0.1228)	(0.0892)
[3]	0.3154	0.2653	0.2353	0.2454	0.2108	0.3206	0.3892	0.3288	0.3321	0.3773
	(0.1315)	(0.1527)	(0.1416)	(0.1289)	(0.1183)	(0.0856)	(0.1147)	(0.0553)	(0.1230)	(0.0889)
[4]	0.2791	0.2362	0.3447	0.2705	0.2978	0.2625	0.3133	0.2898	0.2621	0.3090
	(0.0780)	(0.0810)	(0.0809)	(0.0821)	(0.0822)	(0.0789)	(0.0856)	(0.0747)	(0.0810)	(0.0736)
[5]	0.2701	0.2933	0.2486	0.3147	0.2461	0.2809	0.2077	0.3075	0.3024	0.2681
	(0.0880)	(0.0892)	(0.0887)	(0.0850)	(0.0859)	(0.0592)	(0.0987)	(0.0619)	(0.0909)	(0.0777)
[6]	0.2806	0.3625	0.3061	0.3370	0.3138	0.3415	0.2616	0.3224	0.3223	0.2868
	(0.0981)	(0.0972)	(0.0969)	(0.0994)	(0.0954)	(0.0813)	(0.0942)	(0.0798)	(0.1040)	(0.0894)
[7]	0.2755	0.3033	0.3177	0.2610	0.2679	0.2431	0.2883	0.2715	0.2527	0.2982
	(0.0910)	(0.0706)	(0.0734)	(0.0856)	(0.0660)	(0.0731)	(0.0682)	(0.0669)	(0.0829)	(0.0652)
[8]	0.2806	0.2033	0.2788	0.3051	0.2629	0.3189	0.2451	0.3259	0.2958	0.2605
	(0.0858)	(0.1055)	(0.0977)	(0.0858)	(0.0856)	(0.0439)	(0.0826)	(0.0476)	(0.0830)	(0.0688)
[9]	0.2729	0.2314	0.2997	0.3046	0.2906	0.2431	0.2645	0.2428	0.2941	0.2580
	(0.0772)	(0.1081)	(0.0953)	(0.0767)	(0.0864)	(0.0389)	(0.0786)	(0.0410)	(0.0735)	(0.0587)
[10]	0.3158	0.2428	0.2658	0.3349	0.2692	0.3358	0.3030	0.2718	0.3155	0.2586
	(0.2180)	(0.1747)	(0.1891)	(0.2447)	(0.1467)	(0.1543)	(0.1583)	(0.1394)	(0.2270)	(0.1167)
[11]	0.2712	0.2763	0.2553	0.1922	0.1895	0.3091	0.1825	0.3290	0.1712	0.2533
	(0.2674)	(0.1995)	(0.1952)	(0.2062)	(0.1397)	(0.0673)	(0.1453)	(0.0669)	(0.1902)	(0.0962)
[12]	0.2385	0.3526	0.3298	0.2613	0.2593	0.2561	0.2512	0.2803	0.2383	0.3057
	(0.2765)	(0.2744)	(0.2652)	(0.2665)	(0.1927)	(0.0704)	(0.1972)	(0.0731)	(0.2444)	(0.1194)
[13]	0.3087	0.2949	0.2976	0.3059	0.2441	0.2708	0.2566	0.2995	0.2953	0.3281
	(0.1345)	(0.1231)	(0.1253)	(0.1321)	(0.1039)	(0.0854)	(0.1088)	(0.0817)	(0.1270)	(0.0936)
[14]	0.3160	0.2130	0.1936	0.2354	0.2574	0.2671	0.2485	0.2868	0.2307	0.2691
	(0.1361)	(0.1612)	(0.1539)	(0.1388)	(0.1278)	(0.0691)	(0.1281)	(0.0721)	(0.1331)	(0.1013)
[15]	0.2201	0.3226	0.2821	0.2606	0.2615	0.2545	0.2311	0.2635	0.2465	0.3101
	(0.1355)	(0.2419)	(0.2131)	(0.1492)	(0.1898)	(0.0819)	(0.1737)	(0.0874)	(0.1417)	(0.1157)
[16]	0.2834	0.2429	0.2510	0.2753	0.2971	0.2390	0.3136	0.2681	0.2653	0.2997
	(0.1115)	(0.0927)	(0.0960)	(0.1072)	(0.0819)	(0.0826)	(0.0858)	(0.0764)	(0.1034)	(0.0771)
[17]	0.3052	0.2552	0.2252	0.2352	0.2005	0.3128	0.2792	0.2226	0.2221	0.2706
	(0.1121)	(0.1151)	(0.1100)	(0.1142)	(0.1197)	(0.0610)	(0.1227)	(0.0646)	(0.1185)	(0.0901)
[18]	0.3179	0.3144	0.3551	0.2772	0.3536	0.3144	0.3008	0.3259	0.2600	0.3176
	(0.1114)	(0.2692)	(0.1216)	(0.1269)	(0.1118)	(0.0786)	(0.1776)	(0.0813)	(0.1196)	(0.0998)

Table 5. AE and MSEs for ML and BEs under BSEL and BLINEXL functions of η.

CS	*MLE*	*BE*								
		BSEL			BLINEX					
		ω=0	ω=0.2	ω=0.8	ω=0		ω=0.2		ω=0.8	
					c=1	c=5	c=1	c=5	c=1	c=5
[1]	0.2459	0.1997	0.2264	0.2485	0.2125	0.2214	0.1954	0.1987	0.2058	0.2145
	(0.0218)	(0.0331)	(0.0380)	(0.0383)	(0.0351)	(0.0113)	(0.0291)	(0.0267)	(0.0335)	(0.0205)
[2]	0.1845	0.1984	0.2015	0.1875	0.1925	0.2006	0.2021	0.1987	0.2248	0.2145
	(0.0217)	(0.0126)	(0.0261)	(0.0184)	(0.0112)	(0.0225)	(0.0209)	(0.0267)	(0.0156)	(0.0292)
[3]	0.2154	0.2087	0.2065	0.2045	0.2054	0.2060	0.2192	0.1988	0.2121	0.1987
	(0.0215)	(0.0427)	(0.0316)	(0.0109)	(0.0123)	(0.0256)	(0.0347)	(0.0253)	(0.0130)	(0.0389)
[4]	0.2051	0.2142	0.2147	0.2305	0.2088	0.2025	0.2123	0.2017	0.2021	0.2091
	(0.0280)	(0.0210)	(0.0209)	(0.0221)	(0.0222)	(0.0289)	(0.0256)	(0.0247)	(0.0210)	(0.0236)
[5]	0.2065	0.2123	0.2106	0.1907	0.1961	0.2109	0.2071	0.2075	0.2024	0.2121

	(0.0280)	(0.0292)	(0.0283)	(0.0250)	(0.0259)	(0.0292)	(0.0387)	(0.0119)	(0.0309)	(0.0277)
[6]	0.2150	0.2095	0.2061	0.2150	0.2138	0.1954	0.2016	0.2224	0.2223	0.2068
	(0.0381)	(0.0372)	(0.029)	(0.0394)	(0.0354)	(0.0213)	(0.0342)	(0.0198)	(0.0340)	(0.0294)
[7]	0.2025	0.1953	0.2194	0.2020	0.2210	0.2054	0.2104	0.2046	0.1984	0.1935
	(0.0310)	(0.0206)	(0.0134)	(0.0256)	(0.0060)	(0.0131)	(0.0082)	(0.0069)	(0.0229)	(0.0052)
[8]	0.2065	0.2033	0.2210	0.2051	0.2015	0.9589	0.8551	0.9459	0.9358	0.2005
	(0.0258)	(0.0255)	(0.0327)	(0.0228)	(0.0216)	(0.0029)	(0.0216)	(0.0186)	(0.0210)	(0.0168)
[9]	0.2029	0.2114	0.2015	0.2046	0.2106	0.2094	0.2070	0.2098	0.2047	0.1980
	(0.0312)	(0.0241)	(0.0353)	(0.0267)	(0.0264)	(0.0289)	(0.0386)	(0.0110)	(0.0135)	(0.0187)
[10]	0.2158	0.1938	0.1908	0.2349	0.2054	0.2058	0.2030	0.2118	0.2155	0.2080
	(0.0180)	(0.0247)	(0.0191)	(0.0247)	(0.0127)	(0.0113)	(0.0223)	(0.0154)	(0.0124)	(0.0127)
[11]	0.1912	0.1984	0.1932	0.1920	0.1890	0.1937	0.1828	0.2291	0.1812	0.1904
	(0.0274)	(0.0295)	(0.0152)	(0.0262)	(0.0197)	(0.0173)	(0.0253)	(0.0169)	(0.0102)	(0.0262)
[12]	0.2054	0.2426	0.2298	0.2023	0.2184	0.2011	0.2042	0.2003	0.2047	0.2057
	(0.0165)	(0.0244)	(0.0252)	(0.0265)	(0.0227)	(0.0104)	(0.0272)	(0.0131)	(0.0144)	(0.0114)
[13]	0.1932	0.1949	0.1976	0.2059	0.1902	0.1708	0.1976	0.1995	0.1953	0.2281
	(0.0145)	(0.0131)	(0.0253)	(0.0321)	(0.0039)	(0.0254)	(0.0188)	(0.0217)	(0.0270)	(0.0136)
[14]	0.2010	0.2130	0.1936	0.2354	0.2074	0.2310	0.2240	0.2102	0.2165	0.2024
	(0.0361)	(0.0112)	(0.0239)	(0.0388)	(0.0278)	(0.0191)	(0.0281)	(0.0121)	(0.0331)	(0.0113)
[15]	0.1901	0.1930	0.1902	0.1972	0.1902	0.1945	0.1842	0.2045	0.2065	0.2101
	(0.0255)	(0.0119)	(0.0131)	(0.0292)	(0.0298)	(0.0219)	(0.0237)	(0.0274)	(0.0117)	(0.0157)
[16]	0.2045	0.2064	0.2103	0.2079	0.2107	0.2064	0.2136	0.2081	0.2153	0.2049
	(0.0115)	(0.0227)	(0.0260)	(0.0172)	(0.0219)	(0.0226)	(0.0258)	(0.0264)	(0.0134)	(0.0271)
[17]	0.2052	0.1902	0.1947	0.1932	0.1894	0.2147	0.2065	0.2210	0.2011	0.2006
	(0.0121)	(0.0151)	(0.0100)	(0.0142)	(0.0197)	(0.0110)	(0.0227)	(0.0246)	(0.0185)	(0.0201)
[18]	0.2015	0.2144	0.1951	0.1872	0.1936	0.1944	0.2004	0.1959	0.2054	0.2176
	(0.0114)	(0.0292)	(0.0216)	(0.0269)	(0.0118)	(0.0286)	(0.0376)	(0.0213)	(0.0196)	(0.0298)

Table 6. 95% approximate and credible CIs for α and λ .

CS	*MLE*		*BE*	
	α	λ	α	λ
[1]	1.4471(0.944)	0.0785(0.930)	1.4255(0.972)	0.088(0.962)
[2]	1.3459(0.940)	0.0764(0.932)	1.3058(0.972)	0.0713(0.940)
[3]	1.3239(0.932)	0.078(0.932)	1.2905(0.958)	0.0762(0.938)
[4]	1.3324(0.928)	0.06513(0.916)	1.3160(0.944)	0.0846(0.940)
[5]	1.2838(0.936)	0.0933(0.918)	1.2366(0.932)	0.0939(0.922)
[6]	1.2631(0.926)	0.0945(0.934)	1.2160(0.936)	0.0723(0.926)
[7]	1.3290(0.946)	0.0831(0.946)	1.3151(0.982)	0.0820(0.962)
[8]	1.2482(0.930)	0.0743(0.930)	1.2146(0.964)	0.0849(0.942)
[9]	1.2092(0.918)	0.0857(0.928)	1.5833(0.940)	0.0928(0.940)
[10]	1.1880(0.934)	0.0833(0.940)	1.7445(0.986)	0.0902(0.962)
[11]	1.5486(0.920)	0.0795(0.920)	1.4908(0.982)	0.0815(0.948)
[12]	1.4997(0.918)	0.092(0.930)	1.4558(0.954)	0.0910(0.950)
[13]	1.4008(0.920)	0.082(0.930)	1.3761(0.950)	0.0780(0.932)
[14]	1.3846(0.936)	0.092(0.918)	1.3442(0.936)	0.06513(0.930)
[15]	1.3761(0.934)	0.0880(0.934)	1.3217(0.932)	0.0935(0.926)
[16]	1.3946(0.956)	0.086(0.920)	1.3725(0.966)	0.0941(0.938)
[17]	1.3500(0.948)	0.0969(0.940)	1.3059(0.968)	0.0833(0.946)
[18]	1.3097(0.936)	0.0747(0.950)	1.2545(0.936)	0.0747(0.952)

Table 7. 95% approximate and credible CIs for γ and η .

CS	*MLE*		*BE*	
	γ	η	γ	η
[1]	0.2402(0.938)	0.2829(0.960)	0.252(0.976)	0.1888(0.948)
[2]	0.2164(0.922)	0.2367(0.95)	0.3030(0.966)	0.2213(0.948)
[3]	0.378(0.938)	0.2110(0.938)	0.2522(0.966)	0.2262(0.934)
[4]	0.2513(0.938)	0.2311(0.952)	0.2656(0.946)	0.1846(0.948)

[5]	0.3033(0.932)	0.1855(0.934)	0.3049(0.940)	0.2039(0.938)
[6]	0.2745(0.938)	0.2674(0.944)	0.3134(0.926)	0.2223(0.912)
[7]	0.2331(0.932)	0.1752(0.960)	0.2844(0.966)	0.2420(0.962)
[8]	0.2943(0.934)	0.1644(0.958)	0.2896(0.958)	0.2149(0.942)
[9]	0.2457(0.926)	0.2396(0.956)	0.3311(0.950)	0.2228(0.942)
[10]	0.2533(0.930)	0.1561(0.964)	0.2859(0.958)	0.2002(0.968)
[11]	0.2595(0.918)	0.1969(0.940)	0.3676(0.964)	0.2415(0.940)
[12]	0.252(0.916)	0.2347(0.950)	0.3263(0.946)	0.1960(0.944)
[13]	0.2406(0.938)	0.2734(0.952)	0.3355(0.954)	0.2471(0.950)
[14]	0.2900(0.926)	0.2626(0.950)	0.3270(0.958)	0.207(0.928)
[15]	0.2009(0.940)	0.2563(0.948)	0.327(0.930)	0.1673(0.962)
[16]	0.2602(0.952)	0.2307(0.958)	0.2853(0.954)	0.1860(0.962)
[17]	0.2679(0.920)	0.2218(0.946)	0.3277(0.934)	0.1979(0.926)
[18]	0.2300(0.924)	0.1986(0.948)	0.2548(0.924)	0.1709(0.972)

Table 8. BEs under BSEL and BLINEX for a simulated data.

parameter	*ω*	*BSEL*	*BLINEX*	
			c=1	c=5
α	0.2		1.4317	1.3477
	0.8	1.4522	1.5311	1.4859
λ	0.2	1.5379	0.0831	0.0859
	0.8	0.0694	0.0854	0.0933
γ	0.2	0.0738	0.2699	0.2740
	0.8	0.3683	0.2629	0.3110
η	0.2	0.3767	0.1927	0.2057
	0.8	0.2186	0.1978	0.2024

Appendix

From the log-likelihood function in (13), we have

$$\frac{\partial^2 \ell}{\partial \lambda^2} = -\sum_{j=1}^{2} \frac{m_j}{\lambda^2} = -\frac{m_1 + m_2}{\lambda^2},$$

$$\frac{\partial^2 \ell}{\partial \gamma^2} = \sum_{j=1}^{2} \left(-\frac{m_j}{\gamma^2} - \alpha^{j-1} \lambda \sum_{i=1}^{m_j} W^{-\gamma}\left(x_{ji}, \eta\right) (\log W\left(x_{ji}, \eta\right))^2 \right.$$
$$\left. - \sum_{i=1}^{k_j} R_{ji} \alpha^{j-1} \lambda W^{-\gamma}\left(x_{ji}, \eta\right) (\log W\left(x_{ji}, \eta\right))^2 - A_{m_j} \alpha^{j-1} \lambda W^{-\gamma}\left(x_{m_j}, \eta\right) (\log W\left(x_{m_j}, \eta\right))^2 \right),$$

$$\frac{\partial^2 \ell}{\partial \eta^2} = \sum_{j=1}^{2} \left(-\frac{m_j}{\eta^2} - (\gamma+1) \sum_{i=1}^{m_j} \frac{\frac{1}{x_{ji}^2} e^{\frac{\eta}{x_{ji}}} (e^{\frac{\eta}{x_{ji}}} - 1) - \frac{1}{x_{ji}^2} e^{\frac{2\eta}{x_{ji}}}}{(e^{\frac{\eta}{x_{ji}}} - 1)^2} \right.$$
$$+ \alpha^{j-1} \lambda \gamma (\sum_{i=1}^{m_j} (\frac{1}{x_{ji}^2} e^{\frac{\eta}{x_{ji}}} (e^{\frac{\eta}{x_{ji}}} - 1)^{-\gamma-1} - (\gamma+1) \frac{1}{x_{ji}^2} e^{\frac{2\eta}{x_{ji}}} (e^{\frac{\eta}{x_{ji}}} - 1)^{-\gamma-2}))$$
$$+ \alpha^{j-1} \lambda \gamma \sum_{i=1}^{k_j} R_{ji} ((\frac{1}{x_{ji}^2} e^{\frac{\eta}{x_{ji}}} (e^{\frac{\eta}{x_{ji}}} - 1)^{-\gamma-1} - (\gamma+1) \frac{1}{x_{ji}^2} e^{\frac{2\eta}{x_{ji}}} (e^{\frac{\eta}{x_{ji}}} - 1)^{-\gamma-2}))$$
$$\left. + A_{m_j} \alpha^{j-1} \lambda \gamma (\frac{1}{x_{m_j}^2} e^{\frac{\eta}{x_{m_j}}} (e^{\frac{\eta}{x_{m_j}}} - 1)^{-\gamma-1} - (\gamma+1) \frac{1}{x_{m_j}^2} e^{\frac{2\eta}{x_{m_j}}} (e^{\frac{\eta}{x_{m_j}}} - 1)^{-\gamma-2}) \right),$$

$$\frac{\partial^2 \ell}{\partial \alpha^2} = -\frac{m_2}{\alpha^2},$$

$$\frac{\partial^2\ell}{\partial\lambda\partial\gamma}=\frac{\partial^2\ell}{\partial\gamma\partial\lambda}=\sum_{j=1}^{2}\Bigg(\alpha^{j-1}\sum_{i=1}^{m_j}W^{-\gamma}\left(x_{ji},\eta\right)\log W\left(x_{ji},\eta\right)$$

$$+\sum_{i=1}^{k_j}R_{ji}\alpha^{j-1}W^{-\gamma}\left(x_{ji},\eta\right)\log W\left(x_{ji},\eta\right)+A_{m_j}\alpha^{j-1}W^{-\gamma}\left(x_{m_j},\eta\right)\log W\left(x_{m_j},\eta\right)\Bigg).$$

$$\frac{\partial^2\ell}{\partial\lambda\partial\eta}=\frac{\partial^2\ell}{\partial\eta\partial\lambda}=\sum_{j=1}^{2}\left(\alpha^{j-1}\gamma\sum_{i=1}^{m_j}\frac{1}{x_{ji}}e^{\frac{\eta}{x_{ji}}}W^{-\gamma-1}\left(x_{ji},\eta\right)+\gamma\sum_{i=1}^{k_j}\frac{R_{ji}\alpha^{j-1}}{x_{ji}}e^{\frac{\eta}{x_{ji}}}W^{-\gamma-1}\left(x_{ji},\eta\right)+\frac{A_{m_j}\alpha^{j-1}\gamma}{x_{m_j}}e^{\frac{\eta}{x_{m_j}}}W^{-\gamma-1}\left(x_{m_j},\eta\right)\right)$$

$$\frac{\partial^2\ell}{\partial\lambda\partial\alpha}=\frac{\partial^2\ell}{\partial\alpha\partial\lambda}=-\sum_{i=1}^{m_2}W^{-\gamma}\left(x_{2i},\eta\right)-\sum_{i=1}^{k_2}R_{2i}W^{-\gamma}\left(x_{2i},\eta\right)-A_{m_2}W^{-\gamma}\left(x_{m_2},\eta\right),$$

$$\frac{\partial^2\ell}{\partial\gamma\partial\eta}=\frac{\partial^2\ell}{\partial\eta\partial\gamma}=\sum_{j=1}^{2}\Bigg(-\sum_{i=1}^{m_j}\frac{\frac{1}{x_{ji}}e^{\frac{\eta}{x_{ji}}}}{W\left(x_{ji},\eta\right)}-\alpha^{j-1}\lambda\gamma\sum_{i=1}^{m_j}\left(\frac{1}{x_{ji}}e^{\frac{\eta}{x_{ji}}}W^{-\gamma-1}\left(x_{ji},\eta\right)\right)^2\left(\log W\left(x_{ji},\eta\right)+1\right)$$

$$+\alpha^{j-1}\lambda\gamma\sum_{i=1}^{k_j}R_{ji}\left(\frac{1}{x_{ji}}e^{\frac{\eta}{x_{ji}}}W^{-\gamma-1}\left(x_{ji},\eta\right)\right)^2\left(\log W\left(x_{ji},\eta\right)+1\right)$$

$$+A_{m_j}\alpha^{j-1}\lambda\left(\frac{1}{x_{m_j}}e^{\frac{\eta}{x_{ji}}}W^{-\gamma-1}\left(x_{m_j},\eta\right)\right)^2\left(\log W\left(x_{m_j},\eta\right)+1\right)\Bigg),$$

$$\frac{\partial^2\ell}{\partial\gamma\partial\alpha}=\frac{\partial^2\ell}{\partial\alpha\partial\gamma}=\lambda\sum_{i=1}^{m_2}W^{-\gamma}\left(x_{2i},\eta\right)\log W\left(x_{2i},\eta\right)$$

$$+\sum_{i=1}^{k_2}R_{2i}\lambda W^{-\gamma}\left(x_{2i},\eta\right)\log W\left(x_{2i},\eta\right)+A_{m_2}\lambda W^{-\gamma}\left(x_{m_2},\eta\right)\log W\left(x_{m_2},\eta\right),$$

$$\frac{\partial^2\ell}{\partial\eta\partial\alpha}=\frac{\partial^2\ell}{\partial\alpha\partial\eta}=\lambda\gamma\sum_{i=1}^{m_2}W_1\left(x_{2i},\gamma,\eta\right)+\sum_{i=1}^{k_2}R_{2i}\lambda\gamma\sum_{i=1}^{m_2}W_1\left(x_{2i},\gamma,\eta\right)+A_{m_2}\lambda\gamma W_1\left(x_{m_2},\gamma,\eta\right).$$

Funding

None

ACKNOWLEDGEMENT

None

CONFLICT OF INTEREST

None

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