

Applications for the groups $S.U.T.(2, p)$, where p prime upper than 9

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Abstract The problem of finding the cyclic decomposition (c.d.) for the groups $S.U.T.(2, p)$, where p prime upper than 9 is determined in this work. Also, we compute the Artin characters (A.ch.) and Artin indicator (A.ind.) for the same groups, we obtain that after computing the conjugacy classes, cyclic subgroups, the ordinary character table (o.ch.ta.) and the rational valued character table for each group.

Key Words: Cyclic decomposition, upper triangular subgroup, Artin character, Artin indicator, cyclic subgroups.

1. INTRODUCTION

$G.L.(n, F)$ consisting of all $n \times n$ invertible matrices over the field F . $U.T.(n, F)$ consisting of all $n \times n$ invertible upper triangular matrices over the field F set forms a subgroup. $S.U.T.(n, F)$ obtained from $U.T.(n, F) \cap S.L.(n, F)$, [1].

In this work, we compute the cyclic decomposition (c.d.) and Artin character (A.ch.) and Artin indicator for the groups $S.U.T.(2, 9)$ and $S.U.T.(2, 11)$ and generalize this work for $S.U.T.(2, p)$, where p prime upper than 9.

Many authors [1–6] computed the same work for the groups $SL(2, \mathbb{U})$, $\mathbb{U} = 23, 29, 31, 37$ and 49 , $PSL(2, 31)$ and $PSL(2, 37)$ ($2, \mathbb{U}$), $\mathbb{U} = 23, 29, 31$ and 37 . Although the Authors in [7–12] studied the cases $(6, 6, 4; 0, 0)$, $(8, 8)/(1, 0)$, $(7, 7, 4; 0, 0)$, $(8, 7, 3)$ and $SL(2, 3^8)$, in [13] and [14] the authors studied new algorithm and symmetric generalized.

2. FUNDAMENTAL INFORMATION

Proposition 2.1 [15]:

$$S.U.T.(2, q) = \left\{ \begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix} : a \in F_q^*, b \in F_q \right\}. \text{ Therefore, } |S.U.T.(2, q)| = q(q - 1).$$

Theorem 2.2 [15]:

The group $S.U.T.(2, q)$ has at least $q + 3$ conjugacy classes as illustrated in the following table:

Table 1.

\mathcal{CC}	g	No. of \mathcal{CC}	$ \mathcal{C}_{S,U,T(2,q)}(g) $	\mathcal{C}_g
$\mathcal{T}_0^{(1)}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	1	$q^2 - q$	1
$-\mathcal{T}_0^{(1)}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	1	$q^2 - q$	1
$\mathcal{T}_{01}^{(2)}$	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$	1	$2q$	$\frac{q-1}{2}$
$-\mathcal{T}_{01}^{(2)}$	$\begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix}$	1	$2q$	$\frac{q-1}{2}$
$\mathcal{T}_{0\varepsilon}^{(2)}$	$\begin{pmatrix} 1 & \varepsilon \\ 0 & 1 \end{pmatrix}$	1	$2q$	$\frac{q-1}{2}$
$-\mathcal{T}_{0\varepsilon}^{(2)}$	$\begin{pmatrix} -1 & -\varepsilon \\ 0 & -1 \end{pmatrix}$	1	$2q$	$\frac{q-1}{2}$
$\mathcal{T}_{\hbar,-\hbar}^{(3)}$	$\begin{pmatrix} \alpha & 0 \\ 0 & \alpha^{-1} \end{pmatrix}$	$q-3$	$q-1$	q

where are \mathcal{CC} conjugacy classes in $\mathcal{T}_{\hbar,-\hbar}^{(3)}$: \hbar denote the integer for $\alpha = \varepsilon^\hbar$, where ε is a generator of the group \mathcal{F}_q^* and $\alpha \in \mathcal{F}_q^* \setminus \{1, -1\}$.

Theorem 2.3 [1]:

If G is a cyclic p group. Then $\mathcal{K}(G) = \mathcal{Z}_p$.

Proposition 2.4 [1]:

If G is a cyclic group s.t. $|G| = p^n$, then $\mathcal{K}(G) = \bigoplus_{i=1}^n \mathcal{Z}_{p^i}$.

Theorem 2.5 [15]:

The (o.ch.ta.) of $\mathcal{S.U.T.}(2, q)$, where q is odd.

Table 2

\mathcal{CC}	$\mathcal{T}_0^{(1)}$	$-\mathcal{T}_0^{(1)}$	$\mathcal{T}_{01}^{(2)}$	$-\mathcal{T}_{01}^{(2)}$	$\mathcal{T}_{0\varepsilon}^{(2)}$	$-\mathcal{T}_{0\varepsilon}^{(2)}$	$\mathcal{T}_{\hbar,-\hbar}^{(3)}$
g	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & \varepsilon \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & -\varepsilon \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} \alpha & 0 \\ 0 & \alpha^{-1} \end{pmatrix}$
No. of \mathcal{CC}	1	1	1	1	1	1	$q-3$
$ \mathcal{C}_{S,U,T(2,q)}(g) $	$q^2 - q$	$q^2 - q$	$2q$	$2q$	$2q$	$2q$	$q-1$
\mathcal{C}_g	1	1	$\frac{q-1}{2}$	$\frac{q-1}{2}$	$\frac{q-1}{2}$	$\frac{q-1}{2}$	q
x_\hbar	1	$(-1)^\hbar$	1	$(-1)^\hbar$	1	$(-1)^\hbar$	$\frac{2m\hbar s}{q^{q-1}}$
ϑ_1	$\frac{q-1}{2}$	$-\varepsilon \frac{q-1}{2}$	$\frac{-1 + \sqrt{\varepsilon q}}{2}$	$-\varepsilon \left(\frac{-1 + \sqrt{\varepsilon q}}{2} \right)$	$\frac{-1 - \sqrt{\varepsilon q}}{2}$	$-\varepsilon \left(\frac{-1 - \sqrt{\varepsilon q}}{2} \right)$	0
ϑ_2	$\frac{q-1}{2}$	$-\varepsilon \frac{q-1}{2}$	$\frac{-1 - \sqrt{\varepsilon q}}{2}$	$-\varepsilon \left(\frac{-1 - \sqrt{\varepsilon q}}{2} \right)$	$\frac{-1 + \sqrt{\varepsilon q}}{2}$	$-\varepsilon \left(\frac{-1 + \sqrt{\varepsilon q}}{2} \right)$	0
$x_1 \vartheta_1$	$\frac{q-1}{2}$	$\varepsilon \frac{q-1}{2}$	$\frac{-1 + \sqrt{\varepsilon q}}{2}$	$\varepsilon \left(\frac{-1 + \sqrt{\varepsilon q}}{2} \right)$	$\frac{-1 - \sqrt{\varepsilon q}}{2}$	$\varepsilon \left(\frac{-1 - \sqrt{\varepsilon q}}{2} \right)$	0
$x_1 \vartheta_2$	$\frac{q-1}{2}$	$\varepsilon \frac{q-1}{2}$	$\frac{-1 - \sqrt{\varepsilon q}}{2}$	$\varepsilon \left(\frac{-1 - \sqrt{\varepsilon q}}{2} \right)$	$\frac{-1 + \sqrt{\varepsilon q}}{2}$	$\varepsilon \left(\frac{-1 + \sqrt{\varepsilon q}}{2} \right)$	0

where $\varepsilon = (-1)^{\frac{q-1}{2}}$, $s = 2, 3, \dots, q-2$ and $\hbar = 0, 1, \dots, q-2$.

3. THE $\mathcal{K}(\mathcal{S.U.T.}(2, p))$, WHERE $p = 9, 11$

Following the same idea in [1-6], we find the results for these groups.

3.1 The $\mathcal{K}(\mathcal{SUT}(2, 9))$

$|\mathcal{S.U.T.}(2,9)| = 72$ and the (o.ch.ta.) of $\mathcal{S.U.T.}(2,9)$ is summarized in table 3.

Table 3. (o.ch.ta.) of $\mathcal{S.U.T.}(2,9)$

$c(g)$	$\mathcal{T}_0^{(1)}$	$-\mathcal{T}_0^{(1)}$	$\mathcal{T}_{01}^{(2)}$	$-\mathcal{T}_{01}^{(2)}$	$\mathcal{T}_{0\varepsilon}^{(2)}$	$-\mathcal{T}_{0\varepsilon}^{(2)}$	$\mathcal{T}_{2,-2}^{(3)}$	$\mathcal{T}_{3,-3}^{(3)}$	$\mathcal{T}_{4,-4}^{(3)}$	$\mathcal{T}_{5,-5}^{(3)}$	$\mathcal{T}_{6,-6}^{(3)}$	$\mathcal{T}_{7,-7}^{(3)}$
$ \mathcal{C}_{SUT(2,9)}(g) $	72	7	18	18	18	18	8	8	8	8	8	8
$ c(g) $	1	1	4	4	4	4	9	9	9	9	9	9
x_0	1	1	1	1	1	1	1	1	1	1	1	1
x_1	1	-1	1	-1	1	-1	i	$-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$	1	$-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$	-i	$\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$
x_2	1	1	1	1	1	1	-1	-i	-1	i	-1	-i
x_3	1	-1	1	-1	1	-1	-i	$\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$	-1	$\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$	i	$-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$
x_4	1	1	1	1	1	1	1	-1	-1	-1	1	-1

x_5	1	-1	1	-1	1	-1	-i	$\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$	1	$-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$	i	$-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$
x_6	1	1	1	1	1	1	-1	i	-1	-i	-1	i
x_7	1	-1	1	-1	1	-1	i	$-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$	-1	$\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$	-i	$\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$
θ_1	4	-	1	-1	-2	2	0	0	0	0	0	0
		4										
θ_2	4	-	-2	2	1	-1	0	0	0	0	0	0
		4										
$x_1\theta_1$	4	4	1	1	-2	-2	0	0	0	0	0	0
$x_1\theta_2$	4	4	-2	-2	1	1	0	0	0	0	0	0

After we omit the same similar columns, we gain

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & -4 & 4 & -4 & 4 & -4 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 & -2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\ 4 & -4 & 1 & -1 & -2 & 2 & 0 & 0 \\ 4 & -4 & -2 & 2 & 1 & -1 & 0 & 0 \\ 4 & 4 & 1 & 1 & -2 & -2 & 0 & 0 \\ 4 & 4 & -2 & -2 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The diagonalization of this matrix is

$$\begin{pmatrix} 36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, $\mathcal{K}(S.U.T.(2,9)) = \mathcal{Z}_{36} \oplus \mathcal{Z}_3 \oplus \mathcal{Z}_3 \oplus \mathcal{Z}_3 \oplus \mathcal{Z}_2 \oplus \mathcal{Z}_2 \oplus \mathcal{Z}_1 \oplus \mathcal{Z}_1$

3.2 $\mathcal{K}(S.U.T.(2,11))$

$|S.U.T.(2,11)| = 110$ and the (o.ch.ta.) of $S.U.T.(2,11)$ is summarized in table 4.

Table 4. (o.ch.ta.) of $S.U.T.(2,11)$

$\mathcal{C}(g)$	$\mathcal{J}_0^{(1)}$	$-\mathcal{J}_0^{(1)}$	$\mathcal{J}_{01}^{(2)}$	$-\mathcal{J}_{01}^{(2)}$	$\mathcal{J}_{01}^{(2)}$	$-\mathcal{J}_{01}^{(2)}$	$\mathcal{J}_{2-2}^{(3)}$	$\mathcal{J}_{3-3}^{(3)}$	$\mathcal{J}_{4-4}^{(3)}$	$\mathcal{J}_{5-5}^{(3)}$	$\mathcal{J}_{6-6}^{(3)}$	$\mathcal{J}_{7-7}^{(3)}$	$\mathcal{J}_{8-8}^{(3)}$	$\mathcal{J}_{9-9}^{(3)}$
$ C_{SUT(2,11)}(g) $	110	1	22	22	22	22	10	10	10	10	10	10	10	10
		1												
		0												
$ C(g) $	1	1	5	5	5	5	11	11	11	11	11	11	11	11
x_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
x_1	1	-1	1	-1	1	-1	0.3090 + i	-	-8090 + i	-	-8090 - i	-0.3090 - i	0.3090 - i	8090 - i
							0.9510 + i	0.3090 + i	0.5877 + i		0.5877 + i	0.9510 + i	0.9510 + i	0.5877 + i
							0.9510 + i	0.9510 + i	0.9510 + i		0.9510 + i	0.9510 + i	0.9510 + i	0.9510 + i
x_2	1	1	1	1	1	1	-8090 + i	-8090 - i	0.3090 - i	1	0.3090 + i	-8090 + i	-8090 - i	0.3090 - i
							0.9510 + i	0.9510 + i	0.9510 + i		0.9510 + i	0.9510 + i	0.9510 + i	0.9510 + i

							0.5877	0.5877	0.9510		0.9510		0.5877	
x_3	1	-1	1	-1	1	-1	-8090	8090 - i	0.3090	1	0.3090	8090 + i	-8090	-0.3090
							-i	0.5877	+i		-i	0.5877	+i	-i
x_4	1	1	1	1	1	1	0.3090	0.3090	-8090	-	-8090	0.3090 -	0.3090	-8090 -
							-i	+i	-i	1	+i	i 0.9510	+i	i 0.5877
x_5	1	-1	1	-1	1	-1	0.9510	0.9510	0.5877		0.5877		0.9510	
							1	-1	1	-	1	-1	1	-1
x_6	1	1	1	1	1	1	0.3090	0.3090	-8090	1	-8090	0.3090 +	0.3090	-8090
							+i	-i	+i		-i	i 0.9510	-i	+i
x_7	1	-1	1	-1	1	-1	0.9510	0.9510	0.5877		0.5877		0.9510	0.5877
							-8090	8090 + i	0.3090	1	0.3090	8090 - i	-8090	-0.3090
x_8	1	1	1	1	1	1	0.5877		0.9510		0.9510		0.5877	0.9510
							-i	+i	+i	2	-i	i 0.5877	+i	+i
x_9	1	-1	1	-1	1	-1	0.5877	0.5877	0.9510		0.9510		0.5877	0.9510
							0.3090	-	-8090	-	-8090	-0.3090	0.3090	8090 + i
							-i	0.3090	-i	2	+i	+i	+i	0.5877
							0.9510	-i	0.5877		0.5877	0.9510	0.9510	
θ_1	5	5	$\frac{-1+\sqrt{-11}}{2}$	$\frac{-1+\sqrt{-11}}{2}$	$\frac{-1-\sqrt{-11}}{2}$	$\frac{-1-\sqrt{-11}}{2}$	0	0	0	0	0	0	0	0
θ_2	5	5	$\frac{-1-\sqrt{-11}}{2}$	$\frac{-1-\sqrt{-11}}{2}$	$\frac{-1+\sqrt{-11}}{2}$	$\frac{-1+\sqrt{-11}}{2}$	0	0	0	0	0	0	0	0
x_1, θ_1	5	5-	$\frac{-1+\sqrt{-11}}{2}$	$\frac{1}{2}$	$\frac{-\sqrt{-11}+1}{2}$	$\frac{-\sqrt{-11}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{-11}}{2}$	0	0	0	0	0	0
x_1, θ_2	5	5-	$\frac{-1-\sqrt{-11}}{2}$	$\frac{1}{2}$	$\frac{-\sqrt{-11}-1}{2}$	$\frac{-\sqrt{-11}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{-11}}{2}$	0	0	0	0	0	0

After we omit the same similar columns, we gain

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & -4 & 4 & -4 & -1 & 1 \\ 4 & 4 & 4 & 4 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 10 & 10 & -1 & -1 & 0 & 0 \\ 10 & -10 & -1 & 1 & 0 & 0 \end{pmatrix}$$

The diagonalization of this matrix is

$$\begin{pmatrix} 55 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, $\mathcal{K}(\mathcal{S.U.T.}(2,11)) = \mathcal{Z}_{55} \oplus \mathcal{Z}_3 \oplus \mathcal{Z}_3 \oplus \mathcal{Z}_2 \oplus \mathcal{Z}_2 \oplus \mathcal{Z}_1$

4. THE (A.ch.) AND (A.ind.) FOR $\mathcal{S.U.T.}(2, p)$, WHERE $p = 9, 11$

As the same idea in [1], in [6–10] we find the results for these groups.

4.1 The Result for $\mathcal{S.U.T.}(2,9)$

The group $\mathcal{S.U.T.}(2,9)$ has 8 cyclic subgroups, which are as follows:

$$\langle \mathcal{J}_0^{(1)} \rangle, \langle -\mathcal{J}_0^{(1)} \rangle, \langle \mathcal{J}_{01}^{(2)} \rangle, \langle -\mathcal{J}_{01}^{(2)} \rangle, \langle \mathcal{J}_{0,1}^{(2)} \rangle, \langle -\mathcal{J}_{0,1}^{(2)} \rangle, \langle \mathcal{J}_{2,-2}^{(3)} \rangle, \langle \mathcal{J}_{3,-3}^{(3)} \rangle$$

The rational valued character table of $\mathcal{S.U.T.}(2,9)$ is summarized in table 5.

Table 5. The rational valued character table of $\mathcal{S.U.T.}(2,9)$

$\mathcal{C}(g)$	$\mathcal{J}_0^{(1)}$	$-\mathcal{J}_0^{(1)}$	$\mathcal{J}_{01}^{(2)}$	$-\mathcal{J}_{01}^{(2)}$	$\mathcal{J}_{0,1}^{(2)}$	$-\mathcal{J}_{0,1}^{(2)}$	$\mathcal{J}_{2,-2}^{(3)}$	$\mathcal{J}_{3,-3}^{(3)}$
$ C_{\mathcal{SUT}(2,9)}(g) $	72	72	18	18	18	18	8	8
$ C(g) $	1	1	4	4	4	4	9	9
\mathcal{X}_0	1	1	1	1	1	1	1	1
$\mathcal{X}_1 + \mathcal{X}_3 + \mathcal{X}_5 + \mathcal{X}_7$	4	-4	4	-4	4	-4	0	0
$\mathcal{X}_2 + \mathcal{X}_6$	2	2	2	2	2	2	-2	0
\mathcal{X}_4	1	1	1	1	1	1	1	-1
ϑ_1	4	-4	1	-1	-2	2	0	0
ϑ_2	4	-4	-2	2	1	-1	0	0
$\mathcal{X}_1\vartheta_1$	4	4	1	1	-2	-2	0	0
$\mathcal{X}_1\vartheta_2$	4	4	-2	-2	1	1	0	0

By employing

$$\phi_H \uparrow^{\mathcal{SUT}(2,9)}(\alpha) = \frac{|C_{\mathcal{SUT}(2,9)}(\alpha)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \rightarrow x \\ H \rightarrow \mathcal{SUT}(2,9)}} \phi(\alpha), \text{ where } H \text{ is the cyclic subgroup.}$$

The (A.ch.) is summarized in Table 6.

Table 6. The (A.ch.) table of $\mathcal{S.U.T.}(2,9)$

$\mathcal{C}(g)$	$\mathcal{J}_0^{(1)}$	$-\mathcal{J}_0^{(1)}$	$\mathcal{J}_{01}^{(2)}$	$-\mathcal{J}_{01}^{(2)}$	$\mathcal{J}_{0,1}^{(2)}$	$-\mathcal{J}_{0,1}^{(2)}$	$\mathcal{J}_{2,-2}^{(3)}$	$\mathcal{J}_{3,-3}^{(3)}$
\emptyset_1	72	0	0	0	0	0	0	0
\emptyset_2	18	18	0	0	0	0	0	0
\emptyset_3	9	0	2	0	0	0	0	0
\emptyset_4	2	0	0	9	0	0	0	0
\emptyset_5	2	2	0	0	9	0	0	0
\emptyset_6	2	0	0	0	0	9	0	0
\emptyset_7	2	2	0	0	0	0	9	0
\emptyset_8	2	0	0	0	0	0	0	9

From tables 5 and 6, we get

$$\mathcal{X}_0 = \frac{1}{9}\emptyset_8 + \frac{1}{9}\emptyset_7 + \frac{1}{9}\emptyset_6 + \frac{1}{9}\emptyset_5 + \frac{1}{9}\emptyset_4 + \frac{1}{2}\emptyset_3 + 0.28703703703704 \emptyset_2 - 0.073302469135803\emptyset_1,$$

$$\mathcal{X}_1 + \mathcal{X}_3 + \mathcal{X}_5 + \mathcal{X}_7 = \frac{-36}{9}\emptyset_6 + \frac{36}{9}\emptyset_5 - \frac{36}{9}\emptyset_4 + \frac{36}{9}\emptyset_3 + 0.2222222222222222\emptyset_2 + 0.5\emptyset_1,$$

$$\mathcal{X}_2 + \mathcal{X}_6 = -\frac{18}{9}\emptyset_7 + \frac{18}{9}\emptyset_6 + \frac{18}{9}\emptyset_5 + \frac{18}{9}\emptyset_4 + \emptyset_3 + 0.1111111111111111\emptyset_2 - \frac{64}{441}\emptyset_1,$$

$$\mathcal{X}_4 = -\frac{1}{9}\emptyset_8 + \frac{1}{9}\emptyset_7 + \frac{1}{9}\emptyset_6 + \frac{1}{9}\emptyset_5 + \frac{1}{9}\emptyset_4 + \frac{1}{2}\emptyset_3 + 0.030864197530864 \emptyset_2 - 0.065586419753086\emptyset_1,$$

$$\vartheta_1 = \frac{18}{9}\emptyset_6 - \frac{18}{9}\emptyset_5 - \frac{1}{9}\emptyset_4 + \frac{1}{2}\emptyset_3 + 0.04545454545454 \emptyset_2 - 0.003858024691358\emptyset_1,$$

$$\vartheta_2 = -\frac{1}{9}\emptyset_6 + \frac{1}{9}\emptyset_5 + \frac{18}{9}\emptyset_4 - \emptyset_3 - 0.234567901234568\emptyset_2 - 0.066358024691358\emptyset_1,$$

$$\mathcal{X}_1\vartheta_1 = -\frac{18}{9}\emptyset_6 - \frac{18}{9}\emptyset_5 + \frac{1}{9}\emptyset_4 + \frac{1}{2}\emptyset_3 + 0.4444444444444444\emptyset_2 + 0.031635802469136\emptyset_1,$$

$$\mathcal{X}_1\vartheta_2 = \frac{1}{9}\vartheta_6 + \frac{1}{9}\vartheta_5 - \frac{18}{9}\vartheta_4 - \vartheta_3 + 0.11111111111111111111\vartheta_2 + 0.20216049382716\vartheta_1..$$

Therefore, $72\chi_i = (\mathbb{Z})\vartheta_i, i=1, \dots, 8$. So, $A(\mathcal{S}.U.T.(2,9)) = 72$.

4.2 The Result for $\mathcal{S}.U.T.(2,11)$

The group $\mathcal{S}.U.T.(2,11)$ has 6 cyclic subgroups, which are as follows:

$$\langle \mathcal{J}_0^{(1)} \rangle, \langle -\mathcal{J}_0^{(1)} \rangle, \langle \mathcal{J}_{01}^{(2)} \rangle, \langle -\mathcal{J}_{01}^{(2)} \rangle, \langle \mathcal{J}_{2,-2}^{(3)} \rangle, \langle \mathcal{J}_{3,-3}^{(3)} \rangle$$

The rational valued character table of $\mathcal{S}.U.T.(2,11)$ is summarized in table 7.

Table 7. The rational valued character table of $\mathcal{S}.U.T.(2,11)$

$\mathcal{C}(g)$	$\mathcal{J}_0^{(1)}$	$-\mathcal{J}_0^{(1)}$	$\mathcal{J}_{01}^{(2)}$	$-\mathcal{J}_{01}^{(2)}$	$\mathcal{J}_{2,-2}^{(3)}$	$\mathcal{J}_{3,-3}^{(3)}$
$ C_{SUT(2,11)}(g) $	110	110	22	22	10	10
$ C(g) $	1	1	5	5	11	11
\mathcal{X}_0	1	1	1	1	1	1
$\mathcal{X}_1+\mathcal{X}_3+\mathcal{X}_7+\mathcal{X}_9$	4	-4	4	-4	-1	1
$\mathcal{X}_2+\mathcal{X}_4+\mathcal{X}_6+\mathcal{X}_8$	4	4	4	4	-1	-1
\mathcal{X}_5	1	-1	1	-1	1	-1
$\vartheta_1+\vartheta_2$	10	10	-1	-1	0	0
$\mathcal{X}_1\vartheta_1+\mathcal{X}_1\vartheta_2$	10	10	-1	1	0	0

By employing

$$\phi_H \uparrow^{SUT(2,11)}(\alpha) = \frac{|C_{SUT(2,11)}(x)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \rightarrow x \\ H \rightarrow SUT(2,11)}} \phi(\alpha), \text{ where H is the cyclic subgroup.}$$

The (A.ch.) is summarized in table 8.

Table 8. The Artin characters table of $\mathcal{S}.U.T.(2,11)$

$\mathcal{C}(g)$	$\mathcal{J}_0^{(1)}$	$-\mathcal{J}_0^{(1)}$	$\mathcal{J}_{01}^{(2)}$	$-\mathcal{J}_{01}^{(2)}$	$\mathcal{J}_{2,-2}^{(3)}$	$\mathcal{J}_{3,-3}^{(3)}$
ϑ_1	55	0	0	0	0	0
ϑ_2	22	22	0	0	0	0
ϑ_3	11	0	2	0	0	0
ϑ_4	2	0	0	11	0	0
ϑ_5	2	2	0	0	11	0
ϑ_6	2	0	0	0	0	11

From tables 7 and 8, we get

$$\mathcal{X}_0 = \frac{1}{11}\vartheta_6 + \frac{1}{11}\vartheta_5 + \frac{1}{11}\vartheta_4 + \frac{1}{2}\vartheta_3 - 0.037190082644628\vartheta_2 - 0.106611570247934\vartheta_1,$$

$$\mathcal{X}_1 + \mathcal{X}_3 + \mathcal{X}_7 + \mathcal{X}_9 = \frac{1}{11}\vartheta_6 - \frac{1}{11}\vartheta_5 - \frac{44}{11}\vartheta_4 + 2\vartheta_3 - 0.173553719008264\vartheta_2 - 0.542148760330578\vartheta_1,$$

$$\mathcal{X}_2 + \mathcal{X}_4 + \mathcal{X}_6 + \mathcal{X}_8 = -\frac{1}{11}\vartheta_6 - \frac{1}{11}\vartheta_5 + \frac{44}{11}\vartheta_4 + 2\vartheta_3 + 0.190082644628099\vartheta_2 - 2.003305785123967\vartheta_1,$$

$$\mathcal{X}_5 = -\frac{1}{11}\vartheta_6 + \frac{1}{11}\vartheta_5 - \frac{1}{11}\vartheta_4 + \frac{1}{2}\vartheta_3 - 0.053719008264463\vartheta_2 - 0.106611570247934\vartheta_1,$$

$$\vartheta_1 + \vartheta_2 = -\frac{1}{11}\vartheta_4 - \frac{1}{2}\vartheta_3 + 0.454545454545455\vartheta_2 + 0.10330578585123967\vartheta_1,$$

$$\mathcal{X}_1\vartheta_1 + \mathcal{X}_1\vartheta_2 = \frac{1}{11}\vartheta_4 - \frac{1}{2}\vartheta_3 + 0.454545454545455\vartheta_2 + 0.096694214876033\vartheta_1.$$

Therefore, $55\chi_i = (\mathbb{Z})\vartheta_i, i=1, \dots, 8$. So, $A(\mathcal{S}.U.T.(2,11)) = 55$.

5. CONCLUSION

The c.d. for each group is equal to $K(G) = \mathcal{UJ}(2, q) = \oplus Z q$, where q are $|G|/2$, the orders of the conjugate classes and complete the remainder by 1. Also, the A.ind. is equal to $|G|$.

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CONFLICTS OF INTEREST

None

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