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Parameters Estimation of a Proposed Non-Homogeneous Poisson Process and Estimation of the Reliability Function Using the Gompertz Process: A Comparative Analysis of Artificially Intelligent and Traditional Methods

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ABSTRACT: This work presents a novel approach to enhancing the rate of occurrence of non-homogeneous Poisson processes (NHPP) by utilizing the Gompertz distribution as the rate of occurrence. The primary objective of this study is to determine the parameters of the new process using both traditional methods and intelligent technology, specifically particle swarm optimization (PSO). Additionally, the study aims to estimate the reliability function of the process. The suggested model is simulated to achieve these goals, and the results are compared among various estimation techniques to identify the most accurate estimator. The study demonstrates that when predicting the time rate of occurrence of the proposed Gompertz process and its reliability function, the PSO algorithm outperforms other approaches. Furthermore, this research showcases a practical application utilizing real data from the Mosul power facility. Specifically, the data pertains to the stoppage times of two consecutive units of the Mosul Dam power stations from January 1st, 2021 to January 1st, 2022. Overall, this study introduces a novel process based on the Gompertz distribution to improve the rate of occurrence of NHPP. It employs particle swarm optimization to calculate the process parameters and estimate the reliability function. The superiority of the PSO algorithm is demonstrated through comprehensive comparisons. The practical application using data from the Mosul power facility further validates the effectiveness of the proposed approach.

Keywords: Gompertz Process, Reliability function, Particle Swarm Optimization, Maximum Likelihood Estimation, Simulation.

1. Introduction

The main problem is the non-homogeneous Poisson process's complexity, namely the difficulty of adding a time rate for event occurrences. This issue must be resolved since it makes it difficult to accurately represent phenomena in a various of domains and more difficult to use NHPP to anticipate events over time. The solution to this challenge will have a major impact on the development of stochastic processes, improving the predictive capacity of models and strengthening the simulations' dependability in industries such as finance, telecommunications, and reliability engineering.

Existing research has delved into understanding the time-varying dynamics of NHPP which is used in finance to simulate the frequency of market events or the arrival rate of financial transactions over time[1]. The NHPP is used in telecommunications to simulate the pace at which calls or messages arrive in communication networks[2];[3]. The Markov property, which states that future occurrences solely depend on their current state and not on the process's prior history, is one of the NHPP's key characteristics[4].

The NHPP is particularly helpful for simulating complicated systems and processes because of this feature. Although parameter estimation for the intensity function is difficult due to the intricacy of the intensity function and the time-varying nature of the NHPP, several techniques have been put forth. These include Bayesian inference, genetic algorithms (GA), and neural networks, as well as maximum likelihood estimation (MLE), Bayesian inference, and intelligent approaches [5];[6].

Our proposed approach involves a nuanced method that integrates the derived density function from observed inter-periods between stoppages into the exponential function, capturing the inherent complexity of NHPP. With both intelligent and traditional estimation techniques, this method aims to robustly ascertain the parameters of the time rate

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of events. Additionally, the study explores the estimation of the reliability function and complements the analysis with simulations of event occurrence rates.

This article contributes significantly by offering a novel perspective on the time-varying dynamics of NHPP. The proposed method advances theoretical understanding while providing a practical framework for estimating parameters in a complex stochastic process. By combining both intelligent and traditional estimation methods, the approach offers a holistic and accurate representation of the time rate of events within NHPP. The practical application of this research in predicting downtime in electricity units at the Mosul Dam power station further underscores its potential impact on real data, solidifying its contribution to the broader fields of reliability engineering, finance, and telecommunications.

1.1 Reliability Function

The reliability function also known as the survival function or the complementary cumulative distribution function, is a fundamental concept in reliability engineering and survival analysis. It is used to describe the probability that a system or component will survive or continue to function without failure beyond a given time point. The likelihood that a system or component will continue to perform properly for time longer than a certain time t is the formal definition of the reliability function R(t). The reliability function's equation is[7]:

$$R(t) = 1 - e^{-m(t)}, \tag{1}$$

where the likelihood that the system or component would fail within time t is given by the cumulative distribution function, m(t). The reliability function offers insightful data on a system's or component's long-term behavior. The dependability and durability of different goods and systems, including electrical devices, mechanical parts, infrastructure, and software, are frequently evaluated using this technique in reliability engineering. The reliability function, a subset of statistics, is employed to model and examine the survival periods of people or things. This makes studying survival patterns and calculating hazard rates—which depict instantaneous failure rates at various time points—easier.

1.2 Gompertz Process

The Gompertz distribution is a life-time continuous probability function for nonnegative-valued random variables. This distribution is an important model used in reliability and process research; it is applied in many fields such as health, farming, biology and other sciences. The truncated distribution rescales the "a" parameter to correspond to x = 0 to ensure a proper density function. The distribution function of the Gompertz process is defined as m(t), This depicts the process variable or the average occurrence rate. It is the result of the time rate of occurrence's cumulative function. The inter-arrival process for the Gompertz process follows the distribution as described by Cox [8].

$$f(t) = \lambda(t)e^{-m(t_0)}. \tag{2}$$

The time rate of occurrence, denoted as "λ(t)," in the Gompertz process is defined by the equation

$$\lambda(t) = ae^{bt} \quad , t \geq 0, \ a, b > 0, \tag{3}$$

where "t" is the time and parameters “a” and "b" are positive constants.

The mean value function, denoted as:

$$m(t) = \int_0^t \lambda(u)\, du \qquad , \ 0 < t < \infty \tag{4}$$

$$= \int_0^t ae^{bu}\, du$$

$$= \frac{a}{b}(e^{bt} - 1) .\ 0 \leq t \leq t_0 \tag{5}$$

By substituting equations (3) and (5) into equation (2), we obtain the Gompertz process in terms of its probability density function, denoted as[9]:

$$f(t) = ae^{bt}\, e^{-\frac{a}{b}(e^{bt}-1)} \ ,0 \leq t \leq \infty, a, b > 0, \tag{6}$$

where a and b are parameters that need to be estimated for the intensity function of the Gompertz process. Estimating these parameters "a" and "b" is a critical task in modeling the Gompertz process. And the cumulative function of the Gompertz process is:

$$F(t) = 1 - e^{-\frac{a}{b}(e^{bx}-1)} , \quad 0 \leq t \leq \infty . \tag{7}$$

Thus, the reliability function for this process mathematically is as follows:

$$R(t) = e^{-\frac{a}{b}(e^{bt}-1)} . \tag{8}$$

Thus, the reliability function according to the Gompertz process, when substituting time (t) instead of (x) becomes as follows:

$$R(x) = 1 - e^{-\frac{a}{b}(e^{bx}-1)} . \tag{9}$$

1.3 Particle Swarm Optimization Algorithm (PSO)

The Particle Swarm Optimization (PSO) technique, initially introduced by Eberhart and Kennedy in 1995, is a recent and effective approach to solving optimization problems. It belongs to the family of population-based stochastic optimization algorithms, where particles' positions and velocities are iteratively updated to find the optimal solution [10].

The PSO algorithm can be summarized in the following steps[11]:

- Initialize the position and velocity of each particle randomly.
- Evaluate the fitness value of each particle based on the objective function.
- Update the personal best position (p_{best}) for each particle if it improves the fitness value.
- Update the global best position (g_{best}) by considering the best position among all particles.
- Repeat the above steps until the termination criteria are met

The position and velocity of each particle are updated using the following formulas:

Velocity update:

$$V_i(t+1) = \omega V_i(t) + C_1 r_1 \big(P_i - X_i(t)\big) + C_2 r_2 (G - X_i(t)) \tag{10}$$

Position update:

$$X_i(t+1) = X_i(t) + V_i(t+1), \tag{11}$$

Where $V_i(t)$ represents the velocity vector of particle s and t time; $X_i(t)$ represents the position vector of particles in t time; P_i is the personal best position of particle s;G is the best position of the particle found at present; ω represents inertia weight; C_1 and C_2 are two acceleration constants, called cognitive and social parameters, respectively; r_1 and r_2 are two random functions in the range[12].

1.4 Performance of estimation accuracy

Comparing the accuracy of various parameter estimations that have been acquired is a crucial step. There are several methods in the literature for quantifying the accuracy of these estimations; one of the most widely used ones is the root mean square error (RMSE). The root of the average squared difference between the estimated and actual parameter values, or RMSE, is used to calculate the discrepancies between the estimated and actual parameter values [13]:

Mathematically, the RMSE is defined as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{Q}(\hat{\theta}_i-\theta)^2}{Q}} \tag{12}$$

$\hat{\theta}_i$: Represents the value of the parameter estimated in the iteration i.

θ : Represents the real parameter value.

Q : Represents the number of iterations.

2.Method of Estimation

2.1 Parameter Estimation for the Gompertz using the MLE Method

The following equation may be used to describe the joint probability function of the occurrence times for a non-homogeneous Poisson process (NHPP) whose time rate of occurrences is given by formula (4) [7]:

$$f_n(t_1,t_2,\dots,t_n) = \prod_{i=1}^{n}\lambda(t_i)e^{-m(t_0)}. \tag{13}$$

$$f_n = \prod_{i=1}^{n} a\, e^{bt_i} \cdot e^{-\frac{a}{b}(e^{bt_0}-1)}. \tag{14}$$

Consequently, the Gompertz process probability function for the period (0, t) with the rate time λ(t) is:

$$L = (a)^n\, e^{b\sum_{i=1}^{n}t_i}\; e^{-n\frac{a}{b}(e^{bt_0}-1)}. \tag{15}$$

We utilize the natural logarithm of the maximum function to streamline the computations rather than the maximum function, as denoted by equation (15). This yields the subsequent formula:

$$\ln L = n\ln(a) + b\sum_{i=1}^{n}t_i - \frac{na}{b}\left(e^{bt_0}-1\right), \tag{16}$$

Subsequently, the parameters "a" and "b" of the function λ(t) are estimated through the application of the following equations:

$$\left.\frac{\partial \ln L}{\partial a}\right|_{a=\hat{a}} = 0 \rightarrow \frac{n}{\hat{a}} - \frac{n}{\hat{b}}\left(1-e^{-\beta t_0}\right) = 0\,. \tag{17}$$

For the Gompertz process, the following is the maximum likelihood estimate for the parameter:

$$\hat{\alpha} = \frac{\hat{b}}{(e^{bt_0}-1)}\,. \tag{18}$$

Because the observations in the potential function of the nonhomogeneous Poisson process only come from n, $\sum_{i=1}^{n}t_i$. it is possible to deduce the distribution of parameter b using the conditional distribution of the variable $S=\sum_{i=1}^{n}t_i$ conditional on the number of occurrences n, Finding the probability distribution for parameter b, which reflects the conditional distribution of the variable $S=\sum_{i=1}^{n}t_i$ conditional on the number of occurrences n, is necessary in order to obtain the maximum likelihood estimate for it.. [14]

This entails dividing the potential function of the nonhomogeneous Poisson process in formula (1) as follows in order to determine the probability distribution of the variable S conditional on the number of occurrences "n"[14]:

$$\begin{aligned} f[S|N(t)=n] &= \frac{L}{\frac{e^{-m(t)}}{n!}(m(t))^n} \\ &= \frac{\prod_{i=1}^{n}\lambda(t_i)e^{-m(t_0)}}{\frac{e^{-m(t)}}{n!}(m(t))^n} \\ &= \frac{n!\prod_{i=1}^{n}\lambda(t_i)}{(m(t_0))^n}. \end{aligned} \tag{19}$$

When the event count "n" affects the probability distribution of the variable "S," the sum of occurrences $S=\sum_{i=1}^{n}t_i$ changes into a conditional probability density function:

$$f[S|N(t)=n] = \frac{n!\prod_{i=1}^{n}\lambda(t_i)}{(m(t_0))^n} \quad ;\ n = 0,1,2,\dots \tag{20}$$

The conditional maximum function of parameter "a" (with "b" maintained constant) depending on the event count "n" may be defined as follows in the context of the non-homogeneous Poisson process, where the time rate of occurrence conforms to a Gompertz function as per formula (6):

$$L[S|N(t)=n]=\frac{n!\prod_{i=1}^{n} a\, e^{bt_i}}{\left(\frac{a}{b}(e^{bt_0}-1)\right)^n}$$

$$=\frac{n!b^n e^{b\sum_{i=1}^{n} t_i}}{\left(e^{bt_0}-1\right)^n}\ , b\neq 0 \tag{21}$$

Additionally, the temporal rate of occurrence takes on the following form when b = 0.

$$\lambda(t)=\alpha\ . \tag{22}$$

Hence, the conditional maximum function for parameter "b" can be expressed as follows:

$$L[S|N(t)=n]=\frac{n!\prod_{i=1}^{n}\alpha}{(\alpha t)^n}$$

$$=\frac{n!}{(t_0)^n}\ , b=0 \tag{23}$$

The probability function of $N(t)$ for parameter "b" in the Gompertz process is written as follows if parameter $'a'$ stays constant and is limited to "n" occurrences:

$$L[S|N(t)=n]=\begin{cases}\frac{n!b^n e^{b\sum_{i=1}^{n} s_i}}{\left(e^{bt_0}-1\right)^n} & b\neq 0\\ \frac{n!}{t_0^n} & b=0\end{cases} \tag{24}$$

When b = 0, the probability density function over the time interval (0, t_0) signifies a uniform distribution. On the other hand, if "b" ≠ 0, the probability density function for the same time interval, considered for a random sample size 'n', follows a Truncated Gompertz Distribution from a specific distribution family. This process often pertains to Order Statistics. Where the time rate of occurrence of events, $\lambda(t)$, is solely defined by the Gompertz function and there is a single viewpoint, the probability density function of a random variable "t" can be expressed as follows:

The log-likelihood function for formula (24) is represented as:

$$\ln L=\begin{cases}\ln(n!)+n\ln b+b\sum_{i=1}^{n} t_i+n\ln\left(e^{bt_0}-1\right) & b\neq 0\\ \ln(n!)-n\ln t_0 & b=0\end{cases} \tag{25}$$

The following formula is used to get the derivative of the likelihood function's logarithm for the parameter "b":

$$\frac{\partial \ln L}{\partial b}=\begin{cases}\frac{n}{b}+\sum_{i=1}^{n} t_i+\frac{n\,t_0}{(1-e^{-bt_0})} & b\neq 0\\ -\frac{1}{2}n\,t_0+\sum_{i=1}^{n} t_i & b=0\end{cases} \tag{26}$$

The parameter 'b' can be determined by solving the following equation as accurately as possible:

$$\left.\frac{\partial \ln L}{\partial b}\right|_{b=\hat{b}}=0\ . \tag{27}$$

The equation (26) demonstrates that there is no analytical solution when $b\neq 0$. Consequently, numerical techniques were employed to generate a range of estimated values for the parameter b. Among these techniques, Newton's method played a pivotal role in solving the equations associated with the maximum likelihood method. This method holds significant importance as it enables the estimation of parameters in cases where solutions cannot be obtained through conventional algebraic approaches. Specifically, in the context of the formula for parameter 'b', Taylor's theorem was employed to derive it in the subsequent form:

$$0=\left.\frac{\partial \ln L}{\partial b}\right|_{b=\hat{b}}\cong\frac{\partial \ln L}{\partial b}+\left(\hat{b}-b\right)\frac{\partial^2 \ln L}{\partial b^2} \tag{28}$$

Then

$$\frac{\partial^2 \ln L}{\partial b^2}=\frac{\partial^2 \ln L}{\partial b_i\,\partial b_j}\ ;\qquad i\neq j\ ,\quad i,j=1,2,3,\dots$$

$$\left(\hat{b}-b\right)\frac{\partial^2 \ln L}{\partial b^2}=-\frac{\partial \ln L}{\partial b}\ . \tag{29}$$

Thus, the following expression is the maximum likelihood estimator for the parameter "b" :

$$\hat{b}=b-\frac{\frac{\partial \ln L}{\partial b}}{\frac{\partial^2 \ln L}{\partial b^2}}\ . \tag{30}$$

Formula (30) is transformed into a sequential expression to provide the following outcome:

$$\hat{b}_i = b_{i-1} - \frac{\left.\frac{\partial \ln L}{\partial \beta}\right|_{\beta_{i-1}=\hat{\beta}_{i-1}}}{\left.\frac{\partial^2 \ln L}{\partial \beta^2}\right|_{\beta_{i-1}=\hat{\beta}_{i-1}}} \text{ ; i=1,2,3, …} \tag{31}$$

Formula (26) is then differentiated to get its second derivative in order to complete formula (31), which is an extensive technique of computing the maximum likelihood estimator for parameter "b" using the direct method:

$$\frac{\partial^2 \ln L}{\partial b^2} = -\frac{n}{b^2} - \frac{n\, t^2{}_0\, e^{-bt_0}}{(1-e^{-bt_0})^2} \ . \tag{32}$$

The maximum likelihood estimator for parameter "b" is obtained by substituting the derived equation into formula (30) as follows:

$$\hat{b} = b - \frac{\frac{n}{b}+\sum_{i=1}^{n} t_i + \frac{n\, t_0}{(1-e^{-bt_0})}}{-\frac{n}{b^2}-\frac{n\, t^2{}_0\, e^{-bt_0}}{(1-e^{-bt_0})^2}} \ . \tag{33}$$

We obtain the sequential estimator as follows by converting the formula above into a recursive fashion, similar to the structure in formula (33).

$$\hat{\beta}_i = \beta_{i-1} - \left(-\frac{n}{b} + \sum_{i=1}^{n} t_i + \frac{n\, t_0}{(1-e^{-bt_0})}\right)\left(\frac{n}{b^2} + \frac{n\, t^2{}_0\, e^{-bt_0}}{(1-e^{-bt_0})^2}\right)^{-1} \ . \tag{34}$$

Thus, using the maximum likelihood technique, the maximum likelihood estimates for parameter "b" is:

$$\hat{b}_{MLE} = \hat{b}_i \ . \tag{35}$$

Formula (18) is changed by formula (35) to produce the following result:

$$\hat{\alpha}_{MLE} = \frac{\hat{b}_{MLE}}{e^{\hat{b}_{MLE} t_0} - 1} \ . \tag{36}$$

Then, the estimation of the reliability function for the two parameters of the Gompertz process using the MLE technique will be

$$\hat{R}(t) = e^{-\frac{\hat{a}_{MLE}}{\hat{b}_{MLE}}\left(e^{\hat{b}_{MLE} t} - 1\right)} . \tag{37}$$

2.2 Parameter Estimation for the Gompertz using the PSO Algorithm

This section proposes a novel approach to estimating the Gompertz process parameters using the Particle Swarm Optimization (PSO) algorithm [15]. The main steps for this method are described by the following algorithm:

Algorithm (1)

Step 1: Define the parameters:

- The number of particles should be set to $N = 50$.
- Specify the number of iterations as i_{max}=100.
- Set the acceleration coefficients C_1 and C_2 to 1 (r_1=r_2=0.1).
- Establish the range of inertial weight values, setting the minimum as γ_{min} =0.4 and the maximum as γ_{max}=0.9.

Step 2: Initialize particle positions:

- Randomly generate the position of each particle, representing an estimation of the Gompertz process parameters.
- Use a uniform distribution within the range [0,1] to generate the initial positions of each particle.

Step 3: Initialize particle velocities:

- Generate the initial velocity for each particle using a uniform distribution.

Step4: Set the fitness function in equation (3) and as the RMSE, in which $RMSE = \sqrt{\frac{\sum_{i=1}^{Q}(\hat{\theta}_i-\theta)^2}{Q}}$.

Step 5: Update particle positions and velocities:

- Adjust the parameter estimator β for the Gompertz process by updating the speed (V_j) for each particle using the following equation:

$$V_j^{(i)} = \gamma V_j^{(i-1)} + C_1 r_1 \left[P_{best,j} - X_j^{(i-1)}\right] + C_2 r_2 \left[G_{best,j} - X_j^{(i-1)}\right] ; j = 1,2 \dots, N \tag{38}$$

- Refresh the particle locations X_i per the equation:

$$X_j^{(i)} = X_j^{(i-1)} + V_j^{(i)}; j = 1,2, \dots, N . \tag{39}$$

Step 6: Repeat All steps (2) - (5) are repeated until termination criteria are satisfied.
Step 7: Then we will estimate the reliability function parameter of the Gompertz process using PSO as in the following:

$$\hat{R}(t) = 1 - e^{-\frac{\hat{a}_{PSO}}{\hat{b}_{PSO}}\left(e^{\hat{b}_{PSO} t} - 1\right)} . \tag{40}$$

Following this algorithm (1), the PSO algorithm efficiently estimates the Gompertz process parameters by iteratively adjusting the particle positions and velocities based on the fitness function. The algorithm seeks to minimize the root mean square error by updating the parameter estimator $\hat{a}_{PSO}$, $\hat{b}_{PSO}$ for the Gompertz process until convergence or until the maximum number of iterations is reached.

3. Simulation study

This section explores a simulation study to assess the efficiency of the proposed estimation methods. All these approaches are compared using the generated data from implementing simulation algorithms, and then the most efficient method is identified. Algorithms for generating data are written using the MATLAB program (R2019b). The following explains how the required data are generated and the numerical computations are performed.

3.1 Data Generation

Random samples from the Gompertz distribution function are generated using specific sizes and certain values for the parameters to estimate Gompertz parameters using simulation, and then the parameters are estimated using MLE and the intelligent method PSO. These estimators are compared to see which of them is closer to the known values of the parameters in the proposed model. For this purpose, samples of sizes: n = 25, 50, and 100 are generated from the exponential model, and the MLE and PSO approaches are implemented to determine the parameter estimators. The simulation study shown in Table 1, Table 2, Table 3, and Table 4 shows that the intelligent method, represented by PSO, is more efficient than the classical method.

Table 1. - Simulation-Based Reliability Estimation for the Gompertz Process

n	a	b	$\hat{R}(t)_{MLE}$	$\hat{R}(t)_{PSO}$
25	0.5	0.5	0.3061	0.2681
		0.8	0.2443	0.0731
	0.8	0.5	0.0824	0.0015
		0.8	0.2851	0.1726
	1.2	0.5	0.1676	0.0138
		0.8	0.1777	0.0174
50	0.5	0.5	0.3284	0.0391
		0.8	0.3679	0.2230
	0.8	0.5	0.1011	0.0026
		0.8	0.1447	0.0081
	1.2	0.5	0.2438	0.0727
		0.8	0.3113	0.2994
100	0.5	0.5	0.1885	0.0221
		0.8	0.2955	0.2149

	0.8	0.5	0.1494	0.0090
		0.8	0.2349	0.0603
	1.2	0.5	0.1140	0.0037
		0.8	0.1394	0.0071

Table 2. - RMSE of Reliability Estimates Based on Simulations

n	a	b	$\hat{R}(t)_{MLE}$	$\hat{R}(t)_{PSO}$
25	0.5	0.5	0.0902	0.0899
		0.8	0.0183	0.0151
	0.8	0.5	0.6164	0.6155
		0.8	0.0560	0.0555
	1.2	0.5	1.3444	1.3439
		0.8	0.2316	0.2306
50	0.5	0.5	0.4486	0.4326
		0.8	0.0570	0.0493
	0.8	0.5	1.4061	1.3652
		0.8	0.4084	0.4080
	1.2	0.5	4.7002	4.6989
		0.8	1.0068	1.0036

Table 3. - RMSE of Reliability Estimates Based on Simulations

n	a	b	$\hat{\lambda}(t)_{MLE}$	$\hat{\lambda}(t)_{PSO}$
25	0.5	0.5	1.0109	0.1283
		0.8	9.0841	0.0237
	0.8	0.5	1.4777	0.4420
		0.8	1.1723	0.0987
	1.2	0.5	1.3885	0.0765
		0.8	0.5327	0.5056
50	0.5	0.5	4.8960	0.0284
		0.8	1.9642	0.0510
	0.8	0.5	2.5530	0.0407
		0.8	2.3982	0.0428

	1.2	0.5	0.9106	0.1581
		0.8	1.6860	0.0600
100	0.5	0.5	1.1276	0.1054
		0.8	4.8644	0.0285
	0.8	0.5	0.6156	0.4343
		0.8	0.7436	0.2538
	1.2	0.5	2.8787	0.0374
		0.8	4.9496	0.0283

Table 4. - RMSE of Rate of Occurrence Estimates Based on Simulations

n	a	b	$\hat{\lambda}(t)_{MLE}$	$\hat{\lambda}(t)_{PSO}$
25	0.5	0.5	0.0303	0.0202
		0.8	0.2795	0.0099
	0.8	0.5	0.0425	0.0290
		0.8	0.0222	0.0205
	1.2	0.5	0.0797	0.0489
		0.8	0.0297	0.0269
50	0.5	0.5	0.1336	0.0256
		0.8	0.0543	0.0099
	0.8	0.5	0.0495	0.0486
		0.8	0.0555	0.0268
	1.2	0.5	0.0692	0.0437
		0.8	0.0317	0.0315

4. Application

1- To determine whether the PSO and MLE are applicable for parameter estimation of the Gompertz Process, real data from at the Mosul power station from 1st January 2021 to 1st January 2022 is used.

2- The collected data represent the stoppage times for the units of the Mosul Dam power stations from 1st January 2021 to 1st January 2022.

3- The likelihood function is derived from the probability density function of the Gompertz distribution; this function is used to estimate the parameters using MLE.

4- The parameter for Gompertz is estimated using each of PSO.

5- To evaluate the applicability and effectiveness for each estimate obtained from applying MLE and PSO, simulation is implemented using estimate values.

4.1 Testing Homogeneity in the Gompertz Process

The Gompertz process is considered a non-homogeneous process because its time rate of events depends on the change in time (t), which means that its behavior is affected by time t. Therefore, the Gompertz process is homogeneous when $\lambda = 0.$ and it is non-homogeneous when $\lambda \neq 0$. The following hypothesis is considered to test whether the process is homogeneous or non-homogeneous (Stehlík, 2006):

$H_0: \lambda = 0$

$H_1: \lambda \neq 0$

Utilized Statistical Approach for Hypothesis Testing

$$Z = \frac{-\sum_{i=1}^{n} t_i - \frac{1}{2} n t_0}{\sqrt{\frac{n t_0{}^2}{12}}}, \tag{41}$$

Where:

$\sum_{i=1}^{n} t_i$ is the sum of the accident times for a time period $(0, t_0]$.

n : represents the number of events that occur over a period of time $(0, t_0]$.

Z : represents calculation test.

4.2 Estimation of the GP Rate of Occurrence for the studied Data

The estimates are compared with those obtained using the conventional MLE method using real data, which represent the stoppage times of the units of the Mosul Dam power stations from 1st January, 2021 to 1st January, 2022 in Mosul, Iraq, in order to gauge the efficacy of the intelligent approaches, PSO for estimating the GP parameters. The algorithms were executed with the MATLAB/R2019b software.

Table 5. - MLE and PSO techniques to estimate the Gompertz process parameters using the data from the main power plant

Methods	*Parameters estimator* $\hat{a}$	*Parameters estimator* $\hat{b}$
MLE	0.0083	4.0000e-08
PSO	1.3413	0.1264

The above table shows the estimation of the Gompertz parameters obtained using the two estimation methods: PSO and MLE, shows the best estimated values of the parameters for the Gompertz process.

4.3 Findings and Analysis

To reach the best performance of the estimation methods, we used the root mean square error (RMSE), as specified in formula (12), and we note from the table below that the PSO method describes the time rate of occurrence of events as well as the reliability function, which showed the lowest RMSE value compared to the MLE method.

Table 6. - The Resulted Values for RMSE for Different Methods

Methods	*RMSE*	
	$\hat{\lambda}(t)$	$\hat{R}(t)$
MLE	0.1400	0.1372
PSO	0.0791	0.0036

Figure 1 in the next paragraph can provide further information about how well various estimating techniques function. It makes it easier to see how well the predicted Gompertz process function corresponds to the observed data.

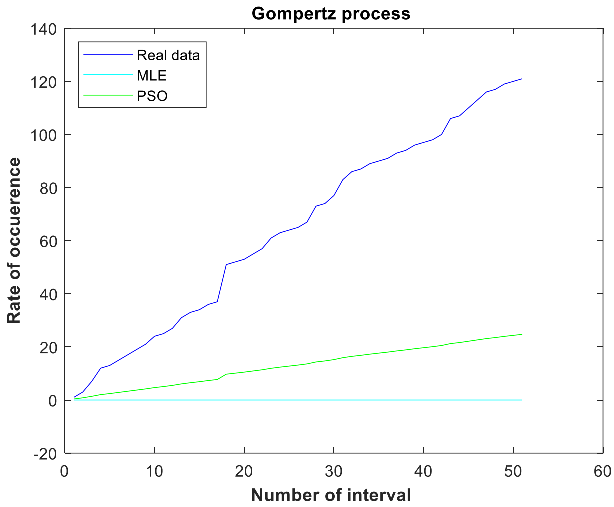

Figure 1. - Applying Several Techniques, Estimating the Total Stoppage Periods for the Units of the Mosul Dam Power Plant

4.4 Discussion

The proposed method distinguishes itself through its innovative incorporation of the Gompertz distribution, coupled with the utilization of particle swarm optimization (PSO) for parameter estimation. Including reliability functions and validation through real data further accentuates its robustness. The efficacy of the method is substantiated by meticulous simulation studies, highlighting its superior performance compared to conventional approaches. Notably, using PSO for parameter estimation is a pivotal aspect, leveraging its adaptability and efficiency in emulating social behaviors observed in birds or fish. This intelligent optimization algorithm excels at handling the intricacies of complex functions, particularly evident in the context of parameter estimation for stochastic processes. The cumulative result underscores the method's potential for delivering precise and dependable predictions of time-dependent event rates.

5. Conclusion

The primary focus of this study was to introduce a highly efficient parameter estimation technique employing PSO algorithms for the non-homogeneous Poisson process (NHPP) model, where the rate of occurrence is described by the exponential function. Our findings indicate that the PSO algorithm provides more accurate parameter estimates and demonstrates faster convergence toward the maximum value of the function compared to the classical Maximum Likelihood Estimation (MLE) method, as evident from the computed RMSE values. Through extensive simulations, we have demonstrated that the PSO algorithm consistently yields efficient results and achieves robust convergence across various parameter values and iterations, outperforming the MLE method. Furthermore, when applied to real data, both the graphical distribution of cumulative downtime durations and the occurrence times, depicted on a logarithmic scale, illustrate that the PSO algorithm enables a linear relationship. This linear relationship suggests the feasibility of modeling such data using the Gompertz function. Overall, our study highlights the effectiveness of the PSO algorithm for parameter estimation in the NHPP model and its potential for accurately modeling and analyzing downtime data in practical applications.

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CONFLICTS OF INTEREST

There is no conflict of interest.

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