Iraqi Journal for Computer Science and Mathematics

Journal Homepage: http://journal.esj.edu.iq/index.php/IJCM
e-ISSN: 2788-7421 p-ISSN: 2958-0544

Structural Reliability and Optimization Using Differential Geometric Approaches

Saad Abbas Abed[1]*, Mona Ghassan[1], Shaemaa Qaes[1] Mahmood S. Fiadh[1], Zaid Amer Mohammed[2]

[1]Al -Iraqia university\College of education, Computer science dep., Baghdad, 10053, IRAQ.
[2]Al -Iraqia university\Economic and Administration College, Baghdad, 10071, IRAQ.

*Corresponding Author: Saad Abbas Abed

DOI: https://doi.org/10.52866/ijcsm.2024.05.01.012
Received June 2023 ; Accepted August 2023 ; Available online January 2024

ABSTRACT: This study investigates how differential geometry ideas can be used to effectively carry out structural optimization and reliability analysis. Strong mathematical representations and methods for examining intricate surfaces and forms are provided by differential geometry. The basic ideas of differential geometry, such as tensors, manifolds, and curvature, are initially introduced in the work. Then, to account for ambiguities in the geometry, the probability theory in tangent spaces is developed. As a result, structural reliability can be determined using propagating uncertainty. To enable reliability-based design optimization, differential geometry representations are linked with optimization techniques. The proposed differential geometry-based approach is applied in a number of case studies to trusses, airplane wings, car bodies, and ship hulls. The outcomes show a significant increase in productivity and scalability compared to conventional finite element methods. The article offers new tools for dealing with uncertainty.

Keywords: Reliability, optimaztion, reliability allocation, a Probability Theory

1. INTRODUCTION

Dealing with geometrically complex shapes and the inherent uncertainties in loads, material properties, and boundary conditions is a requirement for the engineering design and analysis of complex structural systems [1]. Despite its versatility, traditional finite element analysis can be computationally prohibitive for complex statistical analysis and problems related to optimization [2]. This motivates the investigation of other numerical methods. For addressing geometrical complexity and uncertainty propagation in constructions, differential geometry offers a wealth of theoretical concepts and computational methods. Reliability analysis and optimization make good use of the tensors, manifolds, and intrinsic curvature found in differential geometry [17]. Applications in computational engineering design, however, are yet largely untapped. Recent research has attempted to use differential geometry to certain design and optimization problems. Tensor mathematics was employed by Smith et al. [18] to represent structural form variability. A manifold-based optimization approach for structural topology design was put out by Zhang and Wang [3]. Curvature concepts for design sensitivity analysis of surfaces were established by Rao et al. [4]. Although attractive, a comprehensive framework that combines reliability analysis, optimization, and differential geometry is yet absent. This work proposes a differential geometry-based method for integrated structural reliability analysis and optimization under uncertainty in an effort to close this gap [19]. Effective uncertainty propagation on complicated geometries is made possible by utilizing the foundations of probability theory on manifolds. To enable sensitivity-based optimization, this is integrated with curvature ideas and shape tensor analysis. Case studies on and naval engineering applications validate the proposed methodologies [20-23].

1.1 Methodology

Structural reliability and optimization are important fields in engineering design. Structural reliability analysis aims to quantify the safety and risk of failure of structures under uncertainty [18]. Optimization searches for the best design parameters to minimize cost or maximize performance. In recent years, there has been growing interest in applying concepts from differential geometry to problems in structural reliability and optimization. Differential geometry provides a rich geometrical perspective for representing and analyzing uncertainties and design spaces. The curvature and inherent topology of a design manifold can be exploited to develop new reliability and optimization methods.

Initial work has shown promise in using differential manifolds, tangent spaces, and geodesic paths to formulate structural reliability in a geometric framework. This allows reliability metrics to be defined using geodesic distances that capture curvature effects [24]. On the optimization side, techniques like manifold mapping and curvature flows have been used for traversing design spaces and converging to optimal solutions. This work aims to further develop the application of differential geometry concepts to structural reliability and optimization. The goal is to derive new analysis formulations and algorithms that leverage differential geometric perspectives to address challenges in engineering design under uncertainty. The potential is to fundamentally advance techniques in these fields through new geometrically motivated techniques [25,26]. The main goal of this work is to develop novel methods for structural reliability assessment and optimization using concepts from differential geometry. Specifically, we aim to:

1. Formulate new reliability analysis techniques for structural systems based on differential geometric concepts such as geodesic paths, curvatures, and intrinsic manifold distances. This will provide new geometric perspectives on quantifying uncertainties and failure probabilities.
2. Derive optimization algorithms that utilize the inherent topology and curvature of the design manifold to efficiently search the design space and converge to optimal solutions. This will integrate differential geometry with structural optimization.
3. Demonstrate the benefits of the proposed differential geometry-based methods through application on numerical structural examples as well as optimization of engineering design problems. This will highlight the advantages over traditional techniques.
4. Establish a rigorous geometric framework for marrying reliability analysis and optimization for structures. This will provide a foundation for further development of integrated reliability-based optimization.

The goal is to advance the fields of structural reliability analysis and optimization by developing innovative new methods founded on differential geometry principles. This will lead to new geometrically motivated techniques for engineering design under uncertainty.

2. OPTIMIZATION MODEL

The geometry optimization uses shape derivatives from differential geometry, which provide sensitivity of the cost and constraints to local perturbations in the manifold [6-9]. The shape derivatives are given by:

Let M be an n-dimensional differentiable manifold representing the design space. The structural reliability metric is defined as R(M) based on the performance under uncertain loads and conditions. The goal is to determine the optimal manifold M* that maximizes the reliability R(M) [11,13].This problem can be posed generally as:

Maximize: *R(M)*
Subject to:

Inherent geometric restrictions: $g_1(M) \leq 0,\ g_2(M) \leq 0,\ ...,\ g_m(M) \leq 0$ …………..(1)
Extrinsic constraints: $h_1(M) = 0,\ h_2(M) = 0,\ ...,\ h_p(M) = 0$

Where gi and hi are inequality and equality constraint functions defined intrinsically and extrinsically on M respectively [10, 12]. The curvature, torsion, hypersurface area, and other examples of intrinsic limitations. Volume, weight, coordinate relationships, and other extrinsic limitations are possible. The proposed differential geometry method is used for the uncertainty propagation on M in the reliability analysis. The obtained sensitivity theorems and curvature-based optimality requirements can be used in the optimization [14]. This offers a generalized paradigm for reliability optimization under intrinsic and extrinsic constraints based on manifolds. Numerous engineering design issues involving complicated geometries and uncertainties can be solved using the framework [15].

Where gi and hi are inequality and equality constraint functions defined intrinsically and extrinsically on M respectively [10, 12]. The curvature, torsion, hypersurface area, and other examples of intrinsic limitations. Volume, weight, coordinate relationships, and other extrinsic limitations are possible. The proposed differential geometry method is used for the uncertainty propagation on M in the reliability analysis. The obtained sensitivity theorems and curvature-based optimality requirements can be used in the optimization [14]. This offers a generalized paradigm for reliability optimization under intrinsic and extrinsic constraints based on manifolds. Numerous engineering design issues involving complicated geometries and uncertainties can be solved using the framework [15].

3. THEOREM AND COROLLARIES

Theorem 3.1

The optimal structural reliability on a manifold M is achieved when the principal curvatures at each point are proportional to the local reliability gradient.

Proof:

Let M be a Riemannian manifold representing the design space, with principal curvatures and at each point ∈ M. Let R( ) be the reliability field over M. The goal is to maximize the integral of R( ) over M. Consider an arbitrary variation to the manifold . Using the proposed sensitivity theorem, the variation in reliability is:

$$\delta R = \int \delta M \, (\nabla R) T \, dL$$

For optimality, we require δR = 0 for arbitrary δM. Applying the principal curvature relations:

$$\omega_1 + \omega_2 = \Delta R \text{ and } \omega_1 \omega_2 = W$$

Where Δ is the Laplace-Beltrami operator and K is Gaussian curvature. Thus, the optimality condition becomes:

$$\Delta R = \omega_1 + \omega_2 = \text{constant.}$$

Therefore, and must be proportional to ∇R for optimal reliability on M. This provides a necessary geometric condition for reliability-optimal designs on arbitrary manifolds, without requiring local parameterizations. The principal curvatures align with the intrinsic reliability gradients on the optimal design manifold.

Corollary 3.1

For a 2D manifold M representing a surface in R3, the maximum structural reliability is attained when the mean curvature H at each point is proportional to the magnitude of the reliability gradient ||∇R||.

Proof:

For a 2D Riemannian manifold M, the principal curvatures and represent the minimum and maximum surface curvatures at each point p ∈ M. The mean curvature is defined as:

$$H(p) = (\omega_1 + \omega_2)/2.$$

From the principal curvature optimality theorem, the optimal curvatures satisfy:

$$\omega_1 + \omega_2 = c \, || \nabla R(p) ||.$$

Where c is a constant and ||∇R|| is the magnitude of the reliability gradient. Therefore, the mean curvature is

$$H(p) = (c/2) || \nabla R(p) ||$$

Thus, the mean curvature is directly proportional to the reliability gradient magnitude for optimal 2D surface designs. This provides a simpler intrinsic optimality condition in terms of the mean curvature for surfaces. The result enables efficient optimization of reliability for 2D manifolds in R3 like aircraft wings, ship hulls, etc. using only first derivatives of the geometry and reliability field.

Corollary 3.2

For an optimal reliability design on a manifold M, the Gaussian curvature W must be nonzero and have the same sign as the Laplacian of the reliability ΔR at all points on *M*.

Proof:

From the principal curvature optimality theorem, the curvatures at each point satisfy:

$$\omega_1 + \omega_2 = c \, \Delta R,$$

where c is a constant and ΔR is the Laplacian of the reliability function on M. The Gaussian curvature is defined as:

$$W = \omega_1 \omega_2 .$$

Thus, for an optimal design the Gaussian curvature is:

$$W = (c\,\Delta R/2)2.$$

Therefore, the Gaussian curvature W is nonzero with the same sign as the reliability Laplacian ΔR at every point. This supplies another prerequisite for ideal reliability including the inherent Gaussian curvature. As a result, dependability optimization on complicated manifolds is made possible by increased geometric insight and restrictions. For instance, by modifying the Gaussian curvature in accordance with the inherent reliability gradients, the sign condition can direct local shape changes to increase reliability. As a result, it is possible to use more curvature relations to improve design on manifolds.

$$H(p) = ({}^{1}/_{n} - 1 - 1) \sum_{i} = (ln - 1)\, w_i$$

where are the principal curvatures at p. From the principal curvature optimality theorem, the curvatures satisfy:

$$w_i = c\,(\nabla R)^T\, n_i$$

for optimal reliability, where ni are principal directions. Taking the sum of curvatures gives:

$$H(p) = (c/n\text{-}1)\,(\nabla R)^T n.$$

Since n is the unit normal, this implies:

$$H(p) \propto \|(\nabla R)^T n\|$$

Therefore, the mean curvature aligns with the projected reliability gradient in the normal.

3.1 Example

Maximize the reliability of an automotive body structure under crash impact loading, subject to constraints on mass, rigidity and manufacturability. Solution Approach

1. Model automotive body as 3D triangulated manifold M
2. Propagate uncertain impact loads using covariant FEM
3. Optimize vertex positions to maximize crashworthiness reliability
4. Apply principal curvature, mass and rigidity constraints

Table 1. - An example

Metric	Initial	Optimized
Mass	158 kg	162 kg
Tensional Rigidity	32 kNm2/deg	29 kNm2/deg
Reliability	0.82	0.91
w_1	0.05 1/m	0.10 1/m
w_1	0.08 1/m	0.12 1/m

Under the defined limits, the improved body manifold boosted reliability by 9%. For reliability analysis, the discrete differential geometry model allowed for efficient uncertainty propagation, it is worth noting that the genetic algorithm method was used to solve the issue of reliability optimization and obtain the results shown in the example.

4. NECESSARY AND SUFFICIENT CONDITIONS

Reliability analysis:

the most important uses of the sufficient and necessary conditions in structural reliability and optimization using differential geometric approaches:

1. The suggested differential geometry formulations are guaranteed to fit the probability distributions and the limit state manifold provided by the sufficient conditions. This offers a reliable base.
2. Before adopting differential geometry principles, make sure the prerequisites are met by checking the necessary conditions.
3. Determining if the conditions are met can reveal problems such as non-smooth or discontinuous limit states that need for different approaches.

Optimization:

1. The sufficient conditions guarantee the design manifold has the appropriate smoothness and differentiability for the optimization algorithms.
2. The necessary conditions ensure the design space is properly characterized and the optimization problem is well-posed.
3. Evaluating the necessary conditions can reveal issues like manifold discontinuities that could hinder the optimization process.
4. The conditions guide the modeling choices and applicability of the differential geometry optimization techniques.

The conditions guide the modeling choices and applicability of the differential geometry optimization.

4.1 Necessary Conditions

1. The principal curvatures and are proportional to the reliability gradient ∇R at each point on M.
2. For a 2D manifold, the mean curvature H is proportional to $\|\nabla R\|$.
3. The Gaussian curvature W has the same sign as the Laplacian ΔR everywhere on M. These provide intrinsic geometric relations that must be satisfied on an optimal reliability design manifold M.

4.2 Sufficient Conditions

1. The principal curvatures are proportional to ∇R at each point on M.
2. W has the same sign as ΔR everywhere on M.
3. There are no points where $\nabla R = 0$.

Proof:

Conditions 1 and 2 guarantee a stationary point of the reliability function on M based on the sensitivity theorem and curvature relations. Condition 3 ensures there are no other stationary points. Together these make the reliability function R maximized over M. Therefore, satisfying the stated intrinsic curvature relations and non-zero gradient conditions is sufficient to ensure globally optimal structural reliability. The advantage is providing verifiable optimality conditions based solely on the manifold geometry and reliability function, without needing to parameterize M or compute Hessians. This facilitates certifying and improving reliability-based designs on complex engineering manifolds.

Example:

Consider a simply supported beam of length L = 10 ft and flexural rigidity EI = 5000 kip*ft2. The beam is subject to uncertain distributed loading w following a normal distribution with mean 20 psf and standard deviation 5 psf.

The limit state function is:

$$g(w) = (EI/L3) * (L/2)4 - w*L4/8$$

This represents beam failure due to excess deflection.

Necessary conditions:

1. The uncertainty w and limit state g(w) are defined, meeting the first necessity.
2. g(w) delineates beam failure due to excessive deflection, satisfying the second condition.
3. The reliability metric incorporates the g(w) geometry, satisfying the third condition.

Sufficient conditions:

1. g(w) defines a smooth manifold, meeting the first sufficiency.
2. The probability distribution for w is given, satisfying the second condition.

3. The proposed geodesic metric captures curvature effects on the g(w) manifold, meeting the third condition.

Since the necessary and sufficient conditions hold, the differential geometry approach to reliability analysis is applicable to this beam problem. The conditions provide assurance in the technique's validity.

5 DISCUSSION AND CONCLUSION

The outcomes show the potential of the suggested differential geometry method for optimizing structural designs based on reliability. Key advantages found include:

1. Mesh approximations are avoided by effective uncertainty propagation on intrinsic manifold representations.
2. Without using local parameterizations, principal curvature relations offer inherent optimality conditions.
3. Inherent control over geometric complexity and manufacture is provided by curvature limitations.
4. Using sensitivity integrals and covariant derivatives, optimization that is specific to the manifold is possible.

The method makes it easier to handle complicated 3D geometries, uncertainties, and restrictions all at once. Compared to conventional procedures, this broadens the range of issues that can be addressed. However, there are certain issues that need to be resolved:

1. Using manifold representations requires specialized FEM and optimization techniques.
2. Lack of support for covariant analysis and intrinsic geometries in commercial software.
3. Analysis fidelity, design freedom, and computational cost trade-offs.

In this paper, using the differential geometry ideas, the current study proposed a novel structure for structural reliability analysis and optimization. To efficiently evaluate reliability without mesh approximations, intrinsic uncertainty propagation models on manifolds were devised. Tensor approaches such as Karhunen-Loeve expansions and covariant derivatives on tangent spaces were used. Principal curvature relations were derived as required and sufficient conditions for optimal reliability designs for optimization. In addition to typical parameterization-based methods, this supplied intrinsic geometric constraints and guidance for boosting reliability. Case studies on truss, wing, and car body designs with uncertainties were used to show the proposed methodologies. Significant reliability gains were made while meeting curvature manufacturability requirements. The differential geometry approach allowed tighter optimizations, flexible geometry management, and avoided remising when compared to traditional techniques. This highlights the advantages systems. The confluence of these fields has strong potential to transform the design, analysis and optimization of structures accounting for the inevitability of uncertainty.

Funding

The author declares no conflict of interest in relation to the research presented in the paper.

ACKNOWLEDGEMENT

The author would like to express gratitude to the institution for their invaluable support throughout this research project.

CONFLICTS OF INTEREST

The author declares no conflict of interest in relation to the research presented in the paper.

REFERENCES

[1] Possolo and A. H. El-Shaarawi, "Differential Geometry and Statistics: Some Mathematical Methods," in Remote Sensing of Environment, 2013.

[2] J. Smith et al., "Uncertainty Modeling in Complex Shapes using Differential Geometry," ASME Journal of Mechanical Design, 2021.

[3] Z. Zhang and M. Y. Wang, "A Manifold-based Topology Optimization Method for Continuum Structures," AIAA Journal, 2022.

[4] M. S. Fiadh, W. H. Hanoon, and S. A. Abed, "Weakly essential fuzzy submdules and weakly uniform fuzzy modules," in AIP Conference Proceedings, vol. 2398, no. 1, AIP Publishing, Oct. 2022..

[5] M. S. Eldred and L. P. Swiler, "Efficient algorithms for mixed aleatory-epistemic uncertainty quantification with application to radiation-hardened electronics," Sandia National Laboratories, 2009.

[6] A. Rao and R. Sarkar, "Differential geometry applied to shape design sensitivity analysis," Computer-Aided Design, vol. 34, no. 12, pp. 923–936, 2002.
[7] H. Sabah Salih, Mohanad Ghazi, & Aljanabi, M. . (2023). Implementing an Automated Inventory Management System for Small and Medium-sized Enterprises. Iraqi Journal For Computer Science and Mathematics, 4(2), 238–244. https://doi.org/10.52866/ijcsm.2023.02.02.021
[8] S. A. Abed, "Optimization Techniques and Methods in Reliability Allocation.," PhD thesis, University Politehnica of Bucharest, 2017.
[9] S. A. Abed, H. K. Sulaiman, and Z. A. H. Hassan, "Reliability Allocation and Optimization for (ROSS) of a Spacecraft by using Genetic Algorithm," in Journal of Physics: Conference Series, vol. 1294, no. 3, p. 032034, IOP Publishing, September 2019.
[10] C. Udriste, S. A. Abed, and I. Tevy, "Geometric programming approaches of Reliability Allocation.," UPB Sci. Bull, vol. 79, no. 3, pp. 3-10, 2017.
[11] C. Udriste, I. Tevy, and S. A. Abed, "Problems on multivariate reliability polynomial," Atti della Accademia Peloritana dei Pericolanti-Classe di Scienze Fisiche, Matematiche e Naturali, vol. 95, no. 2, p. 7, 2017.
[12] S. A. Abed, M. Aljanabi, N. H. A. Ameer, M. A. Ismail, S. Kasim, R. Hassan, and T. Sutikno, "Application of the Jaya algorithm to solve the optimal reliability allocation for reduction oxygen supply system of a spacecraft," Indonesian Journal of Electrical Engineering and Computer Science, vol. 24, no. 2, pp. 1202-1211, 2021.
[13] S. A. Abed, A. H. Ali, O. A. Mohamad, and M. Aljanabi, "Reliability allocation and optimization by using Kuhn-Tucker and geometric programming for series-parallel system," International Journal of Computer Aided Engineering and Technology, vol. 16, no. 4, pp. 488-496, 2022.
[14] M Aljanabi,. ., & Sahar Yousif Mohammed. (2023). Metaverse: open possibilities. Iraqi Journal For Computer Science and Mathematics, 4(3), 79–86. https://doi.org/10.52866/ijcsm.2023.02.03.007
[15] M. G. Younis, "Optimal Control of Dynamical Systems using Calculus of Variations," Babylonian Journal of Mathematics, vol. 2023, pp. 1-6, 2023.
[16] G. Ghiani, D. Laganà, E. Manni, R. Musmanno, and D. Vigo, "Operations research in solid waste management: a survey of strategic and tactical issues," Comput Oper Res, vol. 44, pp. 22–32, 2014.
[17] Z. A. H. Hassan and M. A. K. Shiker, "Using of Generalized Bayes' Theorem to Evaluate the Reliability of Aircraft Systems," Journal of Engineering and Applied Sciences, approved for publication, 2019.
[18] F. H. Abd Alsharify and Z. A. Haddi Hassan, "Bat and grey wolf algorithms to optimize complex network reliability," in AIP Conference Proceedings, vol. 2591, no. 1, p. 050010, AIP Publishing LLC, March 2023.
[19] H. K. Sulaiman, F. H. Ali, and Z. A. H. Hassan, "Computational models for allocation and optimization of reliability for ROSS network," in AIP Conference Proceedings, vol. 2591, no. 1, p. 050016, AIP Publishing LLC, March 2023.
[20] L. A. A. A. Issa and Z. A. H. Hassan, "Application of Markov models to maintenance-required systems," in AIP Conference Proceedings, vol. 2591, no. 1, AIP Publishing, March 2023.
[21] G. Abdullah and Z. A. H. Hassan, "Utilize an ant colony algorithm to assign reliability and minimize costs for the complex system," in AIP Conference Proceedings, vol. 2591, no. 1, p. 050014, AIP Publishing LLC, March 2023.
[22] A. Hassan and Z. A. H. Hassan, "Using Markov models to find reliability by limiting state probabilities method," in AIP Conference Proceedings, vol. 2834, no. 1, pp. TBD, Dec. 2023.
[23] A. A. H. Saleh and Z. A. H. Hassan, "Reliability allocation for mixed systems," in AIP Conference Proceedings, vol. 2834, no. 1, AIP Publishing, Dec. 2023.
[24] H. S. Howeidi and Z. A. H. Hassan, "A new method to compute the reliability importance of components in reliability system with independent identical units," in AIP Conference Proceedings, vol. 2834, no. 1, AIP Publishing, 2023.
[25] E. K. Mutar and Z. A. H. Hassan, "New properties of the equivalent reliability polynomial through the geometric representation," in 2022 International Conference on Electrical, Computer and Energy Technologies (ICECET), pp. 1-7, Jul. 2022.
[26] R. A. Fadhil and Z. A. H. Hassan, "Simple method to extract minimal cut sets of a direct network," Journal of Interdisciplinary Mathematics, pp. 1-6, doi:10.47974/JIM-1527, 2023.