Economic Order Quantity in a Fuzzy Environment for a Periodic Inventory Model with Variable Demand

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ABSTRACT: The technique of limiting expenditure plays a critical part in an organization's ability to govern the smooth operation of its management system. The economic order quantity is calculated by solving a nonlinear problem, and the best solution is investigated in a fuzzy and intuitionistic fuzzy environment. The overall cost is made up of several factors, such as demand, holding, and ordering costs. The demand and stock-out characteristics are fuzzified using fuzzy and intuitionistic fuzzy numbers. Numerical analysis results show the relation between the two fuzzy numbers on the basis of sensitivity analysis.

Keywords: Economic order quantity (EOQ); triangular intuitionistic fuzzy number (TIFN); triangular fuzzy

1. INTRODUCTION

Fuzzy sets were first introduced by Zadeh [1] in 1965. In 1913, Harris [2] derived the economic order quantity (EOQ) for an inventory model. In 1987, Park [3] applied fuzzy concepts to EOQ models. Zimmerman [4] studied fuzzy sets in the field of operations research. In 1918, Taft [5] investigated the economic production quantity model. Roy [6] examined an EOQ model with limited storage capacity under a fuzzy environment. Chang [7] studied the impact of defective items in inventory management. Hsieh [8] examined an inventory model under fuzzy parameters. In 2010, Zimmermann [9] studied the varying prospectus of fuzzy set theory and its applications to the major fields of science and engineering. Wang [10] evaluated an EOQ model with imperfect quality. Salameh [11] examined an economic production quantity model with imperfect quality. Vujosevi [12] derived an EOQ formula for an inventory model in which the cost is fuzzy. In 2000, Jing [13] varied the demand and order quantity by using fuzzy parameters for a backorder inventory model.

Intuitionistic fuzzy sets were first introduced by Atanassov [14] in 1986. Triangular intuitionistic fuzzy numbers are used to compute imprecise failures. Mijanur [15] demonstrated the various aspects of triangular intuitionistic fuzzy numbers. In 2012, Mahapatra [16] worked on a multiobjective inventory model by using intuitionistic fuzzy numbers. In 2012, Nagoorgani [17] solved a linear programming problem by using intuitionistic fuzzy numbers. Banerjee [18] also solved a multiobjective inventory model by using intuitionistic optimization techniques.

In the current work, EOQ is derived for nonlinear programming, and the components are fuzzified using triangular fuzzy numbers and triangular intuitionistic fuzzy numbers. The signed distance method is used in defuzzification. Sensitivity analysis is conducted to compare the values between the crisp and fuzzy parameters. The rest of the paper is

organized as follows: Section 2 describes the preliminaries required for the study. Section 3 provides the optimal order quantity from the aspect of crisp and fuzzy numbers. Finally, Section 4 summarizes the conclusion.

2. PRELIMINARIES

Definition 2.1. Triangular fuzzy number (TFN)

A triangular fuzzy number $A(a_1, a_2, a_3)$ is said to have the following membership function:

$$\mu_A(x) = \begin{cases} 0 & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0 & x > a_3 \end{cases}$$

Definition 2.2. Intuitionistic fuzzy sets (IFN) [15]

Let $U = \{y_1, y_2, y_3, \dots, y_n\}$ be a finite universal set. An intuitionistic fuzzy set (IFS) $\tilde{B}$ in a given universal set U is expressed in the form

$\tilde{B} = \{ \langle y_i; \mu_{\tilde{B}}(y_i), \omega_{\tilde{B}}(y_i) \rangle : y_i \in U \}$, where the functions

$\mu_{\tilde{B}} : U \rightarrow [0, 1]$ i.e., $y_i \in U \rightarrow \mu_{\tilde{B}}(y_i) \in [0, 1]$ and

$\omega_{\tilde{B}} : U \rightarrow [0, 1]$; i.e., $y_i \in U \rightarrow \omega_{\tilde{B}}(y_i) \in [0, 1]$

respectively describe the degree of membership and the degree of nonmembership of an element $y_i \in U$ satisfying the condition $0 \leq \mu_{\tilde{B}}(y) + \omega_{\tilde{B}}(y) \leq 1$, $\forall y_i \in U$ is known as the intuitionistic condition. The degrees of acceptance $\mu_{\tilde{B}}(y)$ and nonacceptance $\omega_{\tilde{B}}(y)$ can be arbitrary.

2.1 SIGNED DISTANCE DEFUZZIFICATION METHOD

Let (b_1, b_2, b_3) be triangular fuzzy numbers whose signed distance formula are given by

$$\frac{b_1 + 2b_2 + b_3}{4}$$

Let (b_1, b_2, b_3) $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ be triangular intuitionistic fuzzy numbers whose signed distance formula is given by

$$\frac{b_1 + 2b_2 + b_3 + \tilde{b}_1 + \tilde{b}_2 + \tilde{b}_3}{8}$$

3. PROPOSED INVENTORY MODEL

Parameters used

$Q \rightarrow$ order quantity

$I(\alpha) \rightarrow$ investment required to reduce the lost sales fraction

$\alpha \rightarrow$ annual fractional cost of capital investment

$S \rightarrow$ expected stockout

$P \rightarrow$ safety factor

$h \rightarrow$ holding cost per unit per year

$E \rightarrow$ demand

$R \rightarrow$ lead time per week

$E(X - W) \rightarrow$ expected shortage quantity at the end of the cycle

$C(l - t) \rightarrow$ lead time crashing cost

Inventory model

This model constitutes a periodic review model. The total cost is

$$T_C = \alpha I(\alpha) + \frac{1}{Q} [S + C(l - t)] + h \left[P - ER + \frac{EQ}{2} + \alpha E(X - W) \right] \quad (1)$$

Partially differentiating w.r.t Q ,

$$\frac{\partial T_C}{\partial Q} = -\frac{1}{Q^2} [S + C(l - t)] + \frac{hE}{2} \quad (2)$$

Equating $\frac{\partial T_C}{\partial Q} = 0$,

$$\Rightarrow \frac{1}{Q^2} (S + C(l-t)) = \frac{hE}{2} \quad (3)$$

We obtain the optimal order quantity as

$$\Rightarrow Q = \sqrt{\frac{2(S+C(l-t))}{hE}} \quad (4)$$

3.1 INVENTORY MODEL IN FUZZY SENSE

The parameters ' S ' and ' E ' are obtained using triangular fuzzy numbers and the signed distance defuzzification method: (S_1, S_2, S_3) and (E_1, E_2, E_3)

$$T_C = \alpha I(\alpha) + \frac{1}{Q} ((S_1, S_2, S_3) + C(l-t)) + h \left(P - (E_1, E_2, E_3)R + \frac{(E_1, E_2, E_3)Q}{2} + \alpha E(X-W) \right) \quad (5)$$

$$= \alpha I(\alpha) + \frac{1}{Q} (S_1 + C(l-t)) + h \left(P - E_1R + \frac{E_1Q}{2} + \alpha E(X-W) \right), \alpha I(\alpha) + \frac{1}{Q} (S_2 + C(l-t)) + h \left(P - E_2R + \frac{E_2Q}{2} + \alpha E(X-W) \right), \alpha I(\alpha) + \frac{1}{Q} (S_3 + C(l-t)) + h \left(P - E_3R + \frac{E_3Q}{2} + \alpha E(X-W) \right) \quad (6)$$

$$\Rightarrow Q^* = \sqrt{\frac{2(S_1 + 2S_2 + S_3) + C(l-t)}{h(E_1 + 2E_2 + E_3)}} \quad (7)$$

The parameters ' S ' and ' E ' are fuzzified using triangular intuitionistic fuzzy numbers and then defuzzied using the signed distance method.

$(S_1, S_2, S_3)(S'_1, S'_2, S'_3)$ and $(E_1, E_2, E_3)(E'_1, E'_2, E'_3)$

$$T_C = \alpha I(\alpha) + \frac{1}{Q} ((S_1, S_2, S_3)(S'_1, S'_2, S'_3) + C(l-t)) + h \left(P - (E_1, E_2, E_3)(E'_1, E'_2, E'_3)R + \frac{(E_1, E_2, E_3)(E'_1, E'_2, E'_3)Q}{2} + \alpha E(X-W) \right) \quad (8)$$

$$\begin{aligned} &= \alpha I(\alpha) + \frac{1}{Q} [S_1 + C(l-t)] + h \left[P - E_1R + \frac{E_1Q}{2} + \alpha E(X-W) \right], \alpha I(\alpha) \\ &\quad + \frac{1}{Q} [S_2 + C(l-t)] + h \left[P - E_2R + \frac{E_2Q}{2} + \alpha E(X-W) \right], \alpha I(\alpha) \\ &\quad + \frac{1}{Q} [S_3 + C(l-t)] + h \left[P - E_3R + \frac{E_3Q}{2} + \alpha E(X-W) \right], \\ &\quad \alpha I(\alpha) + \frac{1}{Q} [S'_1 + C(l-t)] + h \left[P - E'_1R + \frac{E'_1Q}{2} + \alpha E(X-W) \right], \alpha I(\alpha) \\ &\quad + \frac{1}{Q} [S'_2 + C(l-t)] + h \left[P - E'_2R + \frac{E'_2Q}{2} + \alpha E(X-W) \right], \alpha I(\alpha) \\ &\quad + \frac{1}{Q} [S'_3 + C(l-t)] + h \left[P - E'_3R + \frac{E'_3Q}{2} + \alpha E(X-W) \right] \end{aligned} \quad (9)$$

$$\Rightarrow Q^* = \sqrt{\frac{2(S_1 + 2S_2 + S_3 + S'_1 + 2S'_2 + S'_3) + C(l-t)}{h(E_1 + 2E_2 + E_3 + E'_1 + 2E'_2 + E'_3)}} \quad (10)$$

Table 1. Sensitivity analysis using triangular fuzzy numbers

Sensitivity Variations	S (TFN)	E (TFN)	Crisp Values Q	Fuzzy Values Q^*
- 50%	35 (25, 35, 45)	5 (2.5, 5, 7.5)	1.760682	1.695582
- 25%	52.5 (42.5, 52.5, 62.5)	7.5 (2.5, 7.5, 12.5)	1.732051	1.681940
No Variations	70 (60, 70, 80)	10 (5, 10, 15)	1.717556	1.684488
+ 25%	87.5 (77.5, 87.5, 97.5)	12.5 (7.5, 12.5, 17.5)	1.708801	1.682260
+ 50%	105 (95, 105, 115)	15 (10, 15, 20)	1.702939	1.680774

Table 2. Sensitivity analysis using triangular intuitionistic fuzzy numbers

Sensitivity Variations	S (TIFN)	E (TIFN)	Crisp Values Q	Fuzzy Values Q^*
- 50%	35 (15, 25, 30, 40, 45, 55)	5 (3, 4, 4.5, 5.5, 6, 7)	1.760682	1.684488
- 25%	52.5 (32.5, 42.5, 47.5, 57.5, 62.5, 72.5)	7.5 (5.5, 6.5, 7, 8, 8.5, 9.5)	1.732051	1.680774
No Variations	70 (50, 60, 65, 75, 80, 90)	10 (7, 8, 9, 11, 12, 13)	1.717556	1.678913
+ 25%	87.5 (67.5, 77.5, 82.5, 92.5, 97.5, 107.5)	12.5 (10.5, 11.5, 12, 13, 13.5, 14.5)	1.708801	1.677796
+ 50%	105 (85, 95, 100, 110, 115, 125)	15 (13, 14, 14.5, 15.5, 16, 17)	1.702939	1.677051

□

3.2 NUMERICAL ANALYSIS

The parameter demand, holding cost per unit per year, lead time crashing cost, expected stockout, and lead time crashing cost values are entered in the order quantity model, and the optimal order quantities are compared using sensitivity analysis. $S=70$, $C=2.5$, $h=5$, $l=13$, $E=10$, and $t=10$. Table 1 shows the optimal values obtained when using triangular fuzzy numbers, and Table 2 presents the triangular intuitionistic fuzzy numbers. The demand and expected stockout parameters increase and decrease by 25% and 50%, respectively. The results are tabulated and compared. Tables 1 and 2 show the sensitivity analysis for the triangular fuzzy and triangular intuitionistic fuzzy numbers.

4. CONCLUSION

The optimal order quantity was derived by solving a nonlinear equation to minimize the total cost. Fuzzification was carried out using triangular fuzzy numbers and triangular intuitionistic fuzzy numbers, and the equations were derived accordingly. The signed distance method was used in defuzzification. Figure 1 shows the 3D plot of the crisp, TFN, and TIFN results. The sensitivity analysis showed only a minimal difference between the triangular fuzzy numbers and triangular intuitionistic fuzzy numbers. As shown in Figure 2, the values coincided to a level under -25% variations. Figure 3 shows the comparison of the fuzzy numbers obtained under the signed distance method. Slight variations were noted between the optimal values of the fuzzy numbers.

FIGURE 1. 3D representation between three values of Crisp, TFN, and TIFN

FIGURE 2. Comparative bar chart between crisp, TFN, and TIFN

FIGURE 3. Comparison of observations between TFN and TIFN

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CONFLICTS OF INTEREST

The authors declare no conflict of interest.

REFERENCES

- [1] L. Zadeh, "A, Fuzzy sets," *Inf. Control*, vol. 8, pp. 338–353, 1965.
- [2] F. Harris, . A. S. Operations, and Co. 1913.
- [3] K. S. Park and Fuzzy, "Set Theoretical Interpretation of economic order quantity," *IEEE Trans. Syst. Man. Cybernet SMC*, vol. 17, pp. 1082–1092, 1987.
- [4] H. Zimmermann, "Using fuzzy sets in operational Research," *Eur. J. Oper. Res.*, vol. 13, pp. 201–206, 1983.
- [5] E. Taft, "The most economical production lot," *Iron Age*, vol. 101, pp. 1410–1412, 1918.
- [6] T. K. Roy, M. Maiti, and Fuzzy, "EOQ model with demand-dependent unit cost under limited storage capacity," *Eur. J. Oper. Res.*, vol. 99, no. 2, pp. 425–432, 1997.
- [7] H. C. Chang, "An application of fuzzy sets theory to the EOQ model with imperfect quality," *Computers and Operations Research*, vol. 31, no. 12, pp. 2079–2092, 2004.
- [8] C. Hsieh, "Optimization of fuzzy production inventory models," *Inf. Sci.*, vol. 146, pp. 29–40, 2002.
- [9] H. Zimmermann, "Fuzzy set theory," *Adv. Rev.*, vol. 2, no. 3, pp. 317–332, 2010.
- [10] X. Wang, W. Tang, and R. Zhao, "Random fuzzy EOQ model with imperfect quality items. Fuzzy Optim," *Dec. making*, vol. 6, no. 2, pp. 139–153, 2007.
- [11] M. K. Salameh and H. Lee, "Economic production quantity model for items with imperfect quality," *Int. J. Prod. Econ.*, vol. 64, no. 1, pp. 59–64, 2000.
- [12] M. Vujosevic, D. Petrovi, and Petrovic, "EOQ formula when inventory cost is fuzzy," *Int. J. Prod. Eco.*, vol. 45, no. 1-3, pp. 499–504, 1996.
- [13] J. S. Yao and H. M. Lee, "Economic production quantity for fuzzy demand quantity and fuzzy production quantity," *Eur. J. Oper. Res.*, vol. 109, no. 1, pp. 203–211, 1998.
- [14] T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst*, vol. 20, pp. 87–96, 1986.
- [15] R. S. Mijanur, K. N. Prasun, and M. Pal, "Notes on triangular intuitionistic fuzzy numbers," *Int. J. Math. Oper. Res.*, vol. 5, no. 4, pp. 446–465, 2013.
- [16] N. Mahapatra, "Multi-objective inventory model of deteriorating items with some constraints in an intuitionistic fuzzy environment," *Int. J. Phys. Soc. Sciences*, vol. 2, no. 9, 2012.
- [17] A. Nagoorgani and K. Ponnalagu, "A new approach on solving Intuitionistic fuzzy linear programming problem," *Appl. Math. Sci.*, vol. 6, no. 70, pp. 3467–3474, 2012.
- [18] S. Banerjee and K. Roy, "Solution of single and multi-objective stochastic inventory models with fuzzy cost components by intuitionistic fuzzy optimization techniques," *Adv. Oper. Res.*, no. 1, 2010.